Physics of Turbulence
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Lecture - 14
Craya-Herring Basis: Equations of Motion for an Anticlockwise Triad; Examples

So, this is third slide for Craya-Herring. We will do some more examples to get familiar with the ideas. I am going to work out the formulas, derivation of equations using triad which will go anticlockwise. Yesterday I did clockwise, but now my vectors will go anticlockwise and there will be a minus sign will come. We will go through the derivation and you will get more used to the idea of Craya-Herring basis.

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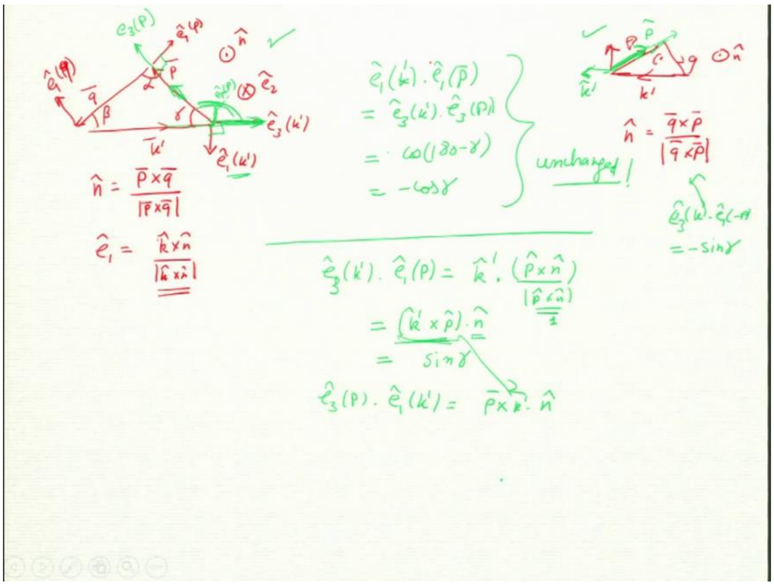

So, I am going to draw the vectors $\mathbf{k}', \mathbf{p}, \mathbf{q}$. They are going anticlockwise now. If you recall, in the last class, I was going clockwise, and one point is that my sequence is $\mathbf{k}', \mathbf{p}, \mathbf{q}$**;** that is my notation. So, I just follow that. Now angles are in front of $\mathbf{k}'$ is α, in front of $\mathbf{q}$ is β, in front of $\mathbf{p}$ is γ.

No, I made a mistake, it will be other way around. In front of $\mathbf{p}$ is β, in front of $\mathbf{q}$ is γ; **k, p, q**, so, α, β, γ. Now here my $\hat{n}$ was $\mathbf{q} \times \mathbf{p}$ **,** if you see the definition, it was yesterdays, by $|\mathbf{q} \times \mathbf{p}|$. So, $\mathbf{q} \times \mathbf{p}$ will be upward. So, $\hat{n}$ was like that. Now I am going to change my definition here.

So, my $\hat{n}$ for this problem, I am going to use $\mathbf{p} \times \mathbf{q}$ and there is a reason for it which I am going to tell you in a minute. So, $\mathbf{p} \times \mathbf{q}$ will be again upward. My $\hat{n}$ is in same direction; it is in fact, as z direction. So, $\hat{n}$ is this way. What is my $\hat{e}_1$ direction? Let's look at for $\mathbf{k}'$, $\hat{e}_1$ was $\hat{\mathrm{k}} \times \hat{n}$ by $|\hat{\mathrm{k}} \times \hat{n}|$and $\hat{k}$ and $\hat{\mathrm{n}}$ are perpendicular to each other. So, this is 1 right. So, $\hat{\mathrm{k}}$ is along $\mathbf{k}$ and $\hat{\mathrm{n}}$ perpendicular. So, $\hat{\mathrm{k}} \times \hat{n}$ is 1; they are unit vector. What is $\hat{\mathrm{k}} \times \hat{n}$? Downward. So, this will go this way. So, this is my $\hat{e}_1(\mathbf{k})$. Now with this definition my $\mathbf{k}'$ is to the right that is my definition last time I said how do I find $\hat{e}_1$? Just go along the vector. So, my $\hat{e}_1$ is always in the same direction either I go clockwise or counter clockwise. If I make that mistake, then I will be changing my $\hat{e}_1$ and I want my $\hat{e}_1$ or $\hat{e}_1$ component to be the same.

So, I will do in one example where it will become clear why should be the same. I want my $\hat{e}_1$ to be same whether for triads where it is going clockwise or counter clockwise, I cannot have two different $\hat{e}_1$ for one and other like my name is my name, you know whether I go to Calcutta or Bombay. So, if I changed my components, then I will be in big trouble with plus minus. So, we don't want to do that, we want to keep $\hat{e}_1$ in the same direction.

So, $\hat{e}_1$ go along the vector to the right, what about $\hat{e}_2$? So, $\hat{e}_3 \times \hat{e}_1$ is $\hat{e}_2$. So, it is downward. So, $\hat{e}_2$ is downward for here and you will find it the same for everything. So, this will be $\hat{e}_1(\mathbf{p})$ and $\hat{e}_1(\mathbf{q})$, a downward, $\hat{e}_1(\mathbf{q})$ ok. Now let's do the dot product of the unit vector. So, let's look at $\hat{e}_1(\mathbf{k}') \cdot \hat{e}_1(\mathbf{p})$, remember this was what was required in my last derivation.

So, when I did, you please look at your notes, you will find that I had $\hat{e}_3 \cdot \hat{e}_1$ and $\hat{e}_1 \cdot \hat{e}_1$, $\hat{e}_2 \cdot \hat{e}_2$ but $\hat{e}_2 \cdot \hat{e}_2$ is always 1. So, that is not a problem, but $\hat{e}_1(\mathbf{k}') \cdot \hat{e}_1(\mathbf{p})$, we will get these things. So, in the table we have this $\hat{e}_3 \cdot \hat{e}_1$ and $\hat{e}_1 \cdot \hat{e}_1$.

So, let us look at how to compute $\hat{e}_1(\mathbf{k}') \cdot \hat{e}_1(\mathbf{p})$. So, $\hat{e}_1(\mathbf{k}')$ is, no sorry, now from here you can see that $\hat{e}_1$ is $\hat{e}_1 \cdot \hat{e}_1$ is same as $\hat{e}_3 \cdot \hat{e}_3$, can I see this or not? So, I rotate my $\hat{e}_1$ by 90 degrees. So, this is 90 degree, this is 90 degree. So, I just left off where basically $\hat{e}_3 \cdot$. So, this is $\hat{e}_3$; now if I want to bring it here. So, $\hat{e}_1(\mathbf{p})$, you bring it here $\hat{e}_1(\mathbf{p})$.

Now, what is the angle between $\hat{e}_1(\mathbf{p})$ and $\hat{e}_1(\mathbf{k}')$; this one and this one; this angle; which is same is this angle they are only offset by 90 degrees both of they are rotated by 90 degrees these are in a plane. So, this is nothing, but this is $\hat{e}_3(\mathbf{k}') \cdot \hat{e}_3(\mathbf{p})$. They just shifted by 90 degrees because everything is all $\hat{e}_1$s have to the right of vector **k**.

So, it is straightforward now, then that is why it is cos of angle between the two vectors, but please remember it is not inner angle, it is an outer angle, you see that this is I have to extend this vector **k,** this one $\hat{e}_3$ and my $\hat{e}_1(\mathbf{p})$ is in that direction, correct, this is $\hat{e}_3(\mathbf{p})$ and this is $\hat{e}_1(\mathbf{k}')$. So, angle is $180^o - \gamma$. So, $\cos(180^o - \gamma)$ which is $-\cos(\gamma)$. You will find the same for counter clockwise as well. So, this is unchanged (Refer Time: 07:27).

But I am doing; $\hat{e}_1$ to the right. So, I am sorry, yes this is I mistake here; $\hat{e}_1(\mathbf{p})$ is going to the right $\hat{e}_1(\mathbf{q})$. Now let us workout $\hat{e}_1(\mathbf{k}') \cdot \hat{e}_1$, now this already done. So, $\hat{e}_3(\mathbf{k}') \cdot \hat{e}_1(\mathbf{p})$, I have this kind of stuff where did it come? So, if you recall at $\mathbf{k}' \cdot \mathbf{u}(-\mathbf{p})$, I have this term $\mathbf{u}(-\mathbf{q})$.

So, I need this. How do I compute this? So, $\hat{e}_3$ is simple, is $\hat{\mathbf{k}}'$ unit vector. Now $\hat{e}_1$, I am going to use my formula which is $\hat{p} \times \hat{n}$ by $|\hat{p} \times \hat{n}|$, but this is one, 90 degree and it is 1.

Now, this one $\hat{k}' \cdot (\hat{p} \times \hat{n})$, I will write this as $\hat{k}' \times (\hat{p} \cdot \hat{n})$. So, this cross product and dot product you can change the order. Now what is $\hat{k}'$ unit vector; is a sin angle; sin of angle between the two and the direction will be again this $\hat{k}'$ going to $\hat{p}$. So, $\hat{k}'$ is going to $\hat{p}$, is upward. So, my screw will move upward, the right-hand screw and that is the same direction as $\hat{n}$.

So, sin of what?

γ ok. Now if you look at the table; well, the table probably you (Refer Time: 09:33) didn't copy; yesterday I had $-\gamma$, $-\sin\gamma$. Now why minus sign was coming? In fact, you can see from here $\hat{e}_3$ was here, I need to compute $\hat{e}_3$ and my $\hat{e}_3$ is here and $\hat{e}(\mathbf{p})$ will be in that direction. So, $\hat{e}_3 \times$, can you see this one? I am drawing this is my $\hat{p}$ and this is; no, I made a mistake this $\hat{k}'$; everybody is able to see that?

Now, what is $\hat{k}' \times \hat{p}$; which direction is it?

"Into the plane"

And so, that is $-\hat{n}$; my $\hat{n}$ does not change direction; the $\hat{n}$ is the same for both this picture as well as that picture. So, for this one $\hat{e}_3(\mathbf{k}') \cdot \hat{e}_1(\mathbf{p})$ is $-\sin\gamma$ ok. So, the minus sign will come for all this, $\mathbf{k}' \cdot \mathbf{u}(-\mathbf{p})$, $\mathbf{k}' \cdot \mathbf{u}(-\mathbf{q})$. So, the first term in the convolution that (Refer Time: 10:57) gets a negative sign when I go counter clockwise.

So, is that idea clear now, you can derive this table. Now Bharath pointed out that there is an asymmetry in the table so, the minus, that is why I started working out. So, minus sign comes because of this, sometimes you get. So, if you go $\hat{k}', \hat{p}$ in that same direction, I get plus, but if I got $\hat{p} \times \hat{k}'$, then what will happen? So, you can see this $\hat{e}_3(\mathbf{p}) \cdot \hat{e}_1(\mathbf{k}')$, what is that? There will be minus sign.

So, it is going to be $\mathbf{p} \times (\mathbf{k}' \cdot \hat{n})$and you can see that this cross product will give $-\hat{n}$; anyway; so, this is origin. So, the table has some nice structure ok.

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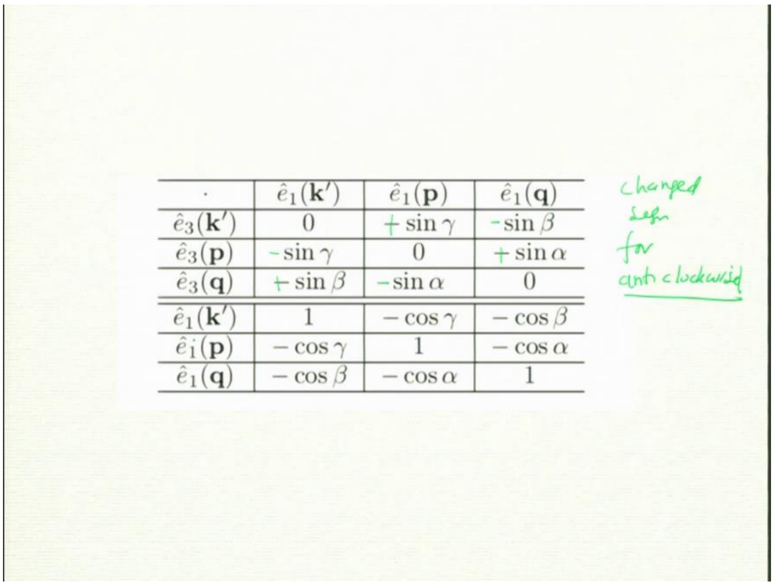

$\cdot$	$\hat{e}_1(\mathbf{k}')$	$\hat{e}_1(\mathbf{p})$	$\hat{e}_1(\mathbf{q})$
$\hat{e}_3(\mathbf{k}')$	0	$+\sin\gamma$	$-\sin\beta$
$\hat{e}_3(\mathbf{p})$	$-\sin\gamma$	0	$+\sin\alpha$
$\hat{e}_3(\mathbf{q})$	$+\sin\beta$	$-\sin\alpha$	0
$\hat{e}_1(\mathbf{k}')$	1	$-\cos\gamma$	$-\cos\beta$
$\hat{e}_1(\mathbf{p})$	$-\cos\gamma$	1	$-\cos\alpha$
$\hat{e}_1(\mathbf{q})$	$-\cos\beta$	$-\cos\alpha$	1

Now, this table was yesterday's table. Now we just change it, the minus becomes plus, plus become minus plus; so, changed sign for anticlockwise ok. Now everything is mechanical, once you have the formula you can compute the equation very easily but we need to keep this in mind, but this one does not change; the second part does not change.

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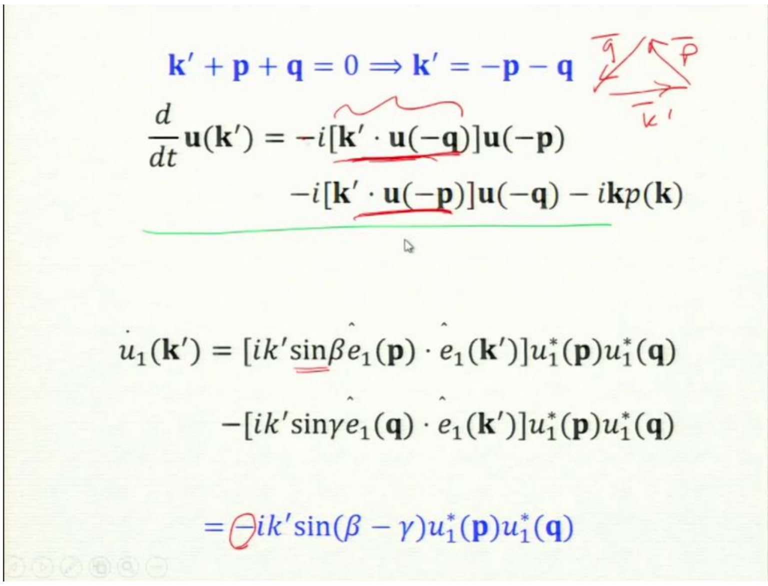

Now, I just go to the equations quickly. So, this is the equations which I did in the last class, this one, till here. So, I will just use the red again ok, but now this one in yesterday's class I had a $-\sin\beta$; the minus here and this was $\sin\beta$, but now it has become $+\sin\beta$. So, this for clockwise. So, this for like this and please remember $\mathbf{k}', \mathbf{p}, \mathbf{q}$ are like that.

So, I just have this sign change and same. So, all these $\hat{e}_3 \cdot \hat{e}_1$, $\hat{e}_3 \cdot \hat{e}_1$ will give a minus sign.

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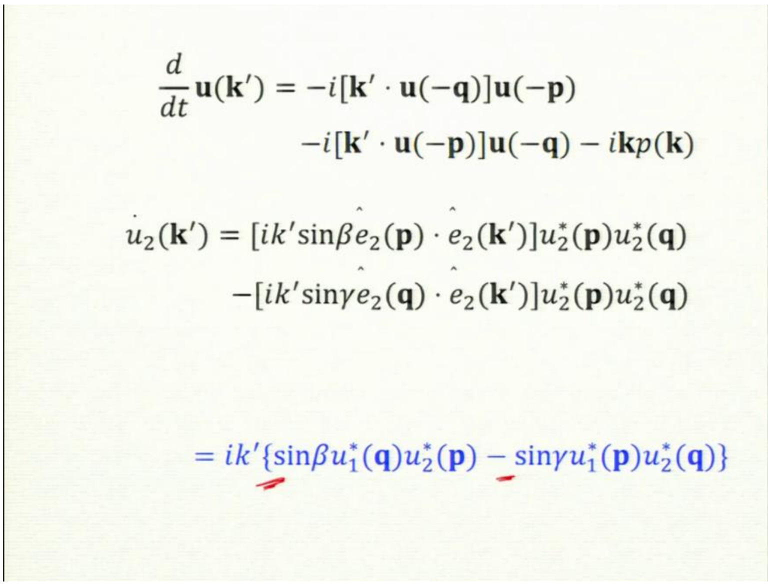

By the way please keep in mind I just want to go back once, this one is not changing, u_1 remains u_1; ok, u_1 should not become $-u_1$ and u_2 should not become $-u_2$; u_2 is into the plane right, it is the direction of $\hat{e}_2$, is downward. Same thing if you see this is a minus sign, this is of by minus sign for clockwise, is that clear to everyone? Now please go through derivation yourself.

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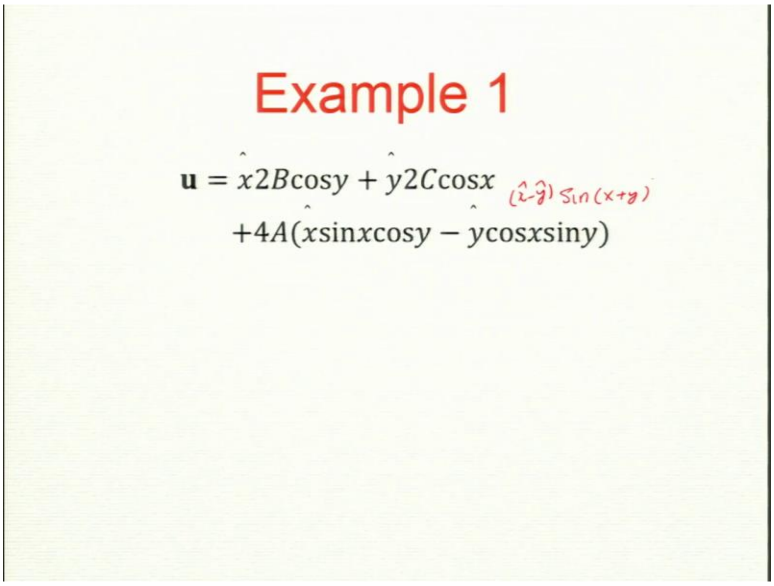

Now, I am going to do this example. So, this example and the previous example is only one difference, this was $\sin(x+y)$. So, in the last example, I had $\sin(x+y)(\hat{x}-\hat{y})$ last time, but now I am changing this. So, this comes from $\sin(x+y)$ and, I had $\sin(x-y)$. So, the number of modes for this problem is more the more modes, do you see that?

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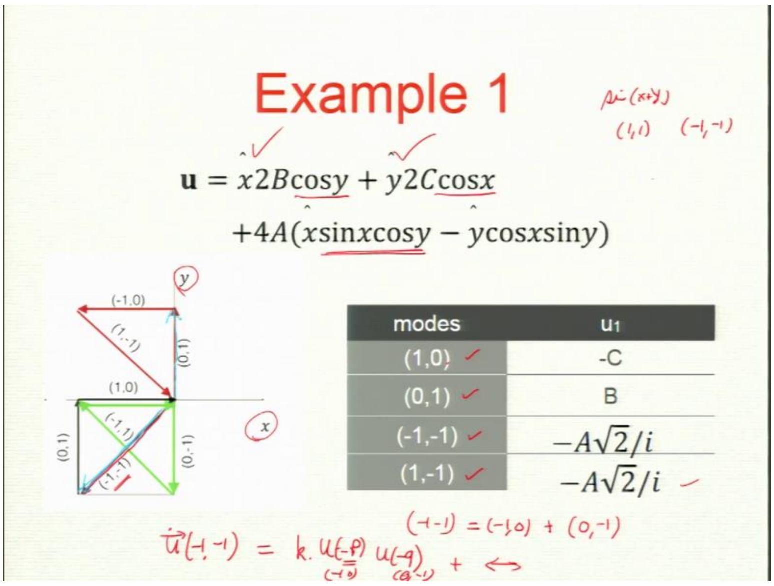

So, last example I had $\sin(x + y)$, it has modes $(1,1)$ and $(-1,-1)$; what about this? (Refer Time: 14:51).

What are the modes which are required to construct $\sin x \cos y$? 4.

$(1,1)$, $(-1,-1)$, $(1,-1)$ and $(-1,1)$, all four are required ok.

So, we did this in the class. So, this is what you just need to apply your Fourier series idea, this not complicated you just have to do it. So, we have done this in the class. So, I am just go and show you the picture, by the way for me, I just copied from my notes. So, I may go bit fast, but you just have to pick up the idea and work them out.

I will give two homework based on this. So, when you work, you have to work them out individually, you can discuss, but you must work them out only then you will learn and the project will be based on these, if you cannot calculate then you cannot do your project ok.

So, my table is here, now I compute u_1, u_2 is 0 because it has no z component, it has x component, y component, no z component. So, there is no u_2, u_2 is into the plane. So, this is my x axis, y axis. So, there is no z axis. So, my u_2 is 0, only u_1. It is a 2D field and the amplitudes are, which I will not work them out, these which we did in the last class; is just identical. Please keep in mind that my vector $\hat{e}_1$ is to the right of vector $\mathbf{k}$; ok, fine. I gave you this, two Python codes, it may help you; (Refer Time: 16:35).

So, now let's look at this triangle. So, I had to look at all the possible triads know which will give me evolution of $(0,1)$ mode, $(1,0)$ mode and $(1,1)$ and minus. So, there are 4 modes sorry there are overall 2;this in complex conjugate, this in complex conjugate this in complex conjugate $(1,1)$, this in complex conjugate; there are 8 modes, but I need to worry about only 4. So, the other one is just complex conjugate of these. So, we do not do it for all 8; you did for half.

Now, I am looking for this evolution of $(-1,-1)$ mode; this one, this is a $(-1,-1)$ mode. So, how many combinations of triangle can come? So, I think there was a question if I have more triads, then I need to just account for all the triads. So, if you see $\dot{\mathbf{u}}(-1,-1)$,right hand side will involve convolution.

So, let us put a vector $\mathbf{u}$. So, this is going be $\mathbf{k} \cdot \mathbf{u}(-\mathbf{p})$. So, I should able to (Refer Time: 18:06) sum them. So that they are 0. So, $(-1,-1)$; I have to construct this equal to what? So, how can you construct $(-1,-1)$? Given these vectors I have no other vectors. So, what are the options? $(-1,0) \oplus (0,-1)$ (Refer Time: 18:26). These are only possibility, I have no other possibility ok. Here I have $(-1,0)$ and $(0,-1)$ and plus flip of these ok, but my formula will take care of it that $\sin(\beta - \gamma)$ takes care of both the combination. So, I have only one choice for this one, is that clear to everyone? For this one, I only have one of them.

What about things like $(0,1)$, this one? How many triads will contribute to this. So, I can erase this one now this is bit of mental jugglery; there is a bit of proof which I have given, but yeah. So, this is a slightly tricky part. So, $(0,1)$, you can see this triangle will contribute, this triangle and there is another triangle here. This is independent triangle; these two triangles, this triangle and this triangle both of them contribute to $(0,1)$. So, there are two triangles; so, I have to add the contribution coming from both the triangles, yes or no?

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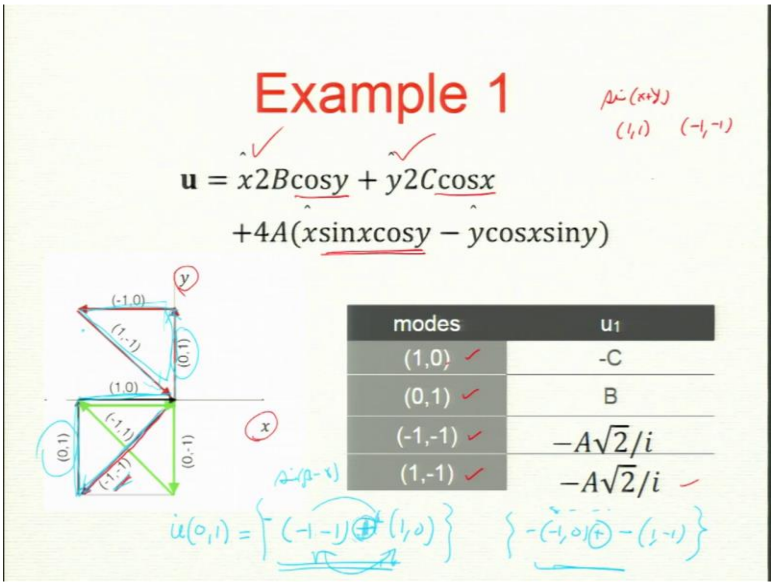

So, if you write $\dot{\mathbf{u}}(0,1)$, then it will involve $(-1,-1)$, coming from here, $\oplus(1,0)$ right. So, if I add these two, I will get $(0,1)$, minus of this; minus of this. So, they add together. So, $\mathbf{k}' = -\mathbf{p} - \mathbf{q}$. So, this becomes $(1,1)$ and this becomes $(-1,0)$. So, that gives you $(0,1)$. This is one triangle and there is another triangle coming from here which will $-(-1,0) \oplus -(1,-1)$.

In fact, I don't do this one, I put this one means non-linear interaction this plus with the circle. So, their non-linear interaction between these modes, is that clear? So, I need to get contribution coming from this one and this one and add them. So, there will be $\sin(\beta - \gamma)$,coming from here and $\sin(\beta - \gamma)$ is coming.

So, there are two different triangles. It turns out the β, γ are kind of common 45, 45, 90 but it could be different. So, you will have two sums; one is coming from one triangle; $\sin(\beta - \gamma)$ and some triangles $\sin(\beta' - \gamma')$ ok.

So, these how we have, if more triads that is how we take care of it, is that clear? But you see this triangle is going, this triangle is going clockwise, and this triangle is going counter clockwise and that is where I have to tell you. So, there you need to worry about the sign, this triangle going clockwise or counter clockwise?

Clockwise, this is for $(0,-1)$ ok.

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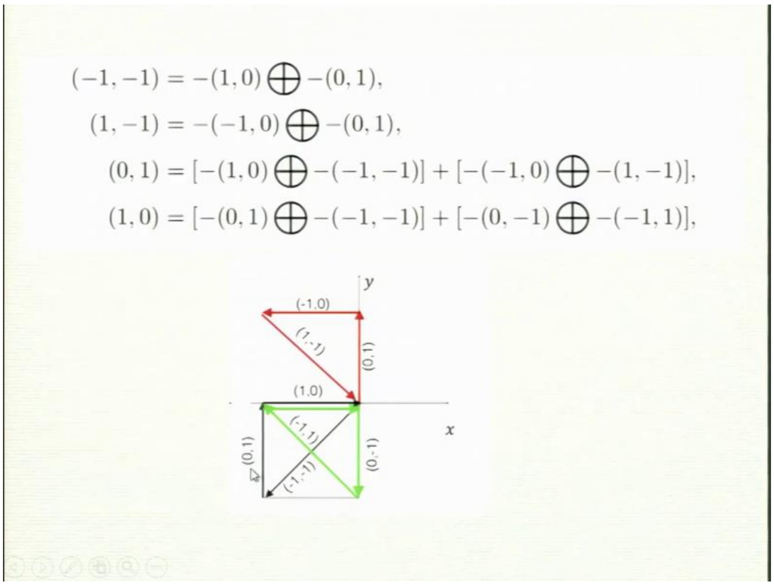

So, let us quickly go through this stuff, $(-1,-1)$, this is only one possibility, $(1,-1)$ in fact, this is only one possibility, but the $(0,1)$ has two possibilities. I just showed you; $(1,0)$ and $(0,1)$ have two possibilities.

Now, when I write down $\dot{u}_1$ of these vectors I will get these.

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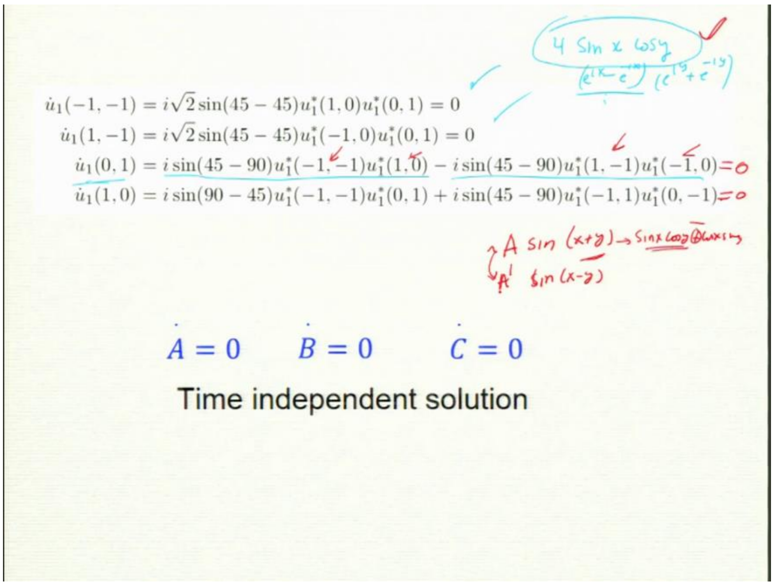

So, this was like last class, in fact they are zeros, but $\dot{u}_1(0,1)$ has one sin of this, other one sin of this. Now since I am using this sin x cos y combination, my amplitudes for $(1,1)$

and $(-1,-1)$ are connected; yes or no? So, I have $\sin x \cos y$. So, you can expand this as $\frac{(e^{ix}-e^{-ix})\,(e^{iy}+e^{-iy})}{4i}$.

So, the relationship between $(1,1)$ mode and $(-1,-1)$ mode. In fact, they are all related with plus minus sign because my structure of the function is this. I mean this is obvious. Now this work you have to do; I mean I simply can't do this on this on the board.

So, this one and this one are related, this one and this one are related. If I do the full thing, it is 0; all are 0 and as a result what happens; $\dot{A}$ is 0 , $\dot{B}$ is 0, $\dot{C}$ is 0. I have more modes but it turns out for this combination my A, B, C are constants. They do not change with time.

So, this is a stationary solution. I should have shown these pictures; I will try to put the movie for this. So, this you see that there are patterns which are constant in time, the patterns which are oscillating in time.

Now, yesterday's pattern, A was this constant, but B and C were changing in time, oscillating. So, that is an oscillatory solution. Present function has more modes, more triads but it becomes simpler. The solution is simple; solution is stationary. Is that clear because it does not change with time. So, pattern is time independent ok.

Now, I can derive this from all these formulas, these are non-linear patterns they are non-linear pattern there is only waves which I cannot change with time, they interact they give energy to each other but they do not change in time. So, there is a coupling, there is a nonlinear interaction, but they do not change with time. We will discuss this energy transfers bit later but you must learn how to derive these equations. So, this is example one.

Now, the example which; well, the homework I would like to give is; make this $\sin\,(x+y)$, $\mathrm{A}'\,\sin\,(x-y)$. So, presently $\sin\,(x+y)$ and $\sin\,(x-y)$ have same amplitude or same or minus amplitude; ok, that I need $\sin x \cos y$.

So, you can see that they must be either same amplitude or of by; is that clear now? (Refer Time: 25:06) These trigonometry formulas I memorize them but the I also require you to memorize. How many of you know this expansion of $\sin\,(x+y)$? Just raise your hand I will not ask you. So, what is it; $\sin x \cos y$ (Refer Time: 26:21) $+ \cos x \sin y$. Here (Refer

Time: 26:26) this one becomes minus ok. So, this is useful. So, if I want sin x cos y then it should be plus right, both of them equal. If I want cos x sin y then I should have minus sign. So, I would like, you to work out this stuff when A and A$'$ are unequal; ok, then see what happens. It is a 2D but there will be something different ok.

Now, let us do another example, example 2 which is this; it is more complicated.

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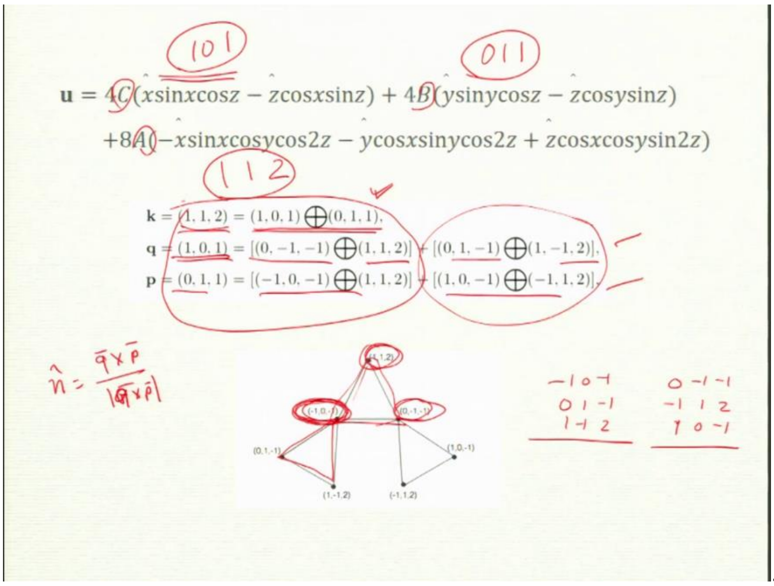

Now I did not do the whole derivation, but I showed that ok, you can compute the nonlinear term and you can derive evolution for A, B, C. $\dot{A}$ was $-\frac{4}{3}$BC, $\dot{B}$ was -2 AB and so. It is there in the book this was done in chapter 3 of the book. Now can you do with Craya-Herring; now all these are worked out in the in chapter 9. So, if I do them what do I get? So, now, my modes what are my modes? Now this is 3D. So, it is not 2D anymore. So, this one gives you what are modes? $(1,0,1)$ and is sin x cos z. So, there are 4 combinations plus minus; this one is $(0,1,1)$ and again 4 combination this one $(1,1,2)$. So, there are 8 modes here, 4 modes here and 4 modes are here. So, it is quite a complex set of modes and in real flows of course, they are tons of them ok. So, there are huge number, but in real flows some matters more, some are like Bill Gates and Steve Jobs and some are small players and some of the big modes are making the patterns. So, if you understand dynamics of big modes and you can understand quite a bit. So, it is important to understand these large scale modes.

So, we will see that bit later. So, I will just make the table. So, I am interested in solving for $\dot{A}, \dot{B}, \dot{C}$, by the way, they are all 8 modes have certain relation. In fact, they are all connected $\pm$ sign. So, if I look at the table. So, we can construct now this is (1,1,2) how will you get? There is only one combination, (1,1,2) can be obtained by (1,0,1) and (0,1,1) nonlinear interaction ok. So, the $\mathbf{k} = \mathbf{p} + \mathbf{q}$. In fact, you have to write just $\mathbf{k}' + \mathbf{p} + \mathbf{q} = \mathbf{0}$ but I am just writing as $\mathbf{k} = \mathbf{p} + \mathbf{q}$.

(1,0,1) can come in two ways this plus this and this plus this. The proof is there in the book ,you will find that in the in the footnote I prove this. There is only two possibilities given this combination and for this again there are two possibilities; only two sums. Now here c do they form of a plane, now this vectors or can I make a single triad or there are two triads; it is obvious in fact, from here.

In fact, this is one triad, this one but these guys form a different triad. In fact, there are the three triads; ok, three of them and how do I make this triangle? This is a triangle. I hope you can see these fonts. So, (1,1,2); so, if I add the add these guys, I should get 0 in a triangle that is a notation I follow.

So, they are interacting; these three guys are interacting; is that ok? (1, 1, 2), (- 1, 0, -1) and this one; this one is obvious, this is the first one but there is another one here. So, (-1 ,0, -1), (-1, 0, -1), (0, 1, -1), (1, -1, 2); if add them, I will get 0 or not? I get 0 here. So, (0, -1, -1), (-1, 1, 2), (1, 0, -1); I get again 0.

So, these are the three triangles for which are connected at (1, 1, 2); by the way this guy (-1, 0, -1) has only two triangles; there is a proof for it ok. I will not reproduce this here, but there is a proof that this mode has only two triangles associated with it. There are not eight triangles there are only two triangles; similarly, here two triangles and these are the triangles in fact, So, you can write down the equation of motion (Refer Time: 31:49) given this, now but can I use this more than one triad so, what should be my $\hat{n}$?

$\hat{n}$, if you recall was $\mathbf{q} \times \mathbf{p}$ by this; for the 2D example just previous one, it turned out that even though they are going clockwise or counter clockwise, I could make $\hat{n}$ along z direction for 2D, I could do it, but can you do this for two triangles in 3D? It can’t be done. For a single triad we can get $\hat{n}$, which is perpendicular to the plane, but a two triad.

So, you see one triangle; well, I cannot really draw one triangle here and other triangle which is like this; well, I mean really they are not triangles, but you can make triangles you can imagine two triangles and $\hat{n}$ will be totally different for the two triangles.

So, you can't have single $\hat{n}$ for the two triads; then idea would be still, I want to apply a similar scheme. So, choose $\hat{n}$ along for one $\hat{n}$ for all of them. So, idea which I propose is choose $\hat{n}$ along z direction, $\hat{z}$ direction ok. It still simplifies; it is less work compared to Cartesian geometry, Craya-Herring is still useful; if you work it out then you will find that it is easier.

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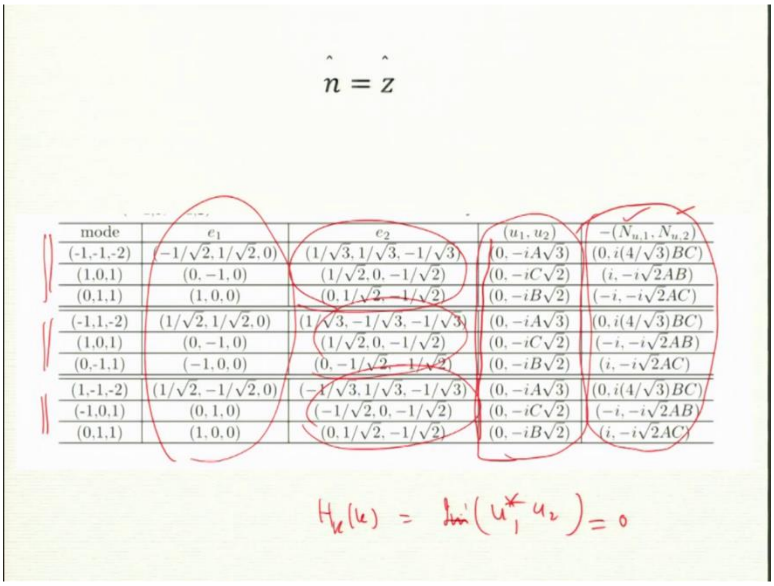

So, $\hat{n}$ is $\hat{z}$ ok. Now, this is the table. So, if you use this some Maple or SymPy (Refer Time: 33:32) and you will find that this is what we get. So, these are $\hat{e}_1$. So, there are three triangles you know; three triangles here and these are $\hat{e}_2$s. Now $\hat{e}_2$s are not $-\hat{z}$; you can see that $\hat{e}_2$s are all changing. In fact, ok, these $\hat{e}_2$s are changing.

Now, u_1, u_2 you see this, this is interesting. There is no u_1, all of them are only u_2 component right, u_1, is 0 for all these. So, is it helical and nonhelical? What is helicity? $H_k(\mathbf{k})$ is imaginary part of $u_1^* u_2$, right. So, it is 0 because u_1 is 0. So, helicity is 0 for all the modes ok.

Now, this is what I use the code. So, I have to use a code then I find that nonlinear term for u_1 and u_2 are these ok.

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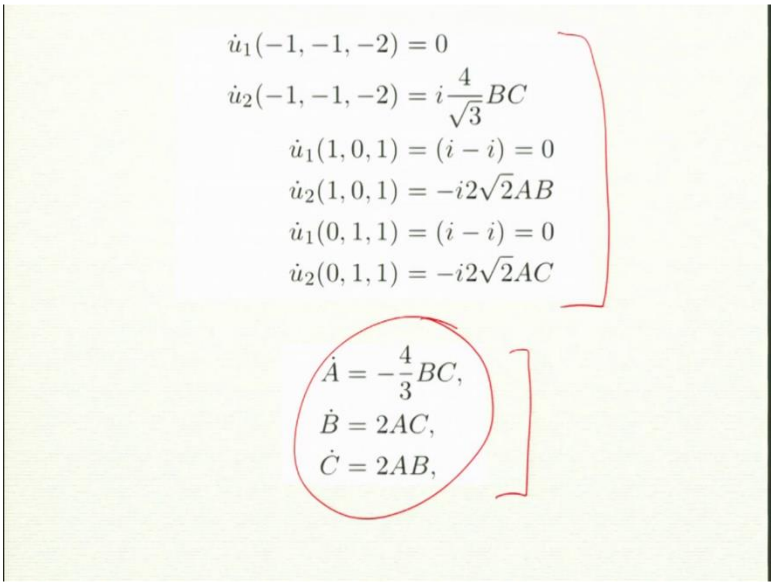

So, now, this work you have to compute them and once you do it then these are the finally, you will get these equations ok; and if you do this then you get these final equations. This needs to be automated; well, I did some automation, but we can use Mathematica, Maple, Python. So, this is what we get, exactly same thing with Cartesian ok. So, I had done this Cartesian first, then I worked out for Craya-Herring.

So, this I am not giving you all the details but one important point is that $\hat{n}$ is not $\mathbf{q} \times \mathbf{p}$ anymore, $\hat{n}$ is chosen because there are more than one triad. If there are a single triad then $\hat{n}$ is fine for even 3D but if there are more than one triad then you need $\hat{n}$ to be one direction and the book has some examples more examples please go through them and then do it patiently, I mean you need one full day or maybe several days to absorb this.

Thank you.