Physics of Turbulence
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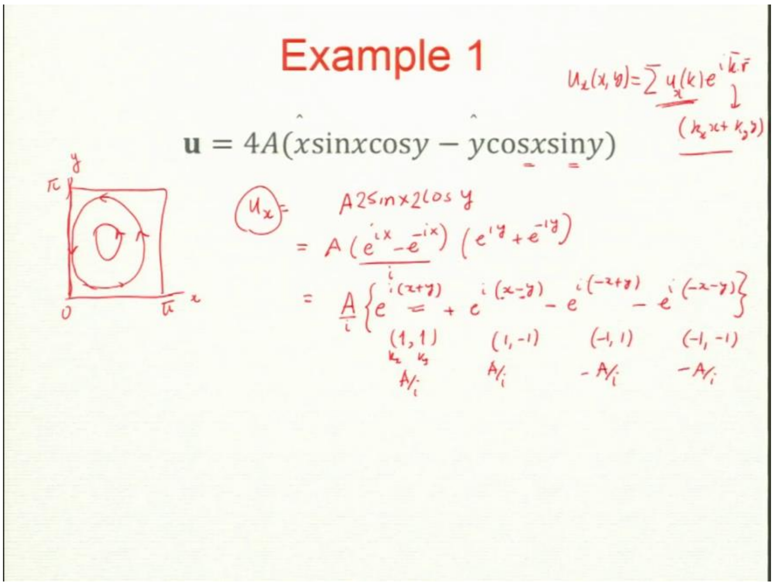
Lecture - 10
Fourier Space Representation Examples

Now, we will work out some examples to just get feel for Fourier Space Representation and the identical examples for we did in real space and there will be some more work in later part where we try to derive some ode's.



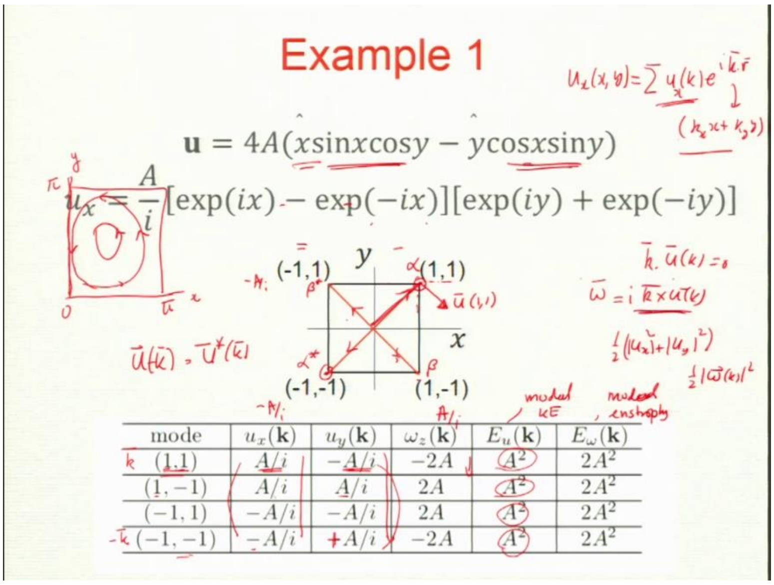
So, Example 1. See the above figure. So, this is what I showed you. So, this is a velocity field in a box of size $\pi \times \pi$. it will some kind of this role. In fact, that is convection role.

Now, how do I put the arrows? So, to get put, so these x axis and y axis. So, at x equal to 0 which is this line my y component I should look at y component. It will here say at y equal to $\pi/2$. The field is going like this counter clockwise and you can draw for various contours or streamlines. These are basically streamlines. Now this was a real space we did this before. We can compute vorticity, you can compute enstrophy and so on.

Now, in Fourier so which are the non-zero Fourier modes for this? How do I do the Fourier transform of this field? It has lot of Fourier modes. In fact, four Fourier modes.

Let us write the first part **u** which is 4 A sin x cos y. Writing $\sin x = \left(e^{ix} - e^{-ix}\right)/2i$ and $\cos y = \left(e^{iy} + e^{-iy}\right)/2$ and comparing this form with the definition of Fourier transform, you can easily see what is kx and ky.

The coefficient are the Fourier modes. When I say Fourier mode means is the amplitude of the Fourier mode.



This is what I have in the table. See the above figure for Fourier modes for respective wave numbers.

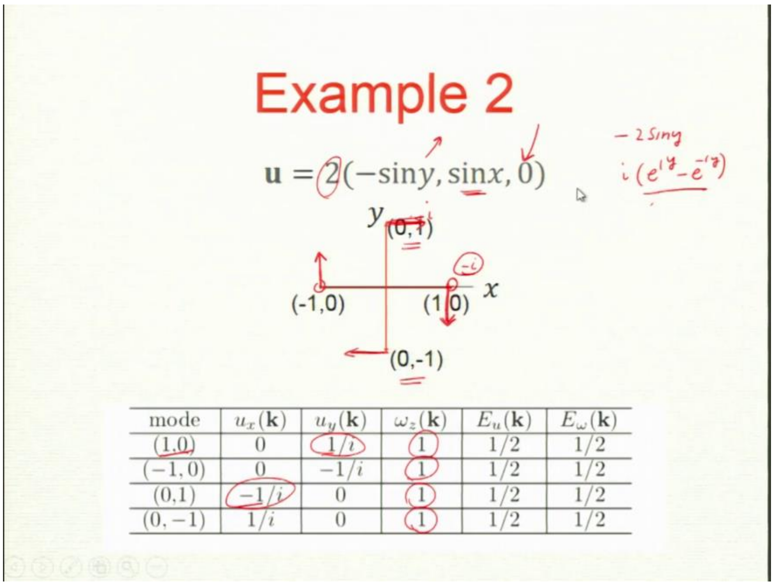
Now check whether do they satisfy the incompressible condition? that means $\boldsymbol{k} \cdot \boldsymbol{u}(\boldsymbol{k})$ must be 0 and is it the case, All Fourier modes satisfy it.

Now, you can compute $\omega_z(\boldsymbol{k})$. it is only z component. What about kinetic energy? We call it modal kinetic energy, energy of a mode. So, instead of saying kinetic energy which is for the whole box, modal kinetic is the mode of energy of a mode. Obtain using $E_u(\boldsymbol{k}) = (1/2)\left(u_x^2(\boldsymbol{k}) + u_y^2(\boldsymbol{k})\right)$. And modal enstrophy using $E_{\mathrm{w}}(\boldsymbol{k}) = (1/2)\omega_z^2$

Now, what about the direction of the velocity field. So, you can easily you can also see this is nice. So, this velocity field has ux component and uy component now these are not in real space, but fine I can also have a Fourier space. (A/i) and (-A/i) for k=(1, 1). uy is the same length same complex length, but a same length with a negative sign.

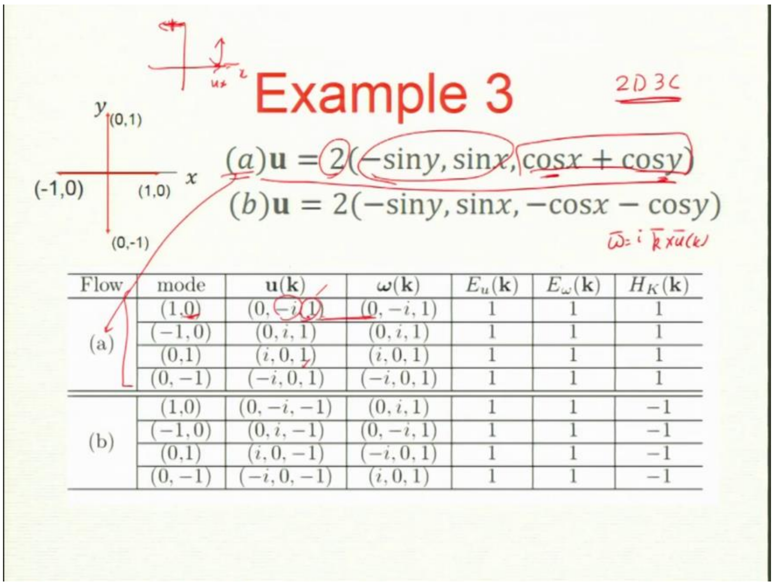
Similarly you can draw for other wave number. Now, reality condition, do they satisfy reality condition? What is reality condition? $\boldsymbol{u}(-\boldsymbol{k}) = \boldsymbol{u}^*(\boldsymbol{k})$.

this table must satisfy these properties, ok.



Now example 2. See the above figure. So, this is again familiar one cyclone anticyclones part of them. So, what are the Fourier modes? So, first identify the Fourier way number. The idea is very similar to last example.

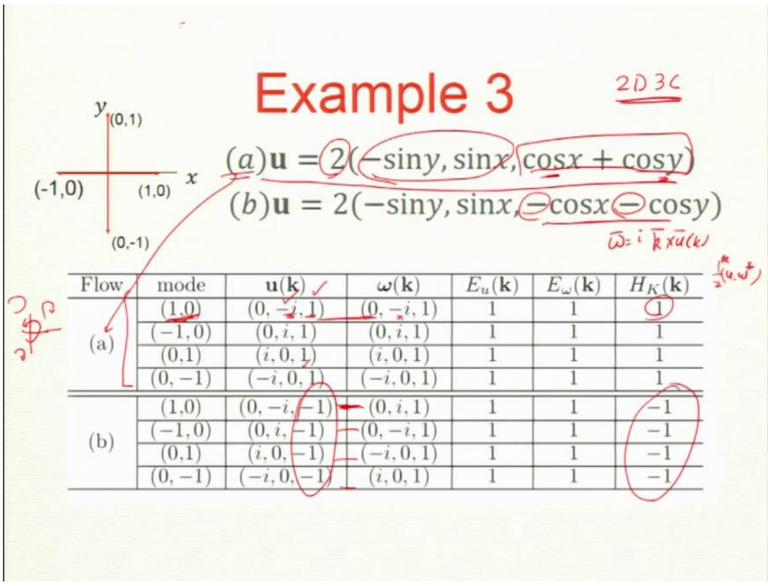
Now, we will do slightly more work, now we will do 3D field. See the below example.



This 3D where the field depends only on x and y. So, it is a very strange kind of Field. This field is called 2D-3C, two-dimensional but three components. So, if I go up in that z direction, then I get basically it is not function of z. So, in same configuration, but it has three components. So, these are third component. So, it is not only 2D field is a 2D, but three component field.

So, this part is exactly same as before this in fact, if you have written is the exactly same, but I added this new uz component.

Note that the filed has maximally helical, it is 1. For Fourier modes, see the figure.



These are simple examples.

Thank you.