Physics of Turbulence
Prof. Mahendra K. Verma
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 13
Craya-Herring Basis: Equations of motion for a Triad

So, using Craya-Herring we will derive equation of motion for a single triad. This is not for general triad, but a single triad, general means multiple triads. I will do for single triad and there is a reason for it why I am focusing on a single triad.

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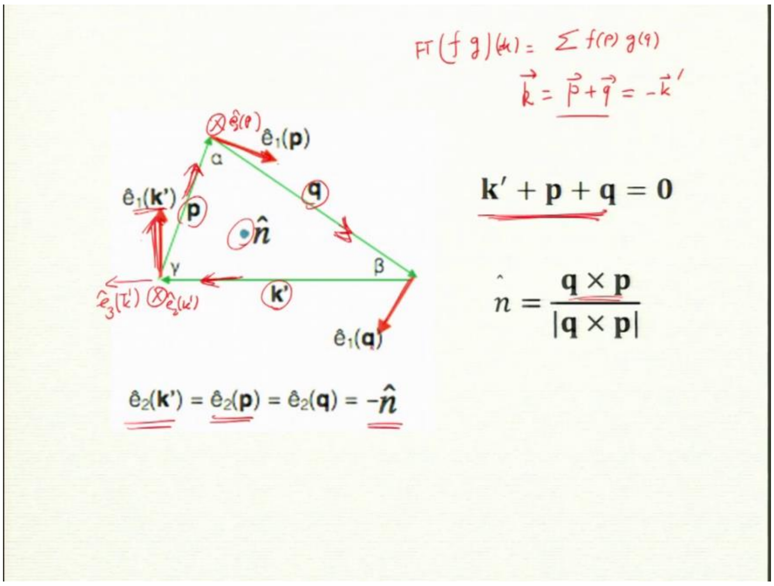

So, this picture again, please focus on this picture. I have three wavenumbers $\mathbf{k'}$**,** $\mathbf{p}$ and $\mathbf{q}$ and my condition is that $\mathbf{k'} + \mathbf{p} + \mathbf{q}$ is 0. So, these are three vectors like this and this, this $\mathbf{k'}$**,** $\mathbf{p}$, $\mathbf{q}$. Is that clear? Now remember I had said for convolution $\mathbf{k} = \mathbf{p} + \mathbf{q}$. So, any product f g, if I do the Fourier transform of this one, I am looking for $\mathbf{k}$**,** this is convolution $f(\mathbf{p})g(\mathbf{q})$ and this also works for cross product, dot product because cross product, dot products involve sum of products.

So, the condition is $\mathbf{k} = \mathbf{p} + \mathbf{q}$. Now it is not symmetric, if I use this definition then $\mathbf{p}$ is equal to $\mathbf{k} - \mathbf{q}$, $\mathbf{q}$ is $\mathbf{k} - \mathbf{p}$. So, they are not symmetric, $\mathbf{k}$ is acts like a boss, so $\mathbf{p} + \mathbf{q}$. So, I want to make it symmetric, all of them have equal footings (Refer Time: 01:49). So, one good idea would be to use this called $-\mathbf{k'}$. So, $\mathbf{k}$ is equal to $-\mathbf{k'}$, then $\mathbf{k'} + \mathbf{p} + \mathbf{q}$ is 0

. Now all of them are in same footings (Refer Time: 02:00). So, $\mathbf{k}'$is $-\mathbf{p}-\mathbf{q}$, **p** is $-\mathbf{k}'-\mathbf{q}$ and **q** is $-\mathbf{k}'-\mathbf{p}$ ok. So, they are all, nobody is boss, they are all equal. And please remember they must add up to 0, vector addition; that means, each component must add up to 0.

Now, this **k, p**, **q** could be 3D vectors, but whatever 3D vectors they may be, **k, p**, **q** will form of plane, because of this condition. So, you can see visually see they will form of plane. So, **k, p**; any two vectors (Refer Time: 02:35) form a plane, but third one, because the third one is minus first minus second, so they will be in the same plane. So, they form a plane, so I am going to use my 2D ideas to solve this problem.

So, basically, we can use 2D physics to workout equation of motion, it simplifies a lot and lot of times we are working with single triad. So, that for, so this idea will work for a single triad, but for other triad if you want to do it, then you have to work only for that particular triad and I will show one example how to deal with it, but this technology may not work. You may have to use $\hat{n}$ vector, so this technology may not work all the time, but if you have single triad this works beautifully ok.

So, let's focus on single triad, we will worry about this technology as and when it is required. So, I will choose $\hat{n}$, so I need $\hat{n}$, I want to use Craya-Herring; that means, I need a $\hat{n}$, without that I can't proceed. So, $\hat{n}$ uses $\mathbf{q} \times \mathbf{p}$. So, **q** is coming this way. So, $\mathbf{q} \times \mathbf{p}$ will be perpendicular to the plane above the plane like this. So, this is $\hat{n}$. Now, you can construct $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ for each **k** vector. So, let's worry about $\mathbf{k}'$ (Refer Time: 04:22).

So, that time you did for a single **k**. Now for single **k**, I said you can choose any **k**. So, that was **k** vector and I was choosing $\hat{n}$, then I could make $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$, but now we have three vectors, I want to choose a $\hat{n}$ which is perpendicular to this plane. So, the advantage will become clear in a minute, but we are doing it for all three of them, it is a single $\hat{n}$ and that is why I think his question also makes relevant. If there are 4 vectors or 5 vectors then this idea will not work ok, but let's not confuse too many things ok, just single triad and we will work with this.

So, now this $\mathbf{k}'$vector. What is $\hat{e}_3$ for this $\mathbf{k}'$ vector? It is in the direction of $\mathbf{k}'$; so, this is my $\hat{e}_3(\mathbf{k}')$. Yes. What about $\hat{e}_1$? $\hat{\mathrm{k}} \times \hat{n}$. So, $\hat{\mathrm{k}} \times \hat{n}$, so my **k** vector is like this, $\hat{\mathrm{k}} \times \hat{n}$. So, which way $\hat{e}_1$ moves, screw moves, right hand screw moves along this direction. So, this is my $\hat{e}_1(\mathbf{k}')$. What about $\hat{e}_2$? $\hat{e}_3 \times \hat{e}_1$. (Refer Time: 05:53).

Into the plane. So, my $\hat{e}_2$ is downward of $\mathbf{k}'$, only $\mathbf{k}'$ ok. So, I have done for $\mathbf{k}'$. What about $\mathbf{p}$? Do the same thing. So, $\hat{\mathrm{p}} \times \hat{n}$, so this $\hat{\mathrm{p}} \times \hat{n}$. So, this will be $\hat{e}_1(\mathbf{p})$. What about $\hat{e}_2(\mathbf{p})$? Into the plane $\hat{e}_2(\mathbf{p})$; so, I have written here; $\hat{e}_2(\mathbf{p})$ is $-\hat{n}$, $\hat{e}_2(\mathbf{k}')$ is $-\hat{n}$ same thing with $\mathbf{q}$. In fact, you do not need to do this algebra all the time or do not keep moving these screws. So, if you just see the direction of vector, right handside. So, the road you know going towards the right, take right. So, if you take right then you will find $\hat{e}_1$, it is easy to see from here. I am going along $\mathbf{k}'$ take right, $\hat{e}_1(\mathbf{k}')$, for $\mathbf{p}$ right and $\mathbf{q}$. So, the $\mathbf{q}$ is going this way, but please keep in mind that they are going clockwise. If it is counterclockwise and there is a problem, there is a something else.

So, these technologies you have to be careful with this. So, this is for clockwise, you arrange your vectors clock 1 2 like this clockwise ok. So, these are tips I am giving you. Now you can work with counterclockwise, but some sign changes happen. So, is that fine? So, if I choose my, I have a triad I choose my $\hat{n}$ which is $\mathbf{q} \times \mathbf{p}$ and then I can construct $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ and I am ready for the next step ok.

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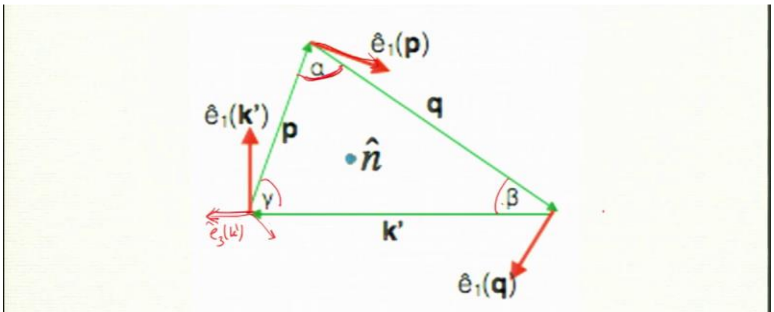

$\cdot$	$\hat{e}_1(\mathbf{k}')$	$\hat{e}_1(\mathbf{p})$	$\hat{e}_1(\mathbf{q})$
$\hat{e}_3(\mathbf{k}')$	0	$-\sin\gamma$	$\sin\beta$
$\hat{e}_3(\mathbf{p})$	$\sin\gamma$	0	$-\sin\alpha$
$\hat{e}_3(\mathbf{q})$	$-\sin\beta$	$\sin\alpha$	0
$\hat{e}_1(\mathbf{k}')$	1	$-\cos\gamma$	$-\cos\beta$
$\hat{e}_1(\mathbf{p})$	$-\cos\gamma$	1	$-\cos\alpha$
$\hat{e}_1(\mathbf{q})$	$-\cos\beta$	$-\cos\alpha$	1

So, I repeat this picture here. Now, I need these dot products, you will need this in the next slide ok. So, you can take the dot, so this $\hat{e}_3$ is in that direction, $\hat{e}_3(\mathbf{k}')$. So, by the way $\hat{e}_3(\mathbf{k}') \cdot \hat{e}_1(\mathbf{k}')$ is 0, they are mutually orthogonal, but I need dot products of $\hat{e}_3(\mathbf{k}')$ with the $\hat{e}_1(\mathbf{p})$; so, this one and $\hat{e}_1(\mathbf{p})$ this one. Now, this table you can just construct. Now this is just bit of work, but I have done this you can use these numbers. So, I need angles, so α, β, γ. Now, this is again a standard notation, in front of $\mathbf{k}'$, the angle is called α; this is front of $\mathbf{k}'$; in front of $\mathbf{p}$, angle is called β and in front of $\mathbf{q}$ angle is called γ.

So, α, β, γ; $\mathbf{k}'$, $\mathbf{p}$, $\mathbf{q}$, so $\mathbf{k}'$ front is α, $\mathbf{p}$ front is β and γ in front of $\mathbf{q}$. Now, this dot product I will not ask you to do, I mean you can do it yourself, but I will not do it here ok. So, $\hat{e}_3 \times \hat{e}_1$, so it is $\hat{e}_3$, $\hat{e}_1$will be in that direction. Now this angle is γ, so you do this. So, you will get $-\sin\gamma$. So, there are quite a few 0s. By the way $\hat{e}_2$, everything is 0, $\hat{e}_2$ is downward. So, $\hat{e}_2 \cdot \hat{e}_3$ is 0, $\hat{e}_2 \cdot \hat{e}_1$ is 0. So, that is a simplification for a triad. So, you will have only $\hat{e}_3 \cdot \hat{e}_1$ and $\hat{e}_1 \cdot \hat{e}_1$. Now, this is algebra. So, let us look at the next slide.

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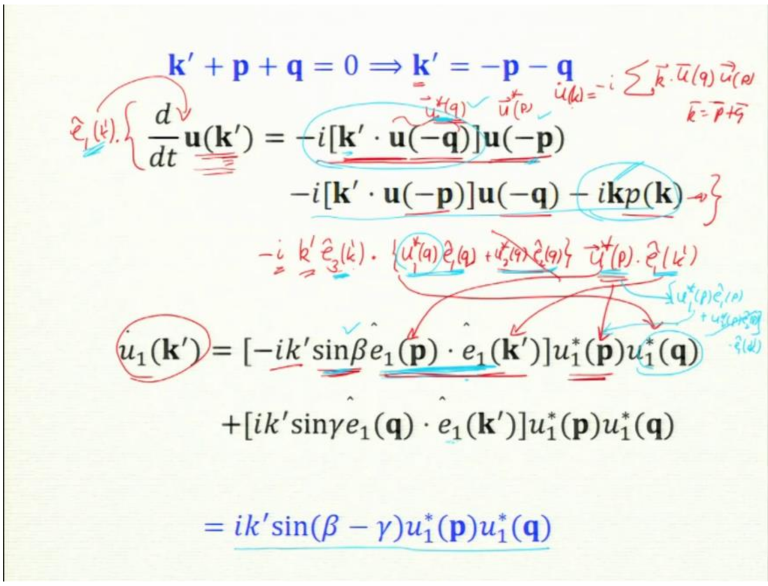

Now, we are ready, getting ready for the deriving equation of motion. So, $\dot{\mathbf{u}}(\mathbf{k})$, there is only one triad. So, this is a convolution know, nonlinear term, you worry about the non-linear term. What is a non-linear term $-i\sum\{\mathbf{k}\cdot\mathbf{u}(\mathbf{q})\}\mathbf{u}(\mathbf{p})$; it is there in chapter 9; that is why you should look chapter 9 for this in more detail. So, you see I have $\mathbf{k}'$. So, this is for $\dot{\mathbf{u}}(\mathbf{k})$, non-linear worry about a non-linear term, but now I put $\dot{\mathbf{u}}(\mathbf{k}')$, so you see $\dot{\mathbf{u}}(\mathbf{k}')$. So, $\mathbf{k} = \mathbf{p}+\mathbf{q}$ for this one. What was $\mathbf{k}$ equal to? It was $\mathbf{p}+\mathbf{q}$. So, what is $\mathbf{k}'$?

$-\mathbf{p}-\mathbf{q}$. So, when I write $\dot{\mathbf{u}}(\mathbf{k}')$, because I want to treat them symmetric. So, I have to write down for $\dot{\mathbf{u}}(\mathbf{k}')$, it will be $-\mathbf{p}-\mathbf{q}$. So, instead of $\mathbf{q}$, I have to put $-\mathbf{q}$, instead of $\mathbf{p}$, I have to put $-\mathbf{p}$ here and this is a pressure gradient. Viscous term, do not worry about it, if we will write; Well, I mean, let us not worry about it right now; is that fine, this equation is ok? So, instead of $\mathbf{k} = \mathbf{p} + \mathbf{q}$, I am using $\mathbf{k}'$ is $-\mathbf{p} - \mathbf{q}$.

Now, take a dot product of this one with $\hat{e}_1$, dot product of the whole equation with $\hat{e}_1(\mathbf{k}')$. Now $\hat{e}_1(\mathbf{k}')$ does not change with time, it changes with $\mathbf{k}'$, but if I choose my $\mathbf{k}$ vectors as well, but even $\mathbf{k}'$ does not change with time. So, this can go inside, it is the important thing to keep in mind. My basis vectors are fixed in time. So, $\mathbf{u}(\mathbf{k}') \cdot \hat{e}_1(\mathbf{k}')$, what is that?

That is u_1 is this guy, dot means $\frac{d}{dt}$. Just focus on the steps, I mean these notes, but I will give the PPT ok, today I will pass it. Just look at the steps. Now I want to compute this. So, I have $-i[\mathbf{k}' \cdot \mathbf{u}(-\mathbf{q})]$.

$\mathbf{k}'$ is not changing with time. In fact, for a given triad, $\mathbf{k}'$ is fixed. So, for example you can think of triad is (0, 1), (1, 0), (1, 1), these are wave numbers. So, wavenumbers don't change with time.

Amplitude of the Fourier thing changes with the time. So, what is the $\mathbf{k}' \cdot \mathbf{u}(-\mathbf{q})$? So, let's look at this one ok. So, just focus on this, k$'$, $\mathbf{k}'$ magnitude, $\hat{e}_3(\mathbf{k}')$. It has only along component along $\hat{e}_3$. Now this $\mathbf{u}(-\mathbf{q})$ has component along $u_1$and u_2, but what about u_2 component, 0, because u_2, I know u_2 is perpendicular. So, that is simplification. When I choose this triad, it goes to 0, before getting into algebra, I am going to write this as $\mathbf{u}^*(\mathbf{q})$ vector, $\mathbf{u}^*(\mathbf{p})$ vector ok; $\mathbf{p}$ is along with this; this I said reality condition. So, this is gone. Now what is $\mathbf{u}^*(\mathbf{q})$ (Refer Time: 13:31).

It has two components $u_1^*(\mathbf{q})\hat{e}_1(\mathbf{q}) + u_2^*(\mathbf{q})\hat{e}_2(\mathbf{q})$ correct so, but this dot product is 0. So, this is gone you are left with $\hat{e}_3(\mathbf{k}') \cdot \hat{e}_1(\mathbf{q})$ and there is an amplitude coming $u_1^*(\mathbf{q})$. Now I can see that from the table know $\hat{e}_3(\mathbf{k}') \cdot \hat{e}_1(\mathbf{q})$ (Refer Time: 14:05) that is why I gave the table and that is true for any triad. So, because α, β, γ, it can be already calculated from a triangle. So, this number in fact, this is sin β.

Now, I will not go back and look in the table. So, $\hat{e}_3(\mathbf{k}') \cdot \hat{e}_1(\mathbf{q})$ (Refer Time: 14:29) is -sin β, so $-i\mathbf{k}'$sin β. So, this is what we got. Now take a dot product of this, $\hat{e}_1(\mathbf{k})$, I want to know, so this is $\mathbf{u}^*(\mathbf{p})$. So, what you got is $\mathbf{u}^*(\mathbf{p}) \cdot \hat{e}_1(\mathbf{k}')$. Now $\mathbf{u}^*(\mathbf{p})$, this one has component along $\hat{e}_1$ and $\hat{e}_2$, but u_2 component goes to 0 anyway.

So, left with $\hat{e}_1(\mathbf{p}) \cdot \hat{e}_1(\mathbf{k}')$, $\hat{e}_1(\mathbf{k}')$ from here, $\hat{e}_1(\mathbf{p})$ will come from here and what is the amplitude will come in front, $\mathbf{u_1}^*(\mathbf{p})$. So, $\mathbf{u_1}^*(\mathbf{p})$ will come from here and $\mathbf{u_1}^*(\mathbf{q})$ has come from here. You can see over, so this is a simple; is everybody following me? So, now, so let's go back. So, I am computing this term. So, $\mathbf{u}(-\mathbf{q})$, I replaced by $\mathbf{u}^*(\mathbf{q})$, number one ok.

So, I first do this one and do this one. So, now I want to compute $\mathbf{u}(\mathbf{k}') \cdot \mathbf{u}^*(\mathbf{q})$. So, $\mathbf{u}^*(\mathbf{q})$ is $u_1^*(\mathbf{q})\hat{e}_1(\mathbf{q}) + u_2^*(\mathbf{q})\hat{e}_2(\mathbf{q})$, I mean this is just a vector. Now I know that $\hat{e}_3 \cdot u_2$ is 0 for any $\hat{e}_2$, all the $\hat{e}_2$ s are downward, $\hat{e}_3$ is in the plane and $\hat{e}_1$ is in the plane. So, that is gone.

So, I get $\hat{e}_3 \cdot \hat{e}_1(\mathbf{q})$, dot product; that is sin β fine, everybody happy, amplitude $u_1^*(\mathbf{q})$ is coming here. No, $\hat{e}_3(\mathbf{k}')$ and $\hat{e}_1(\mathbf{q})$ (Refer Time: 16:29), not saying, that is why you write the argument. (Refer Time: 16:36).

I am telling you these arguments are important. So you have computed this part ok. Now this algebra if you recall computing $\hat{n}$ was quite involved, I had u_x, u_y. So, it is just straight forward, you see now $\mathbf{u}(-\mathbf{p})$, I am taking a dot product, this guy comes dot product with this. So, I get here $\mathbf{u}^*(\mathbf{p}) \cdot \hat{e}_1(\mathbf{k}')$. So, $\mathbf{u}^*(\mathbf{p})$ is again $\mathbf{u_1}(\mathbf{p})$. So, this I write as $u_1^*(\mathbf{p})\hat{e}_1(\mathbf{p}) + u_2^*(\mathbf{p})\hat{e}_2(\mathbf{p})$ dot product with $\hat{e}_1(\mathbf{k}')$. So, $\hat{e}_1(\mathbf{k}')$ dot product with $\hat{e}_2(\mathbf{p})$ is 0. So, this is gone. So, you are left with $\hat{e}_1(\mathbf{k}') \cdot \hat{e}_1(\mathbf{p})$. So, this is what I got here and $u_1(\mathbf{p})$ has come here ok. So, this you just reproducing, just doing it carefully and there are few dot products, but that you take from the table ok.

Now, you can do the same thing for the second term ok, I will show you the result in a minute, what about this term $-i\mathbf{k}p(\mathbf{k}) \cdot \hat{e}_1(\mathbf{k}')$, 0, because this is along $\hat{e}_3(\mathbf{k}')$ and $\hat{e}_1(\mathbf{p})$ is perpendicular to it (Refer Time: 18:03). So, pressure goes away automatically, so you do not need to compute pressure if you do this algebra. So, next step is this one. Now this $\hat{e}_1(\mathbf{p}) \cdot \hat{e}_1(\mathbf{k}')$ is from the table, so that this is the table I computed just read from the table ok.

So, there are two angles product of two sin cosine. So, this is cos γ and this is cos β . So, I get basically sin $(\beta - \gamma)$ ok. Now I am happy with this, I do not know whether you are happy with this. So, all that algebra which we did is just coming from the angles of the triangles and using this, you will see that it is doable in 5 minutes that algebra which is long one can be done in just few minutes. Now this is for u_1; now let us do for u_2, now u_2 will be simpler or complicated more complex.

So, you just take this thing and dot product with $\hat{e}_2$.

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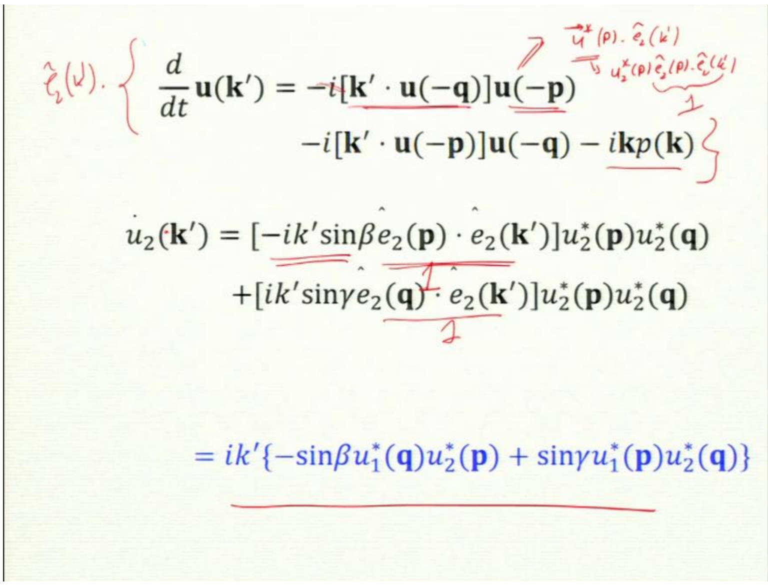

So, I take a dot product to the whole thing again. So, I want to change the color to red. So, dot product, this whole thing by $\hat{e}_2(\mathbf{k}')$ dot. So, this one is pressure, is again gone. $\hat{e}_2(\mathbf{k}')$ is perpendicular to $\hat{e}_3(\mathbf{k}')$. So, this will give you $\dot{u}_2(\mathbf{k}')$. Now, so this you do not need to recompute $\mathbf{k}' \cdot \mathbf{u}(-\mathbf{q})$, is same as before.

Dot product does not change, only thing you need to worry about this $\mathbf{u}(-\mathbf{p}) \cdot \hat{e}_2(\mathbf{k}')$. So, I have few steps written (Refer Time: 20:02). So, instead of product of $\hat{e}_1$, now you have $\hat{e}_2(\mathbf{p}) \cdot \hat{e}_2(\mathbf{k}')$. Remember this was, well this is easy, so this is $\mathbf{u}^*(\mathbf{p}) \cdot \hat{e}_2(\mathbf{k}')$ this what is going to come right from the dot product. Now what is $\mathbf{u}^*(\mathbf{p})$? Now it has u_1 component and u_2 component. What about u_1 component dot u_2 component? 0. So, u_1 and u_2 for all vectors are 0 for all; so, only thing which will give you component is u_2 component. So, $u_2^*(\mathbf{p})\ \hat{e}_2(\mathbf{p}) \cdot \hat{e}_2(\mathbf{k}')$ and what is $\hat{e}_2(\mathbf{p}) \cdot \hat{e}_2(\mathbf{k}')$, $\hat{e}_2$ is always along $-z$ direction. So, it is 1 (Refer Time: 21:01).

So, $\hat{e}_2$; I can close my eyes and just you do not need to read from the table $\hat{e}_2 \cdot \hat{e}_2$ for any pair is 1. So, this becomes 1. So, that is simpler, there is only step which is simpler otherwise. So, this is exactly same as what I did in the previous slide.

Now, this is for the first part now, I can do the second part which is just that. So, these guys are 1; this is also 1. So, I can just put them together ok. So, my $u_2(\mathbf{k}')$ is this. Now of course, you have done (Refer Time: 21:40) only for one of them, you have to do (Refer Time: 21:42) for $\mathbf{u}(\mathbf{p})$ and $\mathbf{u}(\mathbf{q})$. So, there are 6 equations; u_1, u_2 for $\mathbf{k}'$, u_1, u_2 for **p,** and u_1, u_2 for **q** ok.

Now, you please reproduce one of them; well reproduce for u_1 and I have done work, so you can look from the book. So, what is for **p** and what is for **q**. So, we will again go back to our old example.

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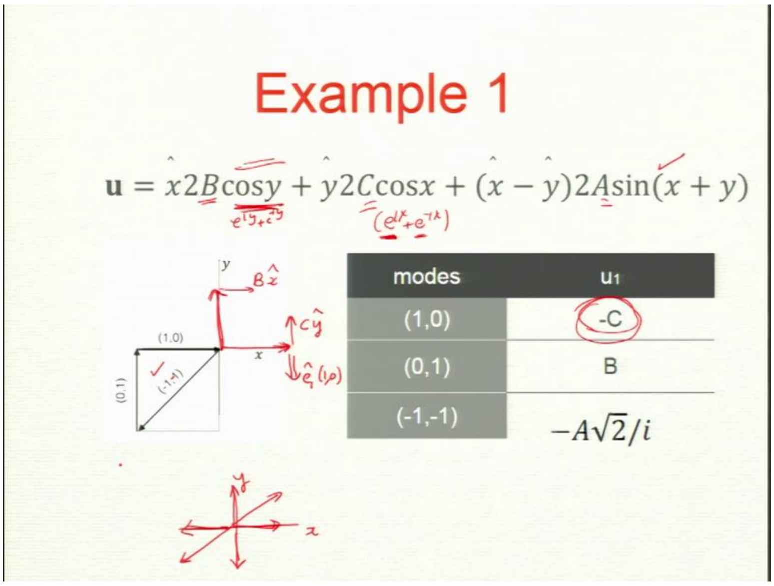

This triangle example, this is that cos y. So, this one I solved with in the last class. So, our idea is to find $\dot{B}$, $\dot{C}$ and $\dot{A}$. Now, they form a triad, the 2 triads. So, some of you, if you are lost then this is another time to regroup you know kind of re-understand. So, this is 2 cos x is $e^{+ix} + e^{-ix}$. I am absorbing two. So, it is k_x is 1, k_y is 0, it is a 2D. So, this is (1,0) and this is a $-(1{,}0)$.

So in fact, if I draw the vector, these are the two vectors. So, this is x axis y axis. So, this one gives you $(0{,}1)$ and $-(1{,}0)$. So, this is $(0{,}1)$ and $-(1{,}0)$. This is another vector, third vector is in that direction now. So, how many triads are there? So, there are interactions among them, so these; I am sorry I forgot this minus. So, these are $(1{,}1)$ here 1, 1. So, they will be like this and they will be like this. So, I want to make the close loop. So, $\mathbf{k} =$

$\mathbf{p} + \mathbf{q}$ is not symmetric. So, you want to make this $\mathbf{k}' + \mathbf{p} + \mathbf{q} = \mathbf{0}$ is close. So, that is why I want.

So, you can see that this is exactly opposite of this and it turns out that equation for this is identical to equation for this (Refer Time: 24:02) because they are complex conjugate of each other for this structure; ok right now, my flow has symmetry, $(1,0)$ and $(0,1)$ are symmetric, either same or complex conjugate. So, it is a simplest non-linear problem. Is that clear to or not clear to everyone? So, this one, this as C and this as C, you see; so, this is now let us find u_1. Now I wrote this u_1, but I need to tell you how to get this.

So, u_1 is along $(1,0)$ know. So, I will draw all again ok. So, $(1,0)$ is this; now $(1,0)$ here. So, it is magnitude it is along y direction right what is the direction for $(1,0)$, my vector is $C\hat{y}$, this I did in the last class ok.

But what is my direction of $\hat{e}_1$ vector. I told one clue, go along the vector turn right. So, this is my direction of $\hat{e}_1$ of $(1,0)$. So, they are opposite. So, my unit vector is downward, my vector is along upward. So, there is a minus sign, what about $(0,1)$? $(0,1)$ is coming from here cos y. So, it is $e^{+iy} + e^{-iy}$. So, my vector is $B\hat{x}$. So, this vector is $B\hat{x}$. So, what is the direction of $\hat{e}_1$ for this vector? (Refer Time: 25:41).

$\hat{x}$, turn right. So, that is why it is positive sign, for case is now we need some more space for $(1,1)$. Now $(1,1)$ vector is like this.

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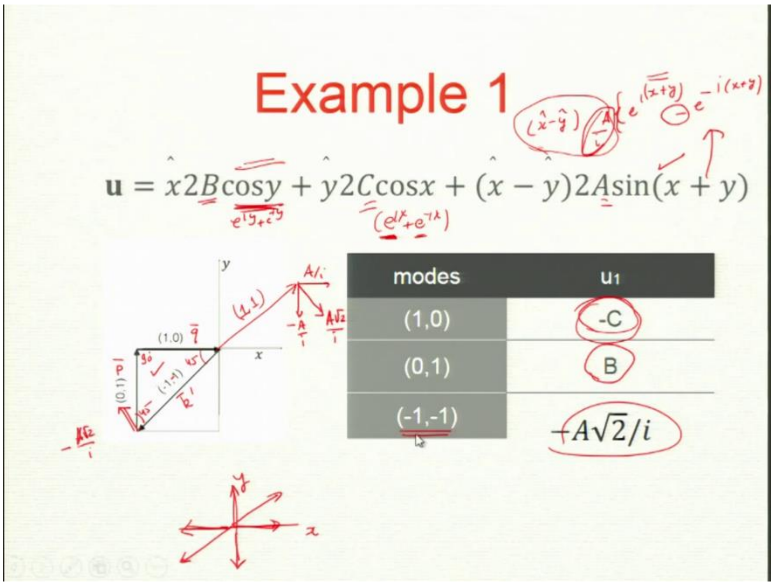

Now, this one here, you have to do more carefully. So, I write this $2A\{e^{+i(x+y)} + e^{-i(x+y)}\}$ by $2i$. So, I am going to pull it out and 2 is absorbed. So, what is, now this is $\hat{x} - \hat{y}$, so I rewrote this in terms of exponential, so $(1,1)$ is here know, this is $(1,1)$ right.

So, Fourier amplitude will be this guy for $(1,1)$. So, what is Fourier amplitude? It has x component as well as y component. So, x component is $\frac{A}{i}$ right $\hat{x}$ this one, fine. y component is negative. So, y component $-\frac{A}{i}$, the resultant vector is (Refer Time: 27:04) in that direction; so, it is $\frac{A\sqrt{2}}{i}$, it is a magnitude $\frac{A\sqrt{2}}{i}$ fine so, these for $(1,1)$ what about $(-1,-1)$ (Refer Time: 27:21). Complex conjugate; so, this will be $-\frac{A\sqrt{2}}{i}$. So, this vector is in that direction. So, the minus sign sitting here you can see that. Is that clear? So, of course, you have to be just careful, but this is straight forward ok, its a labor you know that is why I said you have to do it carefully, so that is why $(-1,-1)$ is $-\frac{A\sqrt{2}}{i}$.

Now, this is my triangle. So, this I call it $\mathbf{k}'$, my notation is I write $\mathbf{k}'$. So, let us just take this from here, I have put them as $\mathbf{k}, \mathbf{p}, \mathbf{q}$ is follow the same thing with notation ok. These I call it $\mathbf{p}$ and this I call it $\mathbf{q}$, so $\mathbf{k}' + \mathbf{p} + \mathbf{q}$ say 0. So, $\mathbf{k}' + \mathbf{p} + \mathbf{q}$ is these are vectors know fine. So, far everybody is with me, so what are the angles α, β, γ. So, this is 90 degree, α is 90 degree, β is 45, γ is 45 ok. So, we have all the ingredients. Now you have to just cook your food, so just get your equation ok. So, α, β, γ and you have in fact, the only one $\hat{e}_1$. So, you do not need to worry about $\hat{e}_2$, you just worry about $\hat{e}_1$ and equation that here $u_1(-1,-1)$.

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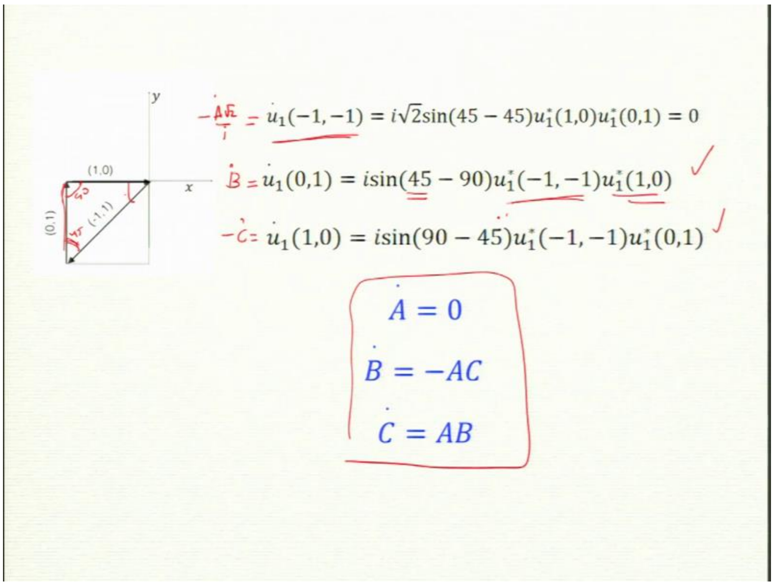

Now you can just read off from here ok. So, this is sin of other two angles. So, for this, actually, there is a rule which I written in the books, so these angle minus this angle ok, so both are 45. So, in fact, $\dot{u}_1(-1,-1)$, this is 0, sin $(45-45)$, 0 and what is $\dot{u}_1(-1,-1)$, I just wrote is $-\frac{\dot{A}\sqrt{2}}{i}$, so dot of this. So, that means Ȧ is 0 ok.

So, it is easy right, I mean once you have this equation then it is a straight forward, you can work out for the other one $\dot{u}_1(0,1)$. Now there are, so this is $(0,1)$, I am talking about. So, the angle is the back angle minus front angle. So, back angle is 45, 45 front angle is 90 and the $\dot{u}_1$, $u_1^*(-1,-1)$, $u_1^*(1,0)$ ok and so this is, you can just do this algebra just copy from the equation and now I know these objects right. These objects are known, so it is $\frac{A\sqrt{2}}{i}$ and C and so on. So, just plug that in and then that gives you this ok. So, you have to just plug in. So, $\dot{u}_1(0,1)$ is what $(0,1)$ is; so, I do not know, go back what is $(0,1)$ can you C right? C or $-$C.

Ḃ and this is $-\dot{C}$ ok. So, you want to be work it out, but you can just do it ok. So, I will not do this algebra. So, this is what we got before and we can do with Craya-Herring in basically one page right. I mean this, once you have that triangle, this triangle put in numbers into it and we can work out more complex examples. So, this for 2D field we can work out for 3D fields. Now I will give some homework, may be slightly difficult one and you can work them out.

Thanks.