Physics of Turbulence
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Lecture - 21
Energy Transfers
Mode-to-mode Energy Transfers

So, we start a new topic today. So, far we started on introduce the Navier-Stokes equation, then we did some Fourier transform representation, then instability and how turbulence comes in a hydrodynamics and convection also. So, I showed you how to get turbulence.

Now, we are ready to start discussing about turbulence ok. So, but the what I am going to tell you today is general, but these tools will be useful to describe turbulent flows. So, I will basically start with energy transfers and tell you about energy flux. So, this is what we plan to do today. So, energy transfers in hydrodynamic flows so it will consist of three big modules, so we will call mode to mode energy transfer.

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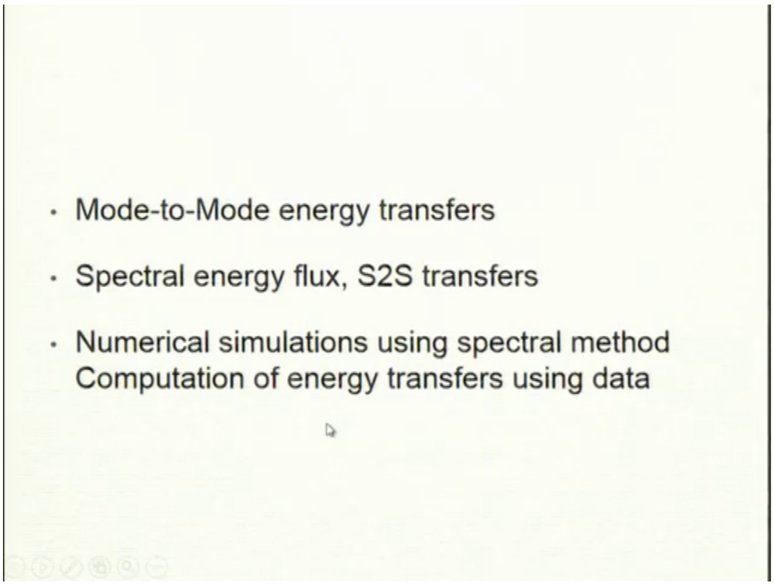

So, how does energy transfer go from one mode to another mode? So, we did see how energy flows, I mean energy is flowing rather modes were oscillating. So, somebody is supplying energy right so that we see from dynamics. Now, we will see from energetic point of view how energy goes from one region to another region, one mode to other mode then we define something called turbulent flux or spectral energy flux and $S2S$ is shell to shell transfer, in turbulence there is energy transfers is quite easy to see.

In linear systems we have waves right and one wave can exist forever if there is no dissipation. Now, if you have many waves, they do not interact then they can exists forever right, I mean in electrodynamics we see many waves they do not interact and they continue to propagate. But as they start interacting then some will lose energy to other or some will transfer energy to other and that leads to cascade of energy or wave breaking or you know so in that is what is what we will do. And third is how to compute them in simulations, so that is what I plan to do for these set of lectures.

So, I will give a new formalism which in fact was developed here is called mode to mode energy transfer ok. So, this we will discuss and that is very useful tool to describe various quantities like energy flux being one of them and also patterns how patterns are formed, we can discuss quite a bit of this using this tool and shell to shell transfer.

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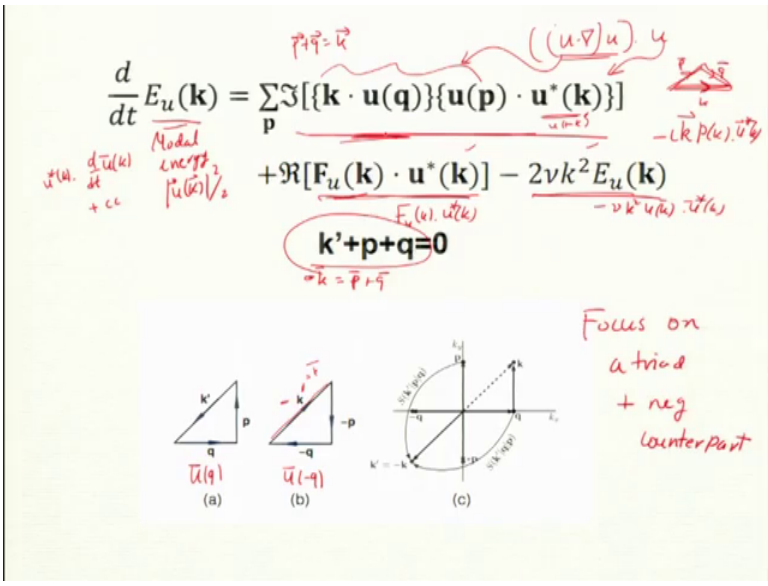

So, let us just start with some simple equations I will derive this, so that is my objective today to derive formula for these transfers. So, you have to recall our discussion in the past. So, I am sure we did enough of this equation so you will immediately recollect them. So, energy of a given Fourier mode and this we call modal energy right, modal energy which is $|\boldsymbol{u}(\boldsymbol{k})|^2/2$, use a vector where $\mathbf{k}$ is a vector right. So, given Fourier mode $\boldsymbol{u}(\boldsymbol{k})$,its energy is $|\boldsymbol{u}(\boldsymbol{k})|^2/2$ and in fact that is a definition.

Now, this energy can change in time, as I said non-linear terms will increase or decrease energy of a mode and these, I derived in the class ok. So, this we did so for lot of Fourier modes the formula is this it comes from a real space $\boldsymbol{u}.\nabla\boldsymbol{u}$ and dot that with $\boldsymbol{u}$. In Fourier space this $\boldsymbol{u}.\nabla\boldsymbol{u}$ becomes the convolution which is $i\ \Sigma_p\{\boldsymbol{k}.\boldsymbol{u}(\boldsymbol{q})\}\boldsymbol{u}(\boldsymbol{p})$. So, this part comes from this one and this term comes from here ok. So, this we did in the class so how many of you remember this, so everybody remembers it knows. So now, this is coming from non-linear term.

Now, as you showed the pressure term does not contribute that this I did in the class. So, pressure term because pressure is $-i\boldsymbol{k}p(\boldsymbol{k}).\boldsymbol{u}^*(\boldsymbol{k})$. So, $\mathbf{k}.\mathrm{u}(\mathbf{k}) = 0$ so pressure term disappears, so pressure goes away. So, this was done when I did Fourier. So, we had to use all those technologies you know all the formulas. So, this is coming from non-linear term pressure does not participate in energy transfers among Fourier modes. We will come to what come to what pressure does with later pressure does something, but in this it is not participate.

Now, external force F_u can feed energy. In fact, we need this like buoyancy or external force, you know external force can also change energy of a mode this again I derived this in earlier classes and viscous term will kill the energy. This comes from $\nu k^2\boldsymbol{u}(\boldsymbol{k})$ and multiply this by $\boldsymbol{u}^*(\boldsymbol{k})$ and it comes here, and this is force this straightforward this is these two are easy terms. In fact, one thing so this $F_u(\boldsymbol{k}).\boldsymbol{u}(\boldsymbol{k})$ and I have to take the real part because to derive this if you remember let me just say this. To derive this, you have to take the Navier-Stokes equation in Fourier space dot this with $\boldsymbol{u}^*(\boldsymbol{k})$ and then add complex conjugate to it ok.

So, this you did in the class, so that is why all the real comes. Now, these two things these two terms are function only of $\boldsymbol{k}$. So, it is function of $F_u(\boldsymbol{k}).\boldsymbol{u}^*(\boldsymbol{k})$ and other one is viscous dissipation at that particular $\boldsymbol{k}$. But these terms involves more modes $\boldsymbol{k}$ as well as $\boldsymbol{p}$, $\boldsymbol{q}$ you know and the condition of course you should remember is $\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{k}=\boldsymbol{0}$ ok. So, this is coming from triad. So, not every mode so well this $\boldsymbol{p},\boldsymbol{q}$ are related, in fact they comes from when I do the convolution I get this $\boldsymbol{k}=\boldsymbol{p}+\boldsymbol{q}$ and of course, there are millions of $\boldsymbol{p}$, $\boldsymbol{q}$ this triads, so that is why I have this some $\boldsymbol{p}$ alright.

Now, this is for this equation was given many years back. In fact, more than 50 years back ok actually right now I did $\boldsymbol{k}'$, so $\boldsymbol{k}' = -\boldsymbol{k}$ ok. So, this is what for this equation it is$\boldsymbol{k} =$

$\boldsymbol{p} + \boldsymbol{q}$, we will encounter $\boldsymbol{k'}$ in the next slide. Now this equation is schematically also is drawn like that. So, by the way I mean I should since I wrote it here, I can replace $\boldsymbol{k}$ by $-\boldsymbol{k'}$, so that I get this one. So, then it becomes this $\boldsymbol{u}^*(\boldsymbol{k})$ becomes $\boldsymbol{u}(-\boldsymbol{k})$ or $\boldsymbol{u}(\boldsymbol{k'})$ ok.

So, as I did tell you that it makes it symmetric. So, the equation becomes symmetric if I write $\boldsymbol{k'} + \boldsymbol{p} + \boldsymbol{q} = \boldsymbol{0}$. Now instead of writing like that this way we can where it is $\boldsymbol{k} = \boldsymbol{p} + \boldsymbol{q}$, so it is not symmetric here this is not symmetric know this one. But here it is becomes symmetric $\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{k'} = \boldsymbol{0}$ they all are going cyclic order or anti cyclic it will also be anti-cyclic or. So, become a reality condition you must also realize that there will be another vector, which is so $\boldsymbol{k'}$ is going this way no, so this I should say $\boldsymbol{k'}$ ok, so $-\boldsymbol{k'}$.

So, I am just changing the sign of so $-\boldsymbol{k'}$ which is equal to $\boldsymbol{k}$. So, I have $-\boldsymbol{k}, -\boldsymbol{p}, -\boldsymbol{q}$. So, we should have two of pair to a reality condition both of them must exist. So, velocity field is real, and I am going to come to this figure c bit later. Now this is a recap of energy equation. Now, let us focus on a single triad. So, I think let us go back to this, so I want to focus on a triad right this triad and its complex or its negative counterpart.

So, this is one triad this one $\boldsymbol{k'}$ $\boldsymbol{p}$ $\boldsymbol{q}$ and this one is the other one which is $-\boldsymbol{k'}$ $-\boldsymbol{p}$ $-\boldsymbol{q}$ and the Fourier mode of these two are related. So, you know if $\boldsymbol{u}(\boldsymbol{q})$ then $\boldsymbol{u}(-\boldsymbol{q})$ is u $\boldsymbol{u}^*(\boldsymbol{q})$ ok. So, they are so they are connected because u is real. Now, for if I just focus on only one triad in its negative counterpart, then this sum becomes simpler this sum will become simpler. Now how will that sum look like.

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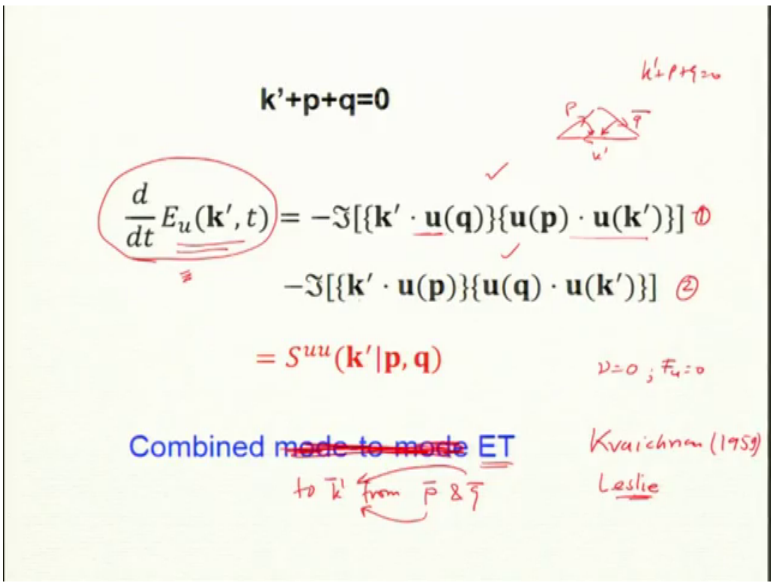

So, it will look like two terms right instead of summing over millions of them now I have a small thing. So, this so I can draw let us draw it like this $\boldsymbol{k}'\boldsymbol{p}\ \boldsymbol{q}$, now left hand side is this one $E_u(\boldsymbol{k}')$ the right hand side is sum remember now $-\Im[\{\boldsymbol{k}'.\boldsymbol{u}(\boldsymbol{q})\}\{\boldsymbol{u}(\boldsymbol{p}).\boldsymbol{u}(\boldsymbol{k}')\}]$. So, I just say that this is one term, but of course I can I also have to get I have to sum over all possible thing. So, instead of $\boldsymbol{q}$ I can also $\boldsymbol{p}$ not can have I have to have $\boldsymbol{p}$ ok. So, both the terms are included in the right-hand side for the single triad.

Student: So, there could be I think $\boldsymbol{u}^*(\boldsymbol{k}')$.

Because I am using this notation $\boldsymbol{k}' + \boldsymbol{p} + \boldsymbol{q}$ is 0. So, there is no $\boldsymbol{u}^*$ if I was making $\boldsymbol{k} = \boldsymbol{p} + \boldsymbol{q}$ then there is a $*$ is that fine to everyone ok. So, this is way to make everybody on the same ground ok. So, in fact, you see bit a later a calculation become much simpler if I make it symmetric. So, is that ok so convolution gives you two terms.

Now, this was derived much earlier by Kraichnan for a single triad please remember a single triad and this is called combined mode to mode transfer and this is Kraichnan 1959 ok. So, ET means I will use this shorthand for energy transfer. Now please keep in mind that I have set viscosity to 0 and my $F_u(k)$ to 0 to simplify this is for simplification. So, we do not want to worry about viscous term, and we do not want to worry about energy coming via external force.

Now, what is interpretation of these two terms in the right-hand side term one and term two, it tells you that how much energy goes into $\boldsymbol{k}'$ so this is rate of change of energy you know. So, energy change per unit time coming from $\boldsymbol{p}$ and $\boldsymbol{q}$ together I mean look energy is changing. So, if this quantity is non-zero; that means, somebody is giving energy like as I said in a wave in a ocean, if a wave is getting energy from non-linear terms they are of this form as long as they are incompressible. So, one assumption we have made is incompressible.

So, it is a combined in mode well actually I should not sorry I made a mistake combined energy terms combined not combined mode combined energy transfer two mode $\boldsymbol{k}'$ from $\boldsymbol{p}$ and $\boldsymbol{q}$; so, this two $\boldsymbol{k}'$ from $\boldsymbol{p}$ and $\boldsymbol{q}$. So, Kraichnan said that they are three people and there are only three people a, b, c or $\boldsymbol{k}', \boldsymbol{p}\ , \boldsymbol{q}$, so $\boldsymbol{k}'$ gets energy from other two. So, according to Kraichnan I can know only how much money or energy this guy is $\boldsymbol{k}'$ is getting from other two, I cannot individually tell how much is getting from b and c.

Student: (Refer Time: 13:30).

Similarly, b will get energy from a and c. So, Kraichnan said I do not know how much a and c will give separately, but I know the total so. In fact, this is what is derived from energy equation that is why it is called combined $\boldsymbol{p}, \boldsymbol{q}$ together giving to $\boldsymbol{k}'$ and this is used by Kraichnan. Kraichnan use it for derive a formula for flux and the derivation is reasonably long its around four pages of his paper in JFM ok (Refer Time: 14:00). There is another book who derive who basically reproduces Leslie this Leslie's book who does this in more words and more systematically.

Now, we this we thought. In fact, there was a there are three of us. So, that there is a student Gaurav many years back in 2000. In fact, 99 2000 and Ishwaran, so there are three of us then we said let us try to see whether we can derive how much goes from $\boldsymbol{p}$ to $\boldsymbol{k}'$ and $\boldsymbol{q}$ to $\boldsymbol{k}'$ separately. So, there has to be three people.

If there is no three people, then this transfer is 0 right. If I turn off any of them then the non-linear term energy transfer is 0, there is the three of them then let us try to decode how much goes from any mode to any mode ok. And it was very useful well fluid it was fluid also is useful for MHD it also become it become more useful it is more useful. So, that was our plan and we call that mode to mode transfer ok.

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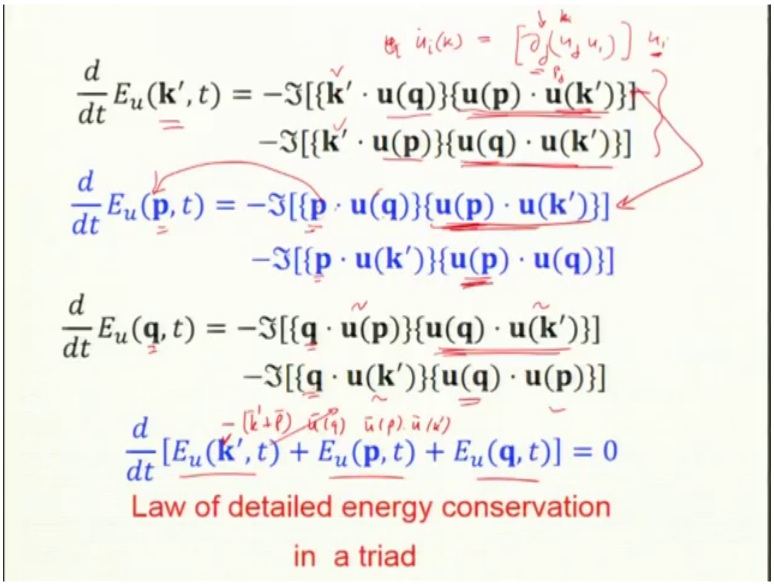

Now, before you go to mode to mode transfer let us do some more analysis. So, I am so I am I am coming back again to usual formula of combined transfer. So, this is for $\boldsymbol{k'}$ know this we you saw in the previous slide you also write for $\boldsymbol{p}$. So, what will $\boldsymbol{p}$ term give you. So, instead of $\boldsymbol{k'}$ you just put $\boldsymbol{p}$ and so $\boldsymbol{u}(\boldsymbol{p})$ will be in the right-hand side.

So, this one thing which is common is $\boldsymbol{p}$ must be a vector wave number $\boldsymbol{p}$ must be dotted with another wave number. So, this is this has to be there for the receiver wave number this is by instruction right, because this is coming from $[\partial_j(u_j u_i)]u_i$. These ∂_j is give you k_j and the k_j is coming from here or if I do for $\boldsymbol{p}$ then this $\boldsymbol{p}$ will come ok. So, wave number of the argument of $\boldsymbol{u}$ must be here by instruction yes or no. So, when I take the Fourier then this becomes k_j for $\boldsymbol{k}$ if I put for $\boldsymbol{p}$ then this becomes p_j.

Student: Right.

Right is straightforward, if you are kind of not following it please stop because this is critical. If I this is the right-hand side u dot dot well I have put $u_i u_i$. So, if I take the Fourier then this $\boldsymbol{k}$ and this Fourier of $\boldsymbol{k}$, so this becomes k_j and that dot product it is coming here. So, this $\boldsymbol{p}$ now $\boldsymbol{u}(\boldsymbol{p})$ will also come this $\boldsymbol{u}(\boldsymbol{p})$, so the $\boldsymbol{u}(\boldsymbol{p})$ is sitting here.

Now, convolution involves basically this and that this term. So, $\boldsymbol{q}, \boldsymbol{k'}$ and $\boldsymbol{k'}, \boldsymbol{q}$, here this was a $*$ you know when I put this $*$ was coming here for $\boldsymbol{k'}$. Now $\boldsymbol{k'}$ is a wave number and convolution involve these two. So, that is the pattern, please relook at it in your notes there is a pattern.

So, similarly we can write for $\boldsymbol{q}$ the two terms. So, this $\boldsymbol{q}$ wave number is coming here and $\boldsymbol{u}(\boldsymbol{q})$ will come here and these are coming from convolution convolution and it is symmetric. So, in fact, we do not need to put any star complex conjugate, so that is the advantage of $k' + p + q = 0$. Now if I sum of these six terms or sum of these three equations what will I get.

Student: 0.

Excellent we get 0 why.

Student: (Refer Time: 18:00).

Why it is 0?

Student: Convolution.

Student: (Refer Time: 18:03) are there.

Excellent, so this is of course, your physical interpretation that this energy is conserved.

Student: (Refer Time: 18:11).

Ok, but we should get it from math as well ok. So, how will I get from math? So, let us collect the term of $\boldsymbol{u}(\boldsymbol{p})\boldsymbol{u}(\boldsymbol{k}')$. So, this $\boldsymbol{u}(\boldsymbol{p})\boldsymbol{u}(\boldsymbol{k}')$ right hand side these are only two terms with product $\boldsymbol{u}(\boldsymbol{p})\boldsymbol{u}(\boldsymbol{k}')$ is there any other term with that none right.

Student: (Refer Time: 18:33).

If I sum these two what will I get?

Student: (Refer Time: 18:35).

So, I get $-\Im[\{(\boldsymbol{k}'+\boldsymbol{p}).\boldsymbol{u}(\boldsymbol{q})\}\{\boldsymbol{u}(\boldsymbol{p}).\boldsymbol{u}(\boldsymbol{k}')\}]$. What is this now arrows I have what will I get what is $\boldsymbol{k}'+\boldsymbol{p}$.

Student: $-\boldsymbol{q}$.

$-\boldsymbol{q}$ and $\boldsymbol{q}.\boldsymbol{u}(\boldsymbol{q})$ is 0.

So, this becomes 0. So, all the six terms we add together in incompressibility condition is invoked here and they give 0, as a result the energy of the three modes together $\frac{d}{dt}$ is 0 and that is how we prove that energy is conserved in a triadic interaction. And, this is having a name is called conservation of detailed conservation of energy in a triad or triadic conservation. So, is not only that energy conserved for all full box.

Student: (Refer Time: 19:51).

But it is also conserved for each triad, so it is more detailed yes.

Student: sir some mode may be part of another triad.

So, that we will do it later that I will come to it, so we are just saying that know only one triad exists

Student: 1 1.

Only one triad exists, and these are physics of single triad ok. If I have more triad then we will do superposition, but right now let us focus on single triad.

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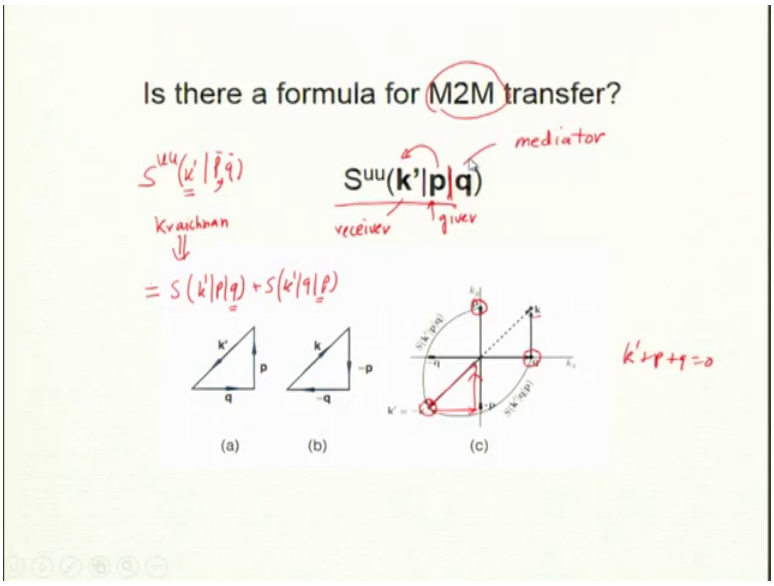

Now, so this is the question which I told slide before, is there a formula for mode to mode transfer this combined transfer. But now if I want to see among these three modes how much x gives to y and y gives to z and so on. So, there are six combinations right and it turns out it is possible and that is this lecture is about. So, if you remember Kraichnan wrote this formula as $S^{uu}(\boldsymbol{k'}|\boldsymbol{p},\boldsymbol{q})$, I did not emphasize it. But is there in a previous slide, so $\boldsymbol{k'}$ is a receiver and $\boldsymbol{p}$ and $\boldsymbol{q}$ are giver. So, that is why there is a comma and this notation by Kraichnan.

But we said well $\boldsymbol{p}$ is a giver so $\boldsymbol{p}$ gives to $\boldsymbol{k'}$, but $\boldsymbol{q}$ is a mediator it is watching it is like a you know it is like a lawyer. So, he does not get, but lawyer will facilitate the transfer. So, this is mediator, mediator, mediator and this is a giver, and this is a receiver. So, remember there is no comma but there is a line here that separates the giver and the mediator. So, first argument is a receiver second is a giver and third is a mediator it is a like the law it is a witness. Of course, $\boldsymbol{q}$ will become a giver at some other formula, but then $\boldsymbol{p}$ will become a mediator.

So, this in fact you can easily see that this is sum of two terms $S^{uu}(\boldsymbol{k'}|\boldsymbol{p}|\boldsymbol{q})$and $S^{uu}(\boldsymbol{k'}|\boldsymbol{q}|\boldsymbol{p})$ right, here $\boldsymbol{q}$ is a mediator but there $\boldsymbol{p}$ will become mediator and $\boldsymbol{q}$ will be giver so that is my definition. If I get from two people, then money I get is some of the money getting from v and c that is law of addition and we want to get a formula for this ok. So, in this diagram, so we have this triad. So, $\boldsymbol{q}, \boldsymbol{p}$ and $\boldsymbol{k'}$ and $\boldsymbol{k'}$ is $-\boldsymbol{k}$ right. So, $\boldsymbol{p} + \boldsymbol{q} = \boldsymbol{k}$, but $\boldsymbol{k'} + \boldsymbol{p} + \boldsymbol{q} = \boldsymbol{0}$ ok, they form in I mean the triangle is not looking so apparent, but you can draw this triangle I mean.

So, you can see that this is $-\boldsymbol{k'}$. So, $+\boldsymbol{q}$ will be going that way and $+\boldsymbol{p}$ that so they close the triangle ok. Now, so we will derive a formula for this for this one and I we call it mode to mode m to m mode to mode ok. So, what is the condition? So, how many so I need some conditions on mode to mode transfer. So, how many equations I am going to get ok.

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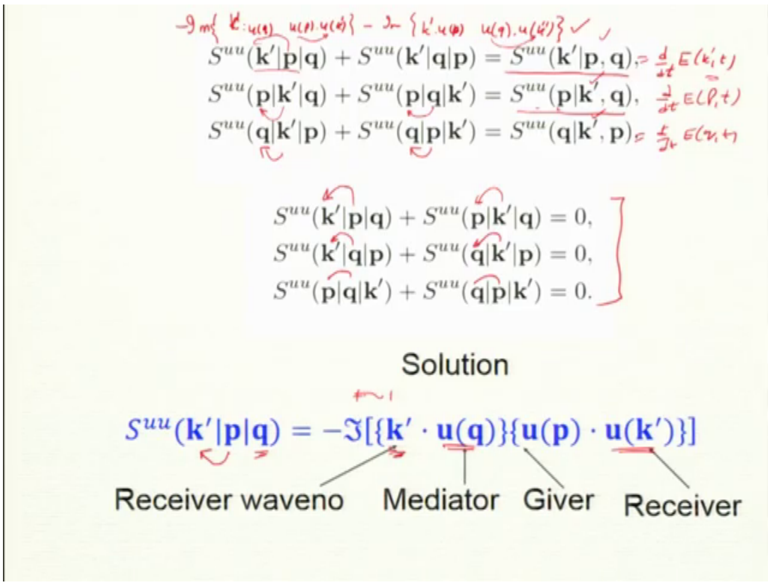

So, we will get the following first three equations. So, what is three equation mean. So, Kraichnan said in the original formula this is the total energy coming into $\boldsymbol{k'}$ from $\boldsymbol{p}$ and $\boldsymbol{q}$ right. This is $\frac{dE_u(\boldsymbol{k'})}{dt}$ viscosity is off, and force is off. So, that has to be as I said is sum of $\boldsymbol{p}$ giving to $\boldsymbol{k'}$ and $\boldsymbol{q}$ giving to $\boldsymbol{k'}$. So, focus of first two arguments and third argument is by definition is a mediator.

Now, second equation comes from energy coming to $\boldsymbol{p}$ from $\boldsymbol{k'}$ and $\boldsymbol{q}$. So, this is coming from $\frac{dE_u(p)}{dt}$. So, that will be $\boldsymbol{p}$ getting from $\boldsymbol{k'}$ and $\boldsymbol{p}$ getting from $\boldsymbol{q}$. Third equation is $\frac{dE_u(\boldsymbol{q})}{dt}$. So, $\boldsymbol{q}$ is a receiver, it receives from $\boldsymbol{k'}$ and $\boldsymbol{p}$.

These are known in computer simulation we can measure them or Kraichnan said well the right-hand side of this equation is this. So, given modes I can write down what is this combined transfer. So, right hand side is known left hand side all the six terms are unknown is the said the six possibilities a from b c b from c a and c from a b. So, I need three more equations right I mean to get six unknowns I need see three more equations.

So, what is the three more equations? So, again by definition of transfers any transfer if I get some from somebody money the other person loses money right. So, is that right I mean two between two people the like I if I get 5 rupees and another person will loss 5 rupees. So, energy transfer from $\boldsymbol{p}$ to $\boldsymbol{k'}$ is equal and opposite of $\boldsymbol{k'}$ to $\boldsymbol{p}$ or if I add them, I get 0, he could also write this as minus of that minus of this term ok. So, just add them you will get 0 just this is again law of exchange. Energy is a scalar quantity and that is how is a number and it should happen like that.

Similarly, $\boldsymbol{q}$ to $\boldsymbol{k'}$ is equal and opposite of $\boldsymbol{k'}$ to $\boldsymbol{q}$ and same thing for $\boldsymbol{p}$ to $\boldsymbol{q}$ is equal and opposite of $\boldsymbol{q}$ to $\boldsymbol{p}$. So, I got six equations and six unknowns. Now I look for a solution. Now this was in fact, the student Gaurav in fact it was first his idea. So, I said we can write down formula for any one of them and it turns out the first one. In fact, is easy to get the formula actually.

Now, is a known (Refer Time: 27:01) side easy to get the formula and it works amazingly it works solution is this. So, energy going from $\boldsymbol{p}$ to $\boldsymbol{k'}$ where $\boldsymbol{q}$ is a mediator is dot product ok. So, I just so $\boldsymbol{k'}$ is a receiver wave number know these are receiver wave number dot this with mediator mode, mediator mode this is a or it is a lawyer mode to mode you know, so this is mediator mode mediator. Then receiver and giver are dot product together. So, this is a receiver $\boldsymbol{u}^*(\boldsymbol{k'})$ and it receives, and giver are dot product together so that is a recipe.

Now, this satisfies equation 1 2 3. If you look at Kraichnan formula it was this plus, in fact I was trying to emphasize when I wrote this formula. So, now I do not want to go back, because it will erase all whatever I written here. So, if you remember so this was this formula was written the right hand just focus on right hand side ok, it was written as $-\Im[\{\boldsymbol{k}'.\boldsymbol{u}(\boldsymbol{q})\}\{\boldsymbol{u}(\boldsymbol{p}).\boldsymbol{u}(\boldsymbol{k})\}] - \Im[\{\boldsymbol{k}'.\boldsymbol{u}(\boldsymbol{p})\}\{\boldsymbol{u}(\boldsymbol{q}).\boldsymbol{u}(\boldsymbol{k})\}]$ ok..

Now, does it satisfy these three formulas? Now can we show this I mean this, so let me just show this just for. In fact, it is quite easy to see in this easily do this, so let me show in the next slide ok.

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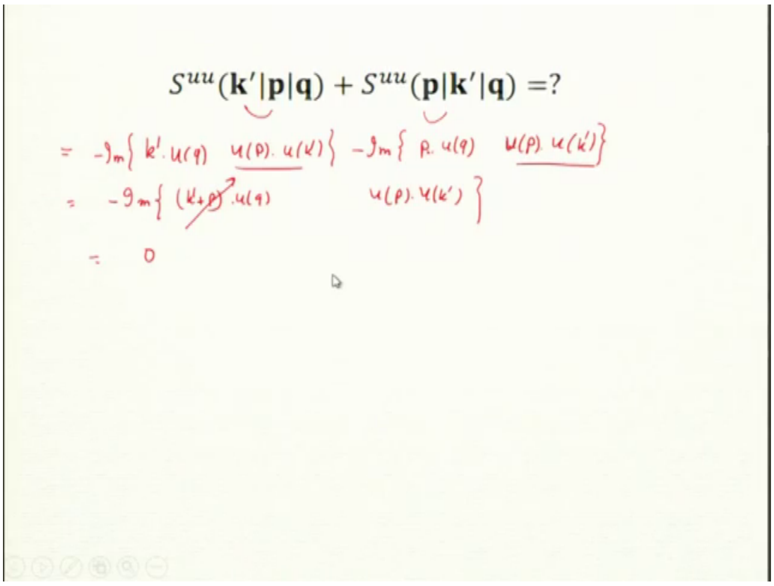

So, is it 0? So, $\boldsymbol{p}$ to $\boldsymbol{k}'$ and $\boldsymbol{k}'$ to $\boldsymbol{p}$ if I add them will I get 0. So, what is this formula? So, $-\Im[\{\boldsymbol{k}'.\boldsymbol{u}(\boldsymbol{q})\}\{\boldsymbol{u}(\boldsymbol{p}).\boldsymbol{u}(\boldsymbol{k})\}]$. I am being lazy I am not writing the vectors I mean just too many vectors and second term is $-\Im[\{\boldsymbol{p}.\boldsymbol{u}(\boldsymbol{q})\}\{\boldsymbol{u}(\boldsymbol{k}).\boldsymbol{u}(\boldsymbol{p})\}]$. For calculation see the slide. it works for all three of them. So, this indeed is a formula.

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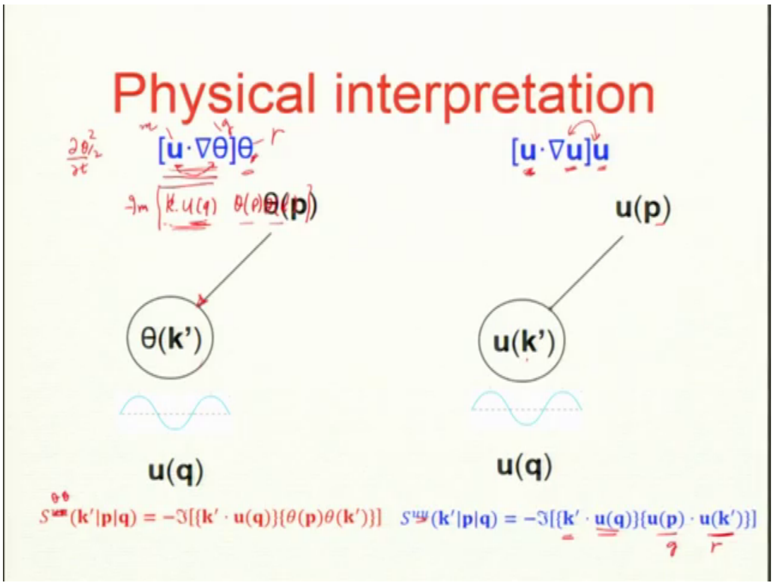

So, it did work it satisfy all the six equations. Now, what is the physical interpretation? Now, we have done bit of temperature stuff know its scalar quantities. Now for scalar quantity is the non-linear term, how does it look like? So, non-linear term is $[\boldsymbol{u}.\nabla\theta]$ right I showed that part ok. Now I have multiplied this by θ for energy. So, this will be part of the $\frac{d}{dt}\theta^2$ yes.

Now, if I look at the in Fourier language, this becomes now dot will of course, you can go inside because incompressible condition; this incompressible condition is critical for this all this will not work for compressible flows which. So, incompressibility is very critical otherwise it spoils the whole show. So, this will come for scalar which I will do it bit later ok. The formula will be$-\mathfrak{I}[\{\boldsymbol{k'}.\boldsymbol{u}(\boldsymbol{q})\}\{\boldsymbol{\theta}(\boldsymbol{p})\boldsymbol{\theta}(\boldsymbol{k})\}$. So, this is the formula. In fact, now we know it. So, well. So, well of course, this some 15 years or 20 years now we have been working on it. So, so these are mediator now these are mediator, and these are receiver, receiver r giver g and this is a.

Student: Mediator.

Mediator this is a lawyer mediator m. So, receiver with mediator receiver wave number is mediator and receiver with giver. Now no dot product there is some ordinary product, now I can draw this in some picture way pictorial way. So, this $\theta(\boldsymbol{k'})$ is trying to receive energy.

So, it can receive from $\theta(\boldsymbol{p})$ and $\theta(\boldsymbol{q})$. So, we are focusing on how much $\theta(\boldsymbol{p})$ gives? So, he is giving you know this arrow and $\boldsymbol{u}(\boldsymbol{q})$ is like it is advecting this is advection.

In fact, the is like in fact, indeed is a advection of perfume θ is carried by $\boldsymbol{u}$. So, this carrying this $\boldsymbol{u}$ is a carrier. So, that is why it is dot product with wave numbers. So, anything which carries is not not transferring, carrier is just like a train you know is taking the two passengers who are exchanging money. So, this is a $\boldsymbol{u}(\boldsymbol{q})$ right now act as a just a mediator or carrier. So, it does not participate in energy transfer it just the carrier or is a mediator in this. So, it is like a wave which is advecting it and one temperature mode gives energy to other temperature mode.

Now, in fact, for temperature is much easy to show and at least make this analogy and is when lot of people are convinced by this by this connection. Now for Navier-Stokes that is why I wrote the formula above for Navier-Stokes equation, you have to identify these two as a receiver and giver. In fact, they are receiver and giver, and this is like a mediator. So, these two-dot product together so that, is why receiver and giver modes are dot product and the mediator mode $\boldsymbol{u}(\boldsymbol{q})$. So, this is a giver receiver they will be dot product and this is a mediator.