Physics of Turbulence
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Lecture - 2
Basic Hydrodynamics: Governing Equations

Okay, so we will start discussion on basic hydrodynamics. So I will discuss incompressible Navier-stokes today and I probably continue in the next set of lectures.

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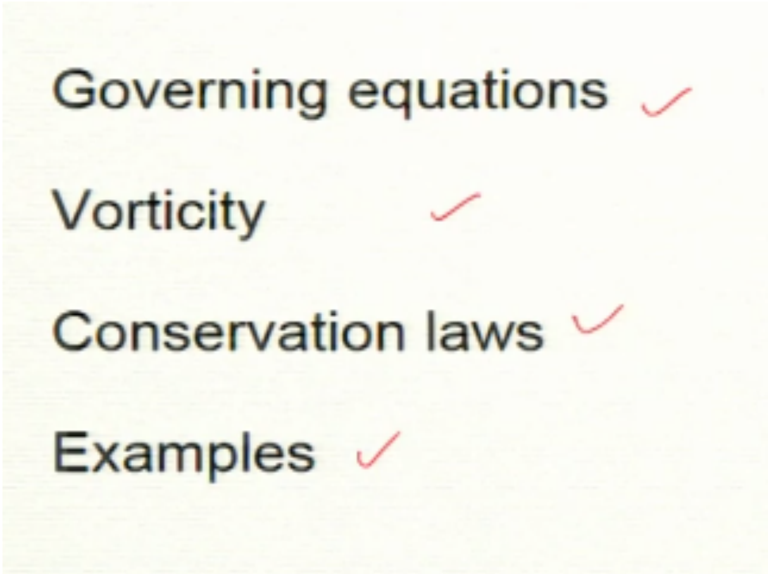

So I will cover these 4 topics. So governing equations, then on vorticity, conservations laws, and illustrate using some examples because I will cover 4 small pieces and I will try to illustrate basic ideas.

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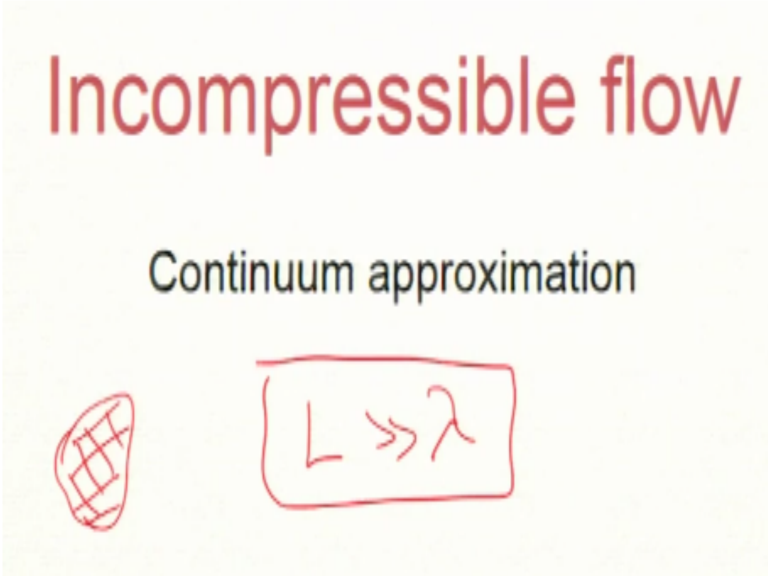

So we go to governing equations okay. So I will brief you on basically Navier-Stokes equation. So continuum approximation. So, I think I did mention about diffusion equation for the molecules you know which were diffusing things, acting on like dust particle or diffusing heat, but we are looking at fluid approximation, I am looking at fluid you know, so fluid velocity. When I say wind is moving, so wind is not molecule which is moving, wind is gross or average state of set of molecules.

So I mean, it is typically I would say you can go up to millimeter packet size. So, this is some kind of a fluid element. I am not looking at molecules. So an assumption is that my distance of interest or my length of interest L must be much bigger than mean free path length okay. So our length scale should not be close to mean free path length, then I start seeing molecules and I am not wanting to look at molecules, that description is different, that description is authentic theory of thermodynamics, so that is not what is of interest for this course.

This course is on fluid and fluid has their own properties, you know these are I did mention for this diffusion equation that we need to consider the fluid motion for the movement of heat or for movement of particles. So please keep in mind that we are interested in continuum approximation okay.

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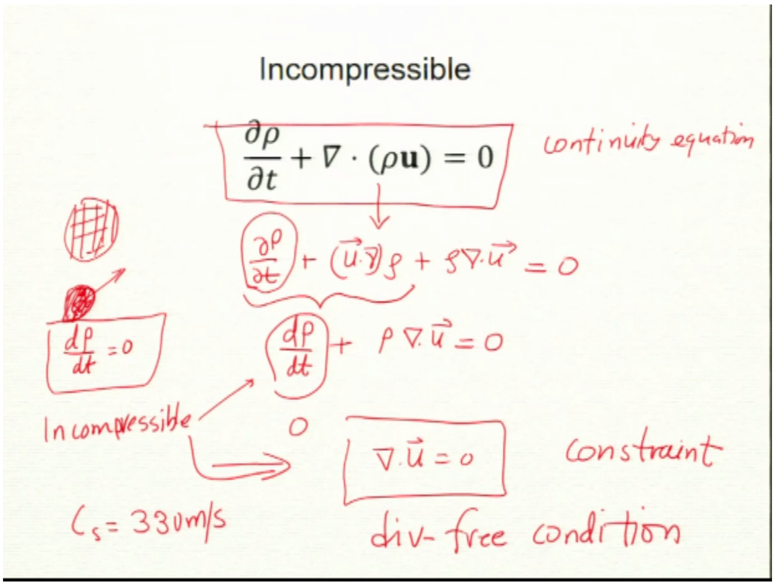

Then, I am going to worry about incompressible flows. For this course, we are not going to do compressible flows which are interesting on their own, I mean most of the astrophysics which Supratik does. So this velocity is very fast. So sound speed is everybody knows no, but if sound speed, if the velocity of fluid is comparable to the sound speed, then we need to worry about density variations. So we assume that our speeds are much lower than the sound speed. In fact, for me the sound speed is infinity, okay.

So, you can see from you can derive that sound speed as infinity. So, this equation is called continuity equation okay, is continuity equation. So it is continuity of mass. This is like if I have some mass here, then rate of change of mass inside this volume will be equal to the flux, no, so from that you can derive this. So again, I will not get into the derivation of this equation, but this equation is basically conservation of mass. Now, we will assume that when the fluid is moving, so I can take any small fluid particle, a parcel, not particle, I would say parcel.

Parcel I mean a packet, which is millimeter size or bigger size, not too big, but ya this is intuitively called parcel. So when the fluid is moving, we assume that the $\frac{d\rho}{dt} = 0$. The density does not change on motion, so that is the density locally could change, but when the fluid is moving, its density is not changing. So, just to see so if the fluid is here in this and when it moves, its volume should not change, right. There are particles inside and if all the particles are inside and that is why it has a mass.

If volume does not change, the density will not change. So $\frac{d\rho}{dt} = 0$ if the fluid on motion does not change its density. So this is called incompressible approximation, this $\frac{d\rho}{dt} = 0$. So let us see what does it imply? So this is in fact assumption of incompressibility, I cannot compress a fluid parcel, this is a packet. So, you can think of 1 cm cube fluid is going around, it is not changing its volume. So let us start from here. Now, I will just do same basic vector calculus. In fact, we need lot of vector calculus, but it will be basic ones.

So here I am going to expand this one, the second term which will be, what is the divergence of scalar multiplied a vector. So this is -- I want to make here -- $\rho\nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla\rho$, this is this term right, I mean this you must have studied this divergence of vector times a scalar. Now we have the first term is $\frac{d\rho}{dt} = 0$. Now I can combine the first 2 terms and what is this one, it is total time derivative of ρ, total means when I go with the fluid. So, this one is Eulerian local time derivative, I don't move with the fluid, but together I go with the fluid.

So these two terms is $\frac{d\rho}{dt}$ and the next one is $\rho\nabla \cdot \mathbf{u}$. Now, incompressible assumption implies that this is $\frac{d\rho}{dt} = 0$, so what does it give you? Incompressible assumption implies, this assumptions implies that $\rho\nabla \cdot \mathbf{u} = 0$, but ρ is positive number, ρ is positive not negative, ρ is always positive number, that means $\nabla \cdot \mathbf{u} = 0$. So, this is my constraint, this is not an equation for time advancing, this is a constraint on velocity okay.

So, velocity must be satisfying this divergence free condition, so this is called div-free condition. So in this course, we will only focus on incompressible fluid, though compressible fluids are also interesting and important. Now, somebody may say well why do you worry about incompressible fluid if it is such a constraining condition. It turns out for most fluid on the terrestrial environment, regular terrestrial environment, these are very good assumptions.

So what is the sound speed for air for example, so again everybody knows for room temperature and normal pressure is around 330 m/s right, so 330 m/s, the sound speed C for air and for water the sound speed is around 1000 m/s. Now, what is the speed typical in regular lab environment or even atmospheric environment? For air, atmospheric environment is 10 km/hr which will translated to 3 m/s for atmospheric flows, so is much smaller than 330 right.

So, the density variations are tiny even though I can see that cycle tube I fill air you know, so air is compressible, but when it is moving, the density variance are not large. For water, hardly I find a flow which is 1000 m/s. So incompressibility is very good assumption for many of the terrestrial applications, for atmosphere, for engineering flows and even for like inside the earth this incompressibility is a good assumption when molten materials inside the earth which is not going very fast okay.

So first thing you should do is this, incompressible flows, okay is that clear. So this is what we will stick ourselves to.

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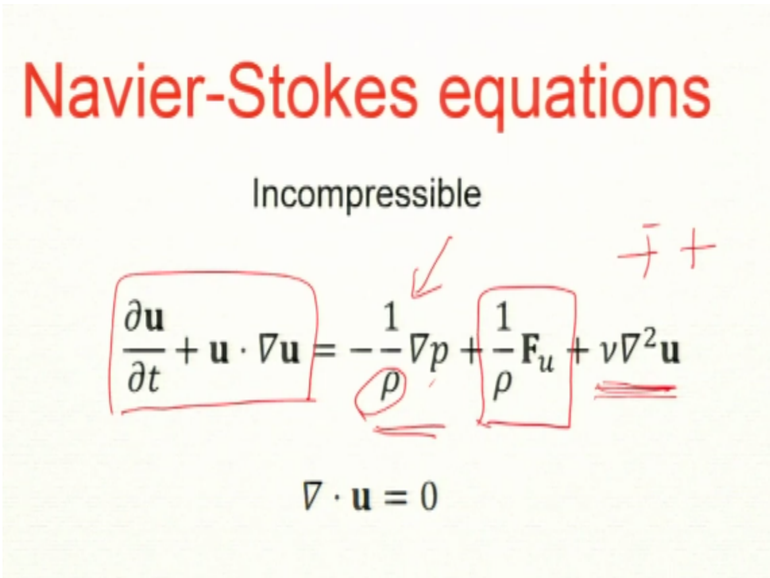

So I am going to say incompressible and this is called Navier-Stokes equation. They are 2 people, Stokes is British and Navier is French. So this is given by these people. So, the form which we are going to use for this course is this is the form. So, I just want to mention that density is constant. So strictly speaking the density while in motion should be constant but with homogenous fluid you know that means my things are not changing when I go from one position to other position on the average, then you can assume the density is constant everywhere, okay that is an assumption, but reasonable assumption okay.

So that is what we would assume. So, in fact, this rho is the density which are assuming to be constant, which is not strictly too, $\frac{d\rho}{dt} = 0$ does not mean ρ is constant, is a total derivative, but fair enough for me, this is the detail which I will not discuss. So this is the acceleration term okay, this is the pressure gradient term. So Newton's law, so this is acceleration, this acceleration is force by mass.

So the 3 forces, one is pressure gradient, so as I said if there is a pressure difference will be in 2 surfaces, there will be a net force and that is we can derive it, in fact the best and cleanest description I find is in Landau's book, chapter 1, okay, so this is very nicely derived in pressure gradient. Viscous term is coming because of viscosity at 2 surfaces, so we will assume this form okay. This is good for incompressible flows, this is reasonably good description and this is the external force.

$\boldsymbol{F}_u$ is the external force, that could be gravity, it could be magnetic force, well buoyancy is gravity or rotation or any other force because external force, not pressure or viscous, in addition to viscous and pressure and as I said incompressibility implies at we have this condition $\nabla \cdot \boldsymbol{u} = 0$, okay. So at least for next set of lectures, this is what I am going to focus on okay.

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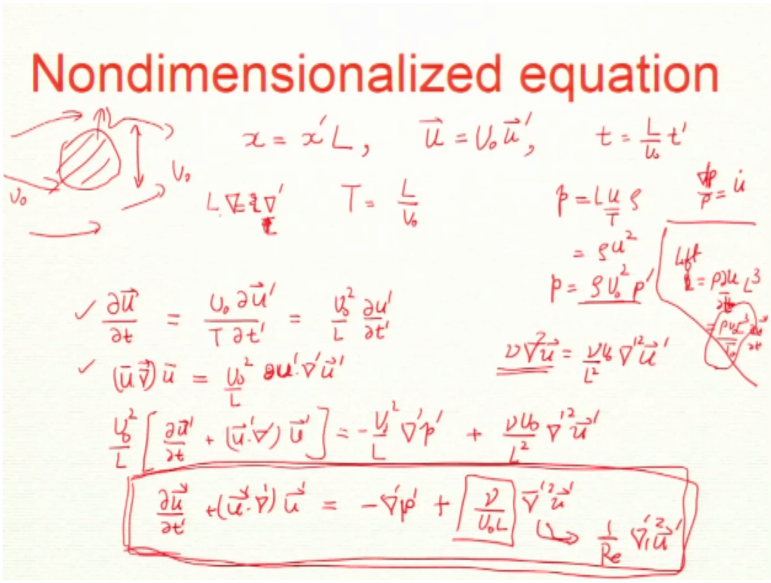

So we need to also worry about nondimensionalization. This plays a very critical role, uh, because we have to deal with systems of different sizes, different velocities and it turns out we can map one system to other system. For the example which we discussed in the last class was aeroplane, no. So aeroplane, a big aeroplane and a small aeroplane has similar properties, in fact we can map one to other if the Reynolds number is the same, there is a trick, so in fact I will show you how to map okay.

So we need to also worry about nondimensionalization. This plays a very critical role uh because we have to deal with systems of different sizes, different velocities and it turns out we can map one system to other system. For the example which we discussed in the last class was aeroplane, no. So aeroplane, a big aeroplane and a small aeroplane has similar properties, in fact we can map one to other if the Reynolds number is the same, there is a trick, so in fact I will show you how to map okay.

So this is very standard example in engineering, the flow pass cylinder. So what is the length scale for this problem? So this diameter of the cylinder is length scale okay. So we will choose that length scale, that diameter is my length scale. So my position *x,* I am going to make $x'L$. So what is the dimension of x'? It is dimensionless because *x* is cm and *L* is cm, x' should have no dimension. What about velocity, what should I choose, **u** which I am using, so you can put a vector if you like.

I am going to choose velocity scale, so velocity would be far away from the cylinder, this is the mean velocity, I can write it here mean velocity no, far away velocity is streaming which is the number. So near the cylinder, velocity is function of *x* and *t*, but far away from the cylinder, I can say I am sending constant velocity and so we want a number, of single number, scale means you want to choose a number, single number for the whole system.

So you know it is my velocity scale, so I multiply this by $\mathbf{u}'$, so $\mathbf{u}'$ and **u** are in the same direction, direction doesn't change, only the numbers have changed and $\mathbf{u}'$ is dimensionless, it has no dimension of cm/s or none of it. What about time, how do I get time. If I have dimension of length and dimension of velocity, then I can immediately construct the dimension of time, so which is what is the dimension of time by the way, *T* will be *L*/*U* not straightforward, so what should I write.

T which is the dimension of second per it could be hours or could be long time, but anyway this *T* has a dimension, I multiply this by $\frac{L}{U_0}$, not multiply, I mean $T = \frac{L}{U_0}t'$. Similarly you can define for pressure, external force, these are other two terms if you saw the last equation, so we can also write down for that. So let us look at pressure. What is the dimension of pressure? So pressure was, so if I look at the last stuff, ∇p.

So let me do some rough work here, $\frac{\nabla p}{\rho}$ was $\dot{\mathbf{u}}$, right. This p and ρ look like similar. So pressure has dimension of $\mathbf{u}/T$ and this length will go up and density right so from here you can see, so this will be ρU^2. L/T is $\mathbf{u}$, so ρ^2, this is the dimension of pressure. So I can easily nondimensionalize it. So I said $p = \rho$ which is constant density $U_0^2 p'$, okay. So you can play this game for all term, in fact, it is very useful game.

Nondimensionalization is one of the important tools for fluid mechanics, plasma physics, and all fields actually, I mean I say that even quantum mechanics you can make nondimensionalization and it helps life very much, especially for simulations; okay this is a separate matter, but we can play this game nicely.

"Professor - student conversation starts" So in this case the ρ is also 1 right. So right now in this equation it is not 1, but I will choose it to be 1, for most of the future discussion after this, I will make rho is one 1 okay, so why carry a symbol, but for this one it is not 1, yes sir, okay. **"Professor - student conversation ends."**

So let us look at few terms, I will not do for all terms, but I just want to do for few terms. So we have one term which is this term, I want to make it nondimensional. So what do I do? Some of you know this very well but I just want to do it for completeness. So $\mathbf{u}$ I am going to write as $U_0\mathbf{u}'$ and what about ∂t? So this $t\partial t'$, correct? So this one, so this term has dimension of, now I can replace this one T, $\frac{U_0^2}{L}\frac{\partial \mathbf{u}'}{\partial t'}$, this is one term. I can look at the next term.

So, I will not do the all algebra, so $\mathbf{u} \cdot \nabla \mathbf{u}$ becomes so the two $\mathbf{u}$'s, so that becomes U_0^2, and ∇ gives you $1/L$, $\frac{1}{L}\nabla$, so $\mathbf{u}' \cdot \nabla \mathbf{u}'$. So I should say what is ∇'. So I can write it here now, I go back. So ∇ has dimension of length, so I should say ∇ is $L\nabla'$, no I made a mistake, this one, so g∇ is dimension of 1/length. So ∇' will be dimensionless okay.

So ∇ has dimension of 1/length, I should multiply by L, that is ∇' okay. That is it okay or divide by L, so either. So $L\nabla$ is ∇' okay. So if I just substitute with Navier-stokes equation, what will I get? I can take look interesting part is that both this acceleration terms have same pre factor in front, $\frac{U_0^2}{L}$, and $\frac{\partial \mathbf{u}'}{\partial t'}$ vectors. I must write the vectors, $\mathbf{u}' \cdot \nabla' \mathbf{u}'$ = now I will not do all the algebra, so I say $\frac{U_0^2}{L}$, you can just shows this, now this I will tell is a homework problem, grad prime p prime this will cancel.

So let us put the viscous term and that I think I am going to do some rough work here. $\nu\nabla^2\mathbf{u}$ what will that be nondimensionally. So ν has dimension of velocity times length, so it is cm^2s^{-1} or m^2s^{-1}. So, ν I will just leave it $\nu\frac{1}{L^2}$, I will not say ν', so ν is a parameter. So $\nabla'^2\mathbf{u}'$ and $\mathbf{u}'$ will give U_0, so that is the term. $\frac{\nu U_0}{L^2}$, so this term is $\frac{\nu U_0}{L^2}\nabla'^2\mathbf{u}'$, okay.

So, in this equation $\frac{U_0^2}{L}$ will cancel, the first 3 terms. So let us do this algebra for the last step dt' prime because $\mathbf{u}' \cdot \nabla'\mathbf{u}' = \nabla' p'$ + ok, ρ has gone, ρ is basically gone here, now this okay so this one ρ doesn't appear because I have replaced dynamic viscosity by kinematic viscosity. Now we got, I have to divide this by $\frac{U_0^2}{L}$ the last term. So what will happen, one U_0 will cancel, U_0 and one L will cancel L.

"Professor - student conversation starts" Sir for a particular problem, uh ha, ν will be constant yes. Yes, so ν is the parameter for the problem. For this particular problem that will be constant. Yes. So ρ is also parameter. Yes ρ is a parameter. For a particular, ya ya ρ is a parameter yes. **"Professor - student conversation ends."**

So in this term, so I mean vicious term is $\nu\nabla^2\mathbf{u}$ is acceleration, this is dimension of acceleration, okay so I am putting acceleration. Now what is this object, does it have dimension or no, inverse Reynolds number and it is dimensionless. One way to check is viscosity has dimension of velocity time length, so velocity time length will they will just cancel. So I found a parameter which is non-dimensional. So, this term, the last term can be written as this one $\frac{1}{\mathrm{Re}}\nabla'^2\mathbf{u}'$.

So this equation is dimensionless and we have one parameter instead of many many parameters. So in your original equation I had velocity, velocity U_0 is a parameter okay which does not appear in any Navier-stokes, but when I solve the problem, U_0 not will be a parameter. Length is a parameter, density is a parameter, ν is a parameter, but now I have only one parameter in the problem and what is my velocity for this problem, which is coming from the left it is 1 unit, well 1 non-dimensional unit okay.

Now, so I will basically in computer we solve this equation or in wind tunnel, wind tunnel is a slightly different game, but this equation we want to solve in computer. Now if I find pressure on this object or pressure gradient or let us say buoyancy, no buoyancy, lift, lift to be some unit okay L, then how do we convert it to a real system. So my this problem gives me a lift, upward lift for this problem it will be some number L. So the real system, I can always convert by this transformation.

So real system what is the dimension of well lift has force, dimension of force, force density, let's say a force density, lift per unit area, per unit volume also okay let us lift. So I will say ρ times acceleration. So, I have some object like this, lift has in fact is a force. So I have to multiply this by volume, so that will be L^3. So I should not see I said where lift, I should use a different pen, but ya so let's say lift, so it has dimension. So you want the lift and force density, force density has dimension of force/volume.

So I will multiply this by L^3. Now, I am getting in terms of ρ primes and u primes, so what do I do, I just multiply this by $\frac{\rho U_0 L^3}{T_0}\frac{d\mathbf{u}'}{dt'}$. So the quantity which I need to multiply pre-factor is this one, $\frac{\rho U_0 L^3}{T_0}$. If I do it, then I will be able to compute lift on the real system. So this is what is done in wind tunnels. So wind tunnel you do, obtain the number for one problem, then by appropriate scaling, so say well I make the length 10 times, velocity 3 times, then what will I by my lift.

So by keeping the same Reynolds number, you must keep this equation required that Reynolds number must be the same, this equation. So that is why our experiment must have the same Reynolds number, then only I can make connection, otherwise I cannot make connection. So the experiment done in wind tunnel is for the same Reynolds number, but I can get the measurement of a lift on a small model, but then I can multiply with the appropriate ratios of lengths, ratios of velocity, I can compute lift on the real aeroplane and it will work okay, so that is what is the importance of scaling.

Now, scaling has one more advantage, well several more advantage, but I will tell you one more advantage. In computer, so suppose somebody wants to compute you know aeroplane is fine but I want to compute for galaxy, no, I want to simulate galaxy. So what is the length of a galaxy, like many many light years. If I put in the centimeter it is going to be huge, let 10^{30} cm or whatever there some number.

Now, putting the numbers in computers is not a good idea, putting large numbers computers don't like, if I also don't like, I can't take up 10^{30} you know, I remember order one numbers. So something like 30, nobody remembers okay. So this is not, so in fact why meter was chosen as a unit you know simple because human being height is 1 m, that is why we choose 1 m as our unit.

Atomic physics use Armstrong as a unit, they do not choose meter as a unit because they can't remember 10^{-9}, 10^{-10}, is a clumsy thing to do all the time, say Armstrong is their unit okay. So similarly as well, in computer you want to keep order one, so I want cylinder of size one that is what computer likes and you work from that okay.

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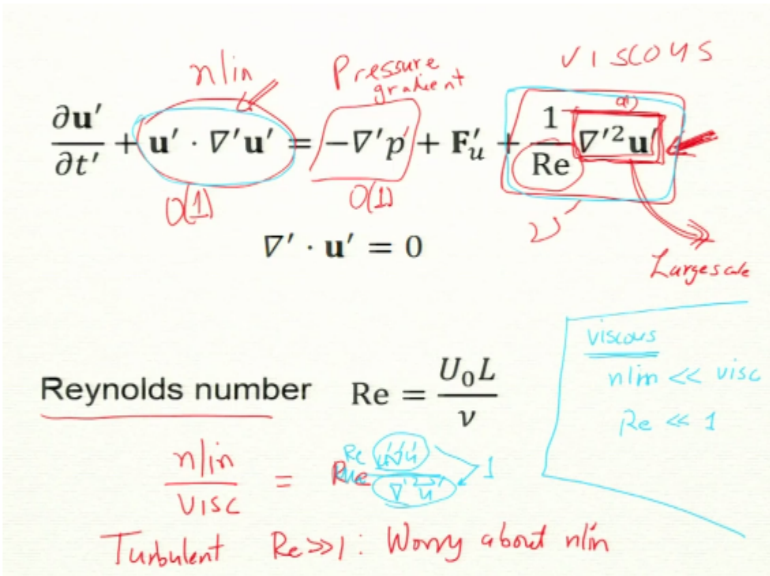

So this is my equation for which I just wrote, so this is the Reynolds number, 1/Re, okay. So in fact, I can also write as ν', nondimensional viscosity if you like. So this ν' has no dimension okay inverse Reynolds number is we can denote by $\nu'$and other equation is very easy to see that nondimensional version is just $\nu'\nabla \cdot \nabla'$ is 0. Of course in future, I will drop the primes.

I will not keep prime, but most of the time we will assume that we are working nondimensional units because it is convenient to work with them and I can always go to real system by multiplying appropriately okay. So, this is called Reynolds number, this is the Reynolds number and you can see one thing that this is a nonlinear term okay. So, this is nlin and what is this term, is a viscous term, and this is the pressure gradient term, pressure, my handwriting is not coming out well, is not very good but is worse here.

"Professor - student conversation starts" So it is p' right. p', you are right, thank you, this is p', yes **"Professor - student conversation ends."**

So pressure gradient. So the nonlinear time of pressure gradient what is, from this equation you can see that there are same order. $\mathbf{u}'$ is order one at large scale, no, so $\mathbf{u}'$, so I must make this remark, $\mathbf{u}'$ will be multi-scale, that means it will be different different values at different different scales and there will be very tiny $\mathbf{u}'$ as well, we will study that, in fact what is velocity at different scales, but in large scale, $\mathbf{u}'$ is order 1, that is what I said no when $\mathbf{u}'$ coming in fact $\mathbf{u}$ from the left is coming as 1.

So $\mathbf{u}'$ is order 1, ∇' is order 1, so in fact this is order 1. Is it happening everybody is agreeing that. What about $\nabla' p'$? that is also order 1. So ∇' is order 1, $p'$is order 1. So this is order 1. What about this term, this is also order 1 okay. In fact, well this is where the, okay now so I think I will not, so this part we will discuss bit later. So which is order 1, so $\mathbf{u}'$ in large scale okay. So, I will only make some loose remarks.

In large scale, ∇' is order 1 and $\mathbf{u}'$ is order 1 okay, so what happens to the ratio of nlin/viscous term, full viscous term. So this is order one 1, this inside inner bracket okay, this inner bracket is order 1 and the order so the ratio will be nonlinear, nlin/full viscous is Reynolds number okay. So, the ratio of n-linear term/viscous term at larger scale is Reynolds number. A small scale of course things, we do not know what is ∇', so that we will discuss that later, but here this is the ratio okay nlin/viscous term is the ratio.

So when system is very nonlinear, we say that this term nonlinear term, this one dominates the viscous term, will dominate the viscous term. So large Reynolds number flows, we need to consider nonlinear term very much. The viscous flows, that means what is viscous flow, viscous term will dominate, so that will be Reynolds number. So let us see turbulent, I must write this here turbulent flows, Re much bigger than 1 and worry about nlin, nlin means nonlinear term okay, nlin, you have to worry about nonlinear term.

What about the viscous regime? Viscous regime, nonlinear term is assumed to be small, so nlin is much small compared to viscous. So what is the Reynolds number for this regime, less than 1, much less than 1 or less than 1 you see okay or less than 1 depends. So, it could be order 1, that is also called laminar okay, so order 1 is also called laminar.

"Professor - student conversation starts" Sir, how have you go this relation nlin/viscous = Re. Okay, so you can see. So this nlin is from here, this box, is order 1. So just look at this, $\mathbf{u}' \cdot \nabla' \mathbf{u}'$. Now what is this one, 1/Re, Re will go up, Re $\nabla'^2 \mathbf{u}'$, right, this is the ratio, I just took the ratio of the two terms. Now what is this term, $\mathbf{u}' \cdot \mathbf{u}'$, how big is the term at large scales, 1, $\mathbf{u}'$ is 1, ∇' is 1, right, because velocity, I said, is 1 unit. So this is 1, what about this term at large scale is also 1. $\mathbf{u}'$ is 1, ∇' is 1. So these 2 ratio is 1. So what is left is Re. **"Professor - student conversation ends."**

So for turbulent regime Re is much bigger one 1 and we need to worry about nlin term. For viscous regime, nlin if Reynolds number is order 1, then both are comparable. If it is much less than 1, then just worry about viscous and there are times when you just worry about viscous, in fact you set nlin term to zero 0 okay, they are so so case looks, but we will most focus on the turbulent regime.

I will do some instability analysis where we will keep the viscous term, this term, but we will basically focus mostly on Reynolds number much greater than 1. This is turbulence course, so we need to worry about nlin term okay.

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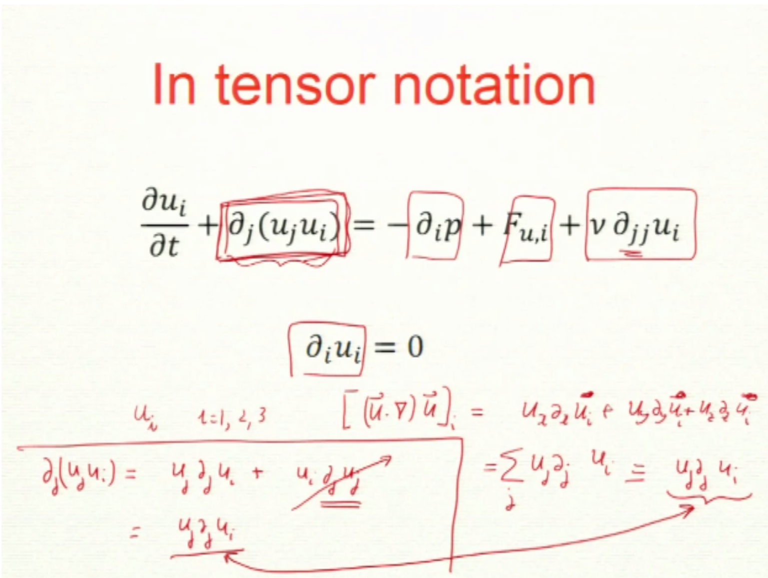

In tensor notation, so this is also very important notation which I will need it for this course. So i is the index. So velocity, you write as **u**, we normally write u_x, u_y, u_z; but instead of that you write u_i and i goes from 1, 2, 3; 1 is x, 2 is y, 3 is z. So, the nonlinear term for incompressible flows will look like this, in fact I will just show you very easily. So, nonlinear term what was it $\mathbf{u} \cdot \nabla \mathbf{u}$ right, so this is **u**, so this $u_x \partial_x \mathbf{u}$ right, so this is, let us write this $u_x \partial_x \mathbf{u} + u_y \partial_y \mathbf{u} + u_z \partial_z \mathbf{u}$.

In terms of tensor notation, I am looking at x component or y component or z component. So, I will put i, so instead of writing vector **u**, I will just replace it by i. This is true for all, either x or y or z, so that is a beauty of the notation. We don't need to write x particularly or y or z, just say i. If you want x, then put x=1, y= 2, z=3 and what is u_x, $u_x \partial_x$, these 3 I can write as $u_j \partial_j$, there is a implicit of the function that is summed, this was first invented by Einstein, at least is attributed to Einstein.

So if I write repeated index, that mean it is summed and this is u_i okay. So it is implicitly assumed that this is summed over j, j will not, this is only function of i, but is not function of j, if this is repeated index that is always summed okay. So we don't write this sum, in fact this is equivalent to $u_j \partial_j u_i$ okay. Now, on incompressible condition, what does that tell you, it tells you that this is incompressible condition is $u_i \partial_i$, $\partial_i u_i = 0$, of course it is assumption that there is a sum. So $\partial_x u_x + \partial_y u_y + \partial_x u_x = 0$.

So, I can use this to simplify this one, in fact write in a more compact notation. So, let us look at now this term okay, this is what I want to discuss. So let us make a box here, $\partial_j (u_j u_i)$, now derivative is acting on the product, so I can use product rule, so what will that be? $u_j \partial_j u_i + u_i \partial_j u_j$, correct? It is the product rule. Of course, you please remember that I had summed more j's. Now what is this term, 0, so this term goes away and this is nothing but $u_j \partial_j u_i$, which is same as what I have it here, these 2 terms are same.

So instead of writing this u dot grad u, it is very convenient many times to write that as a $u_j \partial_j u_i$ okay, especially for conservation laws which I am going to do a bit later, this one is very important notation. The pressure gradient is this one right, along i direction, so this is along i direction and this is the force along i direction and this is viscous term along i direction. Now, I said ∂_j these two j means $\partial_x^2 + \partial_y^2 + \partial_z^2$, which is Laplacian. So this is the tensor notation, which is very useful, especially for conservation laws and some of these derivations okay.

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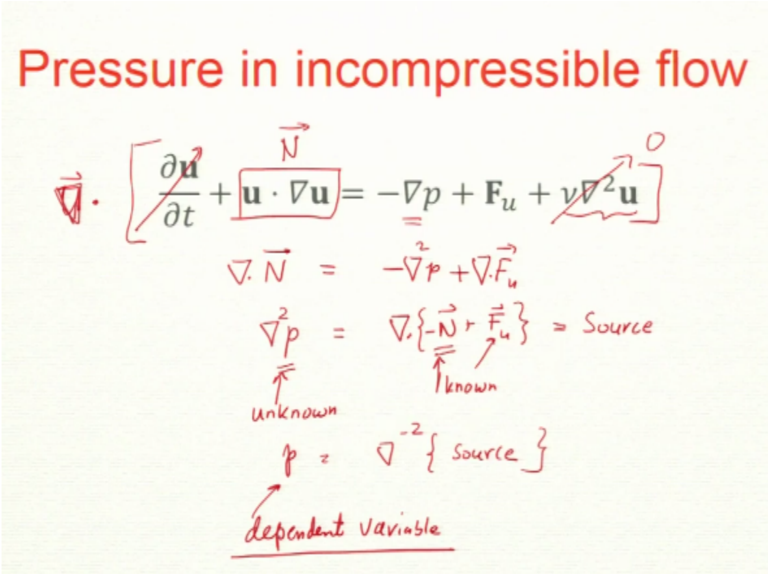

So what about pressure in incompressible flows. It is a very important step. So pressure I can compute from given velocity okay, pressure I can easily compute, how will I do it? So this is my new source equation right. So these are vector equation. So I can take divergence of this equation.

So what happens to the first term 0, right, so this term is gone. What about this term, Laplacian pressure, so this minus Laplacian, it will be the $\nabla^2 p$. I will assume that, well let us keep this $\nabla \cdot \boldsymbol{F}_u$, not let us not assume anything, so this is maybe divergence free. This term you can prove it easily that this term is also 0, both Laplacian and divergence commute okay you can push divergence. So this you can also see that of notation. This term also goes to 0.

Now, this is divergence of the nonlinear term, I will call $\boldsymbol{N}$. So, in fact, this is a notation I will use in later part of the course, these are nonlinear term $\boldsymbol{N}$ vector. So this is what we got okay. So I can rewrite this in ∇p = divergence of uh this is going to write inside $(-\boldsymbol{N}+\boldsymbol{F}_u)$, $\boldsymbol{N}$ is function only of **u**, **u** vector no, so this is known, given the velocity field this one is known to me and force is also known to me, somebody will give me the force. So the RHS is known, this whole thing is I can compute divergence if the fields are known okay.

So, now what is this equation, this is a standard equation in mathematical physics, this is called Poisson equation. So, these unknown, the unknown. So Laplacian of unknown equal to some source term, this we can call it source term, it is a scalar source. So, I can compute pressure given the boundary condition. So my source is known, give me the boundary condition, I can compute force, I can compute the unknown which is p. So p can be computed, in fact is written an $\nabla^{-2}$of the source.

In fact, this is exactly same what we do for electrostatics. So given the charge density, I can compute potential right, so this is what is done in electrostatics or magnetostatics. This is one of the standard equation, which is very useful equation okay, so this is highly used in electrodynamics and also in fluid mechanics. So pressure will be, incompressible fluid, pressure is not an independent variable, pressure is a dependent variable, this is a dependent variable, depends on what, depends on velocity field and force field, is a dependent variable okay. So, we will compute pressure by this trick.

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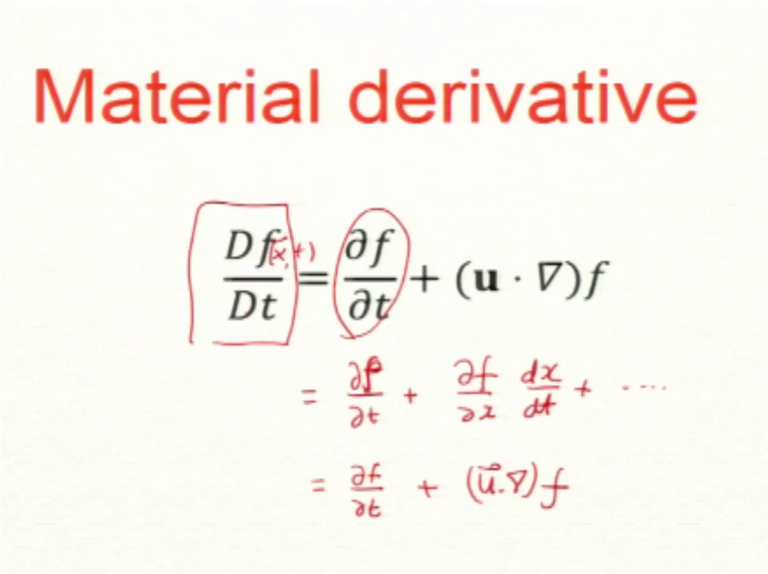

ow, I also need this definition for my future derivation. So this material derivative which I did discuss some while ago, this is called total derivative. It has moved with the fluid, then how f changes is function of time, so this is total derivative $\frac{Df}{Dt}$, is partial derivative plus material derivative right, and the proof is easy. So f is function of **x** and t. So, $\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{dx}{dt}$ and similarly for y … of 3 components.

So this $\frac{dx}{dt}$ is velocity, we are moving the fluid, so this will give you what I got above, no not ρ, this is f. so df this is true for, so this is $\mathbf{u} \cdot \nabla f$. So f could be also vector or f could be scalar. So f could be energy, f could be U^2, f could be in fact magnetic fields, no, or f could be velocity field itself okay. So this ends my first part which is basic equation. So, I will close now, thank you.