Physics of Turbulence
Prof. Mahendra K. Verma
Department of Physics
Indian Institute of Technology – Kanpur

Lecture - 1
The Turbulence Problem

This course is on physics of turbulence. We will discuss the physics aspects; I will describe a bit of them in today's class. So let me just first describe what is the turbulence problem at least from the course's perspective.

So first, why study turbulence. So, I am going to show you that turbulence is a very important process for lot of applications including what happens in day-to-day life.

Example: Room heater: Let us take this example of room heater. So, we have this heater in the left and you are here in the cold winter night. You come close to the heater to warm yourself. Now, let us say the distance between you and the heater is 1 m. Now, we want to solve this problem; how long will it take for the heat to reach you. Well, one obvious way to solve this problem is to start with the diffusion equation;

$$\frac{\partial \theta}{\partial t} = \kappa \nabla^2 \theta$$

Here, θ is temperature and κ is the thermal diffusion coefficient. Heat diffuses by this κ. So, one important point to remember is that this equation has exact solution. We can solve this exactly by Fourier series; in fact Fourier gave the first solution for this equation, but we will not solve it exactly in today's discussion. We will just do an order of magnitude estimate. I am only interested in *approximate* time, not exact time.

So, I can just take this first part which has magnitude $\frac{\theta}{T}$, T is the time scale . So, T is what I am looking for; how long will it take for the heat to travel 1 m. Now, I will do an order of magnitude estimate for the second term also. So, dimensionally, θ will go at the top, ∇ is $1/L$, so the second term will be $\frac{\kappa\theta}{L^2}$. Thus, θ cancels out and we get basically an equation for T;

$$\frac{1}{T} = \frac{\kappa}{L^2}$$

We need this parameter κ, I know L, I am looking for T, but what is κ? κ can be looked up in your book on statistical physics. κ for air is $10^{-5}m^2s^{-1}$. We can also derive it from kinetic theory, but we will not get too much into that aspect. So, we have

$$T = \frac{L^2}{\kappa}.$$

Now, plugging in the values, we get

$$T = 10^5 s.$$

This value is very large; I can't wait for $10^5 s$ to warm myself up! So there is some problem with our approach.

Now this is only one example, but we can also think of diffusion of perfume; how long will it take to diffuse 1 m. We can proceed with an identical analysis, except kappa will be replaced by diffusion coefficient. So what has gone wrong, so what has gone wrong here is that I am missing one important term in this equation . So the term which is missing here is the fluid property of air. So in this room, right now molecular diffusion is not dominant factor for transmission of heat. What is important is this new term $\mathbf{u} \cdot \nabla\theta$. $\mathbf{u}$ is the local velocity of air. This is the realistic version; the diffusion equation is not the correct description. Now, we would like to see how big is this term compared to $\kappa\nabla^2\theta$. So I can again do order of magnitude estimate;

$$\frac{\mathbf{u} \cdot \nabla\theta}{\kappa\nabla^2\theta} \approx \frac{UL}{\kappa}.$$

Note that this is not exact calculation, but these are often done in fluid mechanics. Now what is U for air in this room? I will estimate from cigarette smoke; you got to just look at with some focus, you will find that the smoke will probably travel 10 cm in 1 s , but it could be faster too, I mean this is only a magnitude, order of magnitude.

So, U is 0.1 *m/s* (I am using only MKS right now or SI unit), L is same -- 1 m, and κ is $10^{-5}m^2s^{-1}$, so

$$\frac{\mathbf{u} \cdot \nabla\theta}{\kappa\nabla^2\theta} \approx \frac{UL}{\kappa} \approx \frac{0.1 \times 1}{10^{-5}} \approx 10^4.$$

Thus, the term is bigger than diffusion term. In fact, this number we will also discuss later it is for Peclet number, Peclet number, but we will discuss that later. So what will be the time now? Diffusion time is too small, so I can ignore this term; we get an estimate of time as

$$\frac{\partial \theta}{\partial t} \approx -\mathbf{u} \cdot \nabla \theta \implies \frac{\theta}{T} \sim \frac{\theta U}{L}$$

$$T \approx \frac{L}{U} = \frac{1}{0.1} \approx 10s$$

In fact, you can easily see what the answer should be. So, I should equate $\frac{\partial \theta}{\partial t}$ to $\mathbf{u} \cdot \nabla \theta$. So using the same order of magnitude estimate, *T* will be *L/U* which is 10 sec, so this is correct estimate. So what has helped the diffusion of heat is the flow term. Now we will discuss later, I am using only LS, capital L which is large length scale 1 m, but if you look at air, which I am going to show you one slide bit later, that there structures within structures.

So we are looking at only one scale which is the large scale; and at that scale we get this equation. Now so what do we learn from this example. So fluid flow is important to study , we can't simply use thermodynamics blindly and this process is not only in air, but also in stars, planets

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So I just want to summarize the lessons learnt from this example. The flow term is very important and this is what we will focus in this course. So, the dust particle in the room is advected by **u**; it is not by diffusion, not the Brownian motion. It is contrary to what is taught in textbooks. Brown's experiment, in fact, was done in still water where **u** is zero or is very close to zero. So, the Peclet number for the experiment for Brown would be close to zero. So the nonlinear term can be dropped. Brown could drop the nonlinear term, he didn't need to worry about it , but in many experiments, real-life scenarios or even astrophysical scenarios $\mathbf{u} \cdot \nabla \theta$ is important so it can be ignored.

One important point which we will discuss a bit later is that this mixing with diffusion term $\kappa \nabla^2 \theta$ was very slow, but what made it more efficient or what made it faster -- this mixing -- was the $\mathbf{u} \cdot \nabla \theta$ term and that, in fact, is the turbulence. The Peclet number, nonlinear by viscous diffusion term, it is like Reynolds number . So, Reynolds number is for the velocity field in Navier-Stokes equations, but Peclet number is like Reynolds number for scalar quantity. If that term is bigger, the Peclet number is bigger than 1, much bigger than 1, then the flow is turbulent. So the nonlinear term, which is much bigger than the diffusion term, is improving mixing, right now mixing of heat.

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There are other examples; I will give only a couple, I mean the turbulence is everywhere, this is quite true. In atmospheres of planets and stars you will find most of the time the Reynolds number, or the nonlinear term compared to diffusion term, is very large. So weather, climate, diffusion of pollution, the nonlinear terms play a big role. But the interior, we go inside, inside the earth there is a part where Reynolds number is quite large and magnetic field is being generated in that region; its called outer core and there too things are turbulent. Industrial applications, from the huge applications, in fact most of these appliances like air conditioner there will be turbulence inside. When the steel is being formed in furnaces, flow is fairly turbulent .

So for all of astrophysics, galactic and planetary environment we need to study turbulence. It turns out that it is very difficult, I mean some of it to be we don't know how to solve it, there only order of magnitude estimates and, for example, the magnetic field generation, which we will cover some of it in the class, in the earth or in sun, remains an unsolved problem. We understand some of it, but not all of it. One simple reason is that linear equations have exact solutions, but nonlinear equations do not have exact solution, most of the time, some special ones have, but most of them don't have exact solutions and there is no generic property. In Laplace equation, there is certain property, but not for nonlinear equations.

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So, now I hope you are motivated enough to see the important problem. So, we will see what is the problem .

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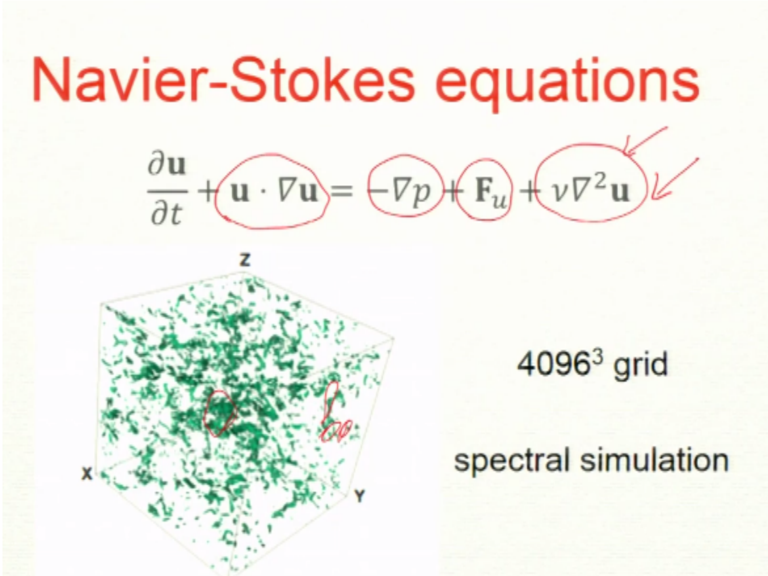

So in this example, I will just say that we are looking for this Navier-Stokes equation . So I am just illustrating the Navier-Stokes equation. $\mathbf{u} \cdot \nabla \mathbf{u}$, is the nonlinear term, similar to $\mathbf{u} \cdot \nabla \theta$. This special gradient -- I am going to describe it a bit later, but I just want to give you some idea. $\boldsymbol{F_u}$ is the forcing term and $\nu \nabla^2 \mathbf{u}$ is viscous term . So, we will discuss in more detail, but I just want to tell what the turbulence problem is. So, these are various terms and this is the picture of vorticity magnitude (see above figure).

So, what is vorticity? It is the curl of $\mathbf{u}$. So, this is of important quantity which we will discuss in this course. Here, we can see the structures, probably not very clearly, but there are bigger structures here and there are smaller structures, then the still smaller structures, then smaller smaller structures. So, the structures of different scales. So, this curl of $\mathbf{u}$, so it is not homogenous, normally we assume that density is homogenous, but this is not homogenous.

So, this is a multi-scale problem, and we will discuss it a bit later, the dissipation occurs at a smaller scale. So, something happens at very small scale. Small scale is important; specially for singular problems. So length going to 0 you need to consider properly. This viscous term is important, this plays a very important role, but some phenomena happens at large scale, some happens small scale, so there is a lot of inter-scale interactions.

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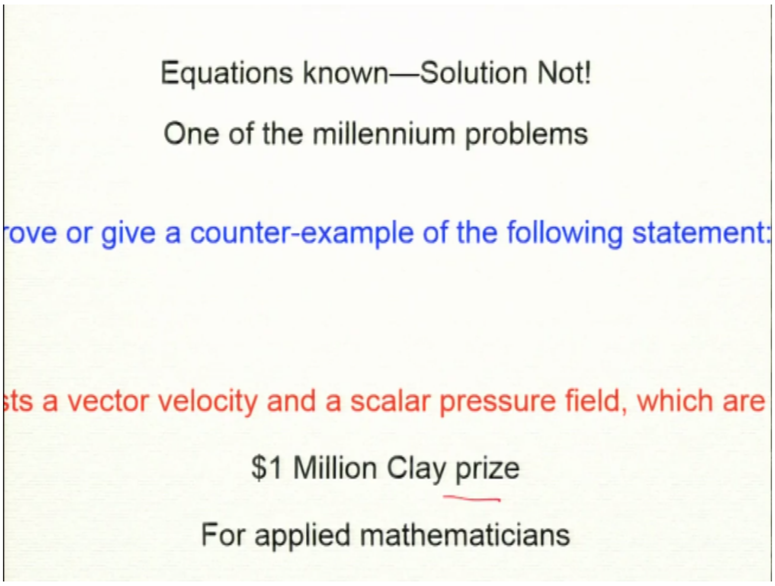

So in this problem, which we will study in this course, the equation is known but solution is not. In fact, I will just state one of the problems which can give you million dollars if you solve it is called one of the millennium problems. There are 7 millennium problems and this is for Navier-Stokes or in fact for all your problems. So prove or give counter-example of the following statement; given the velocity field and scalar field, there exists a solution for Navier-Stokes equation. We don't even know whether there exists a mathematical solution. Finding it is different matter, but one needs to prove the existence of a solution. By the way, not every equation has a solution. There are equations for which in ODE, ordinary differential equations, we know their condition for having a solution, whereas some equations do not have solutions. So, it is not even clear whether Navier-Stokes equations has a solution or not.

So this one million dollar prize is for proving there exists a solution. You don't need to find the solution, but just prove that there is a solution or if it doesn't exist, then you show that it doesn't exist. So this is the statement. Of course, this is what applied mathematicians are working on and not my field. We don't even know whether there exist a solution and this problem is basically at small scale, this is a similar problem. ν tending to 0 is a big issue .

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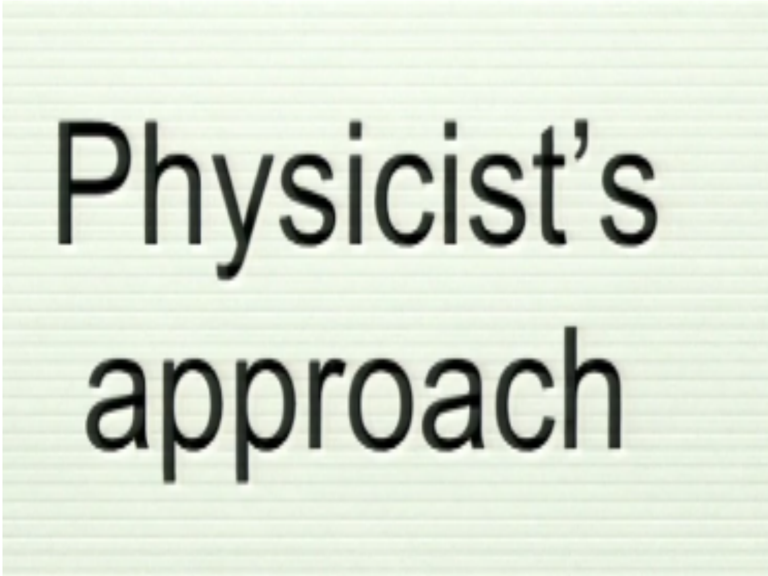

So, I will go to the next slide which is physicist's approach, which is what we will do in this course.

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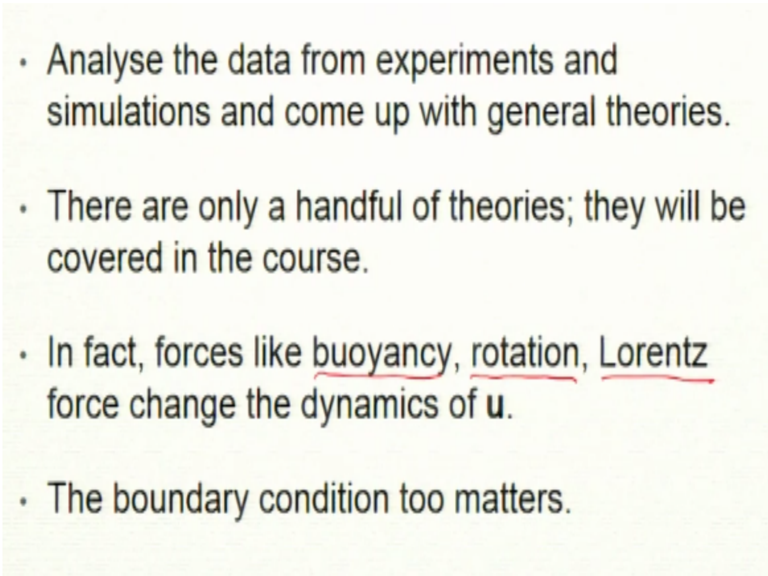

So basically here in physics we will, like other parts of physics, analyze the data from experiments and simulations and come up with general theories. So I will construct theory from first principles, that is my objective. There are some models that are derived from first principle; we build the model from the data fit.

So these are theories. So as I said there are only handful of theories, Kolmogorov theory is one of them. In 2-D turbulence, we have some theories, but they are very few and we will cover some of these in this course.

So we want to consider a law which will work for all turbulent systems . In fact, so that is the dream, in fact that is what people want to state, the universal law of turbulence. So it turns out from my experience for last many years is that there is no universal theory of turbulence. If you put forces, then the law changes quite a lot. There are some basic tools, but law changes dramatically. So for example for hydrodynamic turbulence without any external force so that $\boldsymbol{F_u}$ term is zero, then there is a law. Kolmogorov theory is basically $\boldsymbol{F_u} = 0$ in the initial range, but I do another system like heat the fluid from the bottom, then we have buoyancy. Then the law changes dramatically, it is not the same law. If I have put rotation, then the law is also very different. If I put magnetic field, then in fact we get a different behaviour. So it looks like this problem, in fact lot of problems in nature, the equation may be the same or equation may have slight difference, like Schrodinger equation. Schrodinger equation is same. Of course, the potential term is different for different system, but if I put different potential, I get a differential solution for different system. So, there is no universal theory even in quantum mechanics. You have one equation which works for many systems. Similarly, here we have one form of equation where $\boldsymbol{F_u}$ is again changed. Thus, there is no universal theory for turbulence. We have good common tools, but not universal theory, one theory for everything .

Also, boundary condition. So we will see some examples like walls, if you put walls on the side, then physics is very different. So like convection, we heat from the bottom, walls have very important effect. This is like channel flow, where wall has significant effect .

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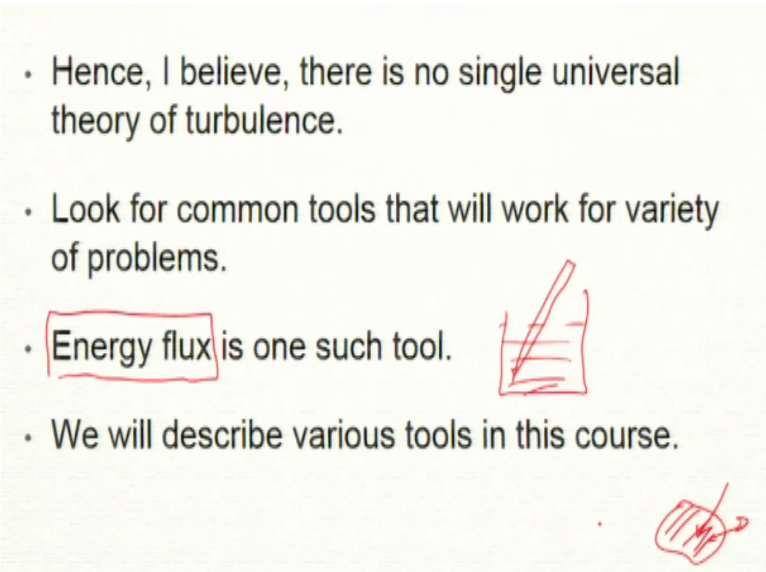


So as I said there is no single universal theory of turbulence. However, we do not know one single theory, does it mean that we don't even attempt this problem? No, we should attempt, but we should understand different aspects of the different phenomena or different forces. So this is what we will do in this course. We will develop general tools, but then we will apply this tool to different systems and try to derive properties of those systems.

I will tell you some very nice tools which will employ for studying all these systems. We will not cover in big detail, but we will cover in some detail, the basic aspect. So we look for common tools and apply them to variety of problems. So this is what I like is called energy flux. So one example which is given is that you take a bucket of water you know water and stir this water by a rod. So I supply energy at large scale, I am forcing it, so I am supplying energy at large scale, bucket scale and this water will flow down, this energy will flow down the small scale.

So there is a transfer of energy from large scale to small scale. So this is called energy flux in turbulence language or in this field. It is not like flux of electromagnetic radiation, so there is energy transferred per unit area per unit time; you know; so that is very different thing. The energy flux is energy flow across scales and there is a way to quantify it and that is what I will show in this course, how to quantify this energy transfer. In fact, I may have this next line, so we will describe this is one tool, but we will also have other tools.

One example which I like to give so before I go. So, I like this example of finance model in this connection. What is finance model, so this is a money supply by the central government you know, so this basically they distribute budget. So they get money from elsewhere, but they basically supply money to, so what they supply they give to a state, state to districts, district to blocks, village, so it flows from large scale, which is the the central government scale, to small scales.

So this is what is a multi-scale problem, it is not a single scale problem and it is not flowing in real space, it is really flowing across scales. So you should not visualize this in real space. If you see visual real space, you cannot get this flow. So this is in fact very natural in Fourier space, Fourier decomposes different scales, which what we will do. This is the analogy: the money flow is like energy flux, but I also want to make a remark then in lot of physics like the thermodynamic is there a flow of energy across scales or what are the scales in thermodynamics.

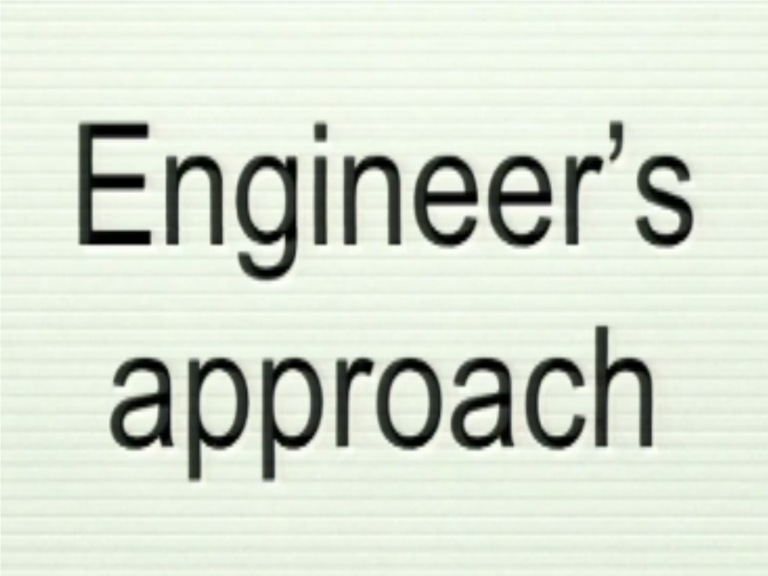
“Path mean area, so scales are not large scale.” “No, thermodynamics is basically hinging on molecular interparticle, well in collision length, so it is, the temperature is defined as average quantity but it is only one scale length scale which is mean free path length.”

So, there an energy cascade or is there energy flow, so if you look at one region of here, in real space, but there are many ways to define , so just look at molecules here, the number of molecules are approximately same as molecules at different times let us focus only one region, let us think of region like this.

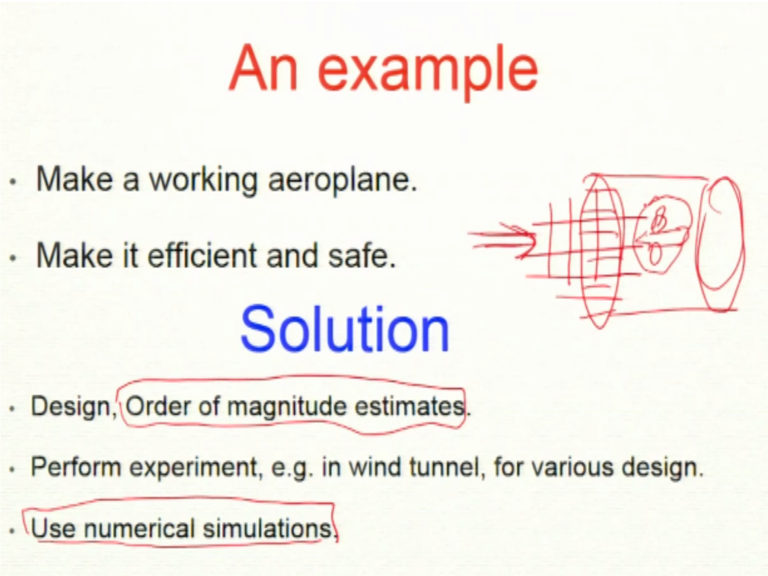
Number of molecules on the average do not change. An energy coming and energy going out should be equal. There is some energy flowing around, but there is no net flow of energy at any scale, in fact you can look at small scale, large, in fact there is no scale, there is only single scale, it is a single scale problem and there is no energy flow from one scale to other scale, so is an equilibrium system. Equilibrium systems do not have this transfer from one scale to other scale. This turbulence is a non-equilibrium problem.

There is a supplied scale and there is a dissipation of the scale. It is a very different system, but there are other systems in physics; by the way thermodynamics according to me in this room is not really applicable. We define temperature of course, but this room a non-equilibrium room, is not, thermodynamics plays a small role, important role but small role, but earthquake is an example of non-equilibrium systems.

oney supply as I said or biological systems, it takes energy, then does things, it is all non-equilibrium systems. So there are, I mean these require different tools and the tools of thermodynamics do not work in many of these applications. **(Refer Slide Time: 25:56)**



Okay, what about engineer's approach. So this approach is quite different. So I will give, start with an example, of course very important, I mean this is in fact they really do much more thing that what physicists can get to. **(Refer Slide Time: 26:10)**



So let us take an example, make an aeroplane, working aeroplane. Now so this is what I want to do and if you succeed, then you make an efficient and safe aeroplane. So this is one objective which the engineers achieved. So the Wright brothers, by the way they didn't really get the aeroplane by accident, even if you really see the story, that will be inspiring. So they were not from MIT or Harvard, basically they had a shop, but they had set up their own wind tunnel.

Everybody knows what is wind tunnel, so some of you the physics students may not know. So wind tunnel is, so I need to see whether aeroplane will fly or not, no, so people do experiments in IIT itself some in this kind of there is a wind tunnel, aerospace department, a big tube you know that is a wind tunnel. So wind tunnel will have this tube, okay, I really can’t draw this well, you know, and one end of the tube has this grid, okay.

So this grid is one end of the tube, is that clear and you push air from the left, very hard fast speed air and you put aeroplane in here, okay, this put small aeroplane, you can’t put a big aeroplane, you put a small aeroplane and then we can make measurements of this on this aeroplane. So you can make measurements on the stresses on the wings, how much lift is there on the wing, so all these experiments are done in with small aeroplanes and the interesting property of Navier-Stokes equation.

If we can measure the forces and so on on this small plane, then we can extrapolate for the same Reynolds number to large aeroplane, okay. So this is called self-similarity, okay, the some of it you have learnt in your other course which I taught myself, okay, so this is similarity. So you can extrapolate the forces on the larger aeroplane and they are doing all the experiments in their small garage, okay, and so, but they are not looking to solve turbulence problem.

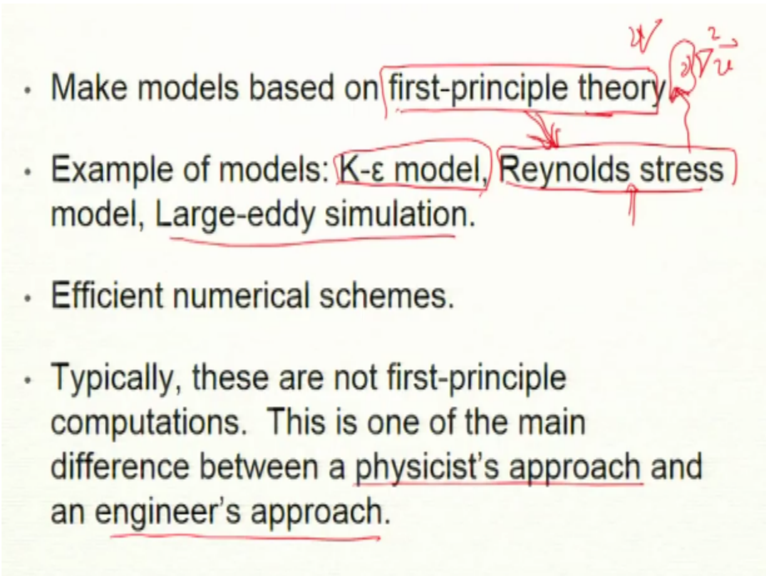
Please make sure, these physicists are trying to say well what is the force from the first principle, what is the property, they want to say where I want to make aeroplane, flying object, case objective is different, a working machine, of course with lot of intuition, so it requires big knowledge of fluid mechanics. It is not that you just put the model and it will work. This design, there are infinite number of ways we can make wings, but you should have intuition to make a good wing which will work.

So look for solution. So one thing will be first design and order of magnitude estimates, so this really plays a very important role, order of magnitude estimates. So how big it should be. If it is too tiny, then there won’t be enough force, so all that stuff or event and then we do experiments. I mean, so the engineer will do experiment, for example the wind tunnel experiment, okay for various designs. To make the real one, takes money, huge money and also effort.

So idea is to make small experiments, then make a big one, make several big ones and see which one works best. Also now-a-days, these computers are become very useful, earlier they were not, well they were not very heavily used. I mean, Wright brothers did not have a computer, computers came much later in 50s, 1950s, but now it is heavily used. In fact, I am told a Boeing has like the number may be wrong, but they say that equal money for experiment and equal money for simulations, huge amount of money goes into simulations.

In fact Indian spacecraft or rockets use the simulations to see whether how would this, but I don't know what to say about missile, even this, this, all these things will fly rockets or aeroplanes, okay. So, this is every important for engineers also and physicist we rely heavily on computer simulations for understanding the flow and making models, but we can see the approach is different.

Here they are not looking for first principle theories, but it is not that everything just gets work. So there are simulations, you know, you need to solve the Navier-Stokes equation. So I want to solve flow around aeroplane here and the Reynolds number for the aeroplane could be large near the wings, it could be 10,000. **(Refer Slide Time: 31:36)**



Now if I do first principle calculation, I require very big grid which we will do estimate in the course a bit later, but we don't, we should do that much grid it will take years of computed time. So these guys will basically use various models, so the viscous term, viscosity term the $\nu\nabla^2\mathbf{u}$, this was the term if you recall $\nu\nabla^2\mathbf{u}$, $\nu\nabla^2\mathbf{u}$ this is viscous term. So technically, they alter this ν, that is I am not interested in small scale but I am looking at large scale ν.

So there are various models, in fact the Reynolds stress model will help you to model ν for realistic flows at large scale or Large-eddy simulation, this is one simulation, one tool, so this is also used for modeling ν, okay. So, we will do some of it, so but you can see the idea, so this first principle calculation is not what engineers are interested to solve, but by the way engineers also come in different variety and physicist also come different variety.

So some engineers are interested in first principle because they basically should be in physics department in some sort and some physicists are working on all these stuff. So it is now a mix. Lot of interdisciplinary and cross talks, but to make this model of Reynolds stress, we require this first principle theories. So these theories are built typically by physicist, so this connection and then of course innovated by good numerical scientists, okay. Is that clear? The connection? I hope so.

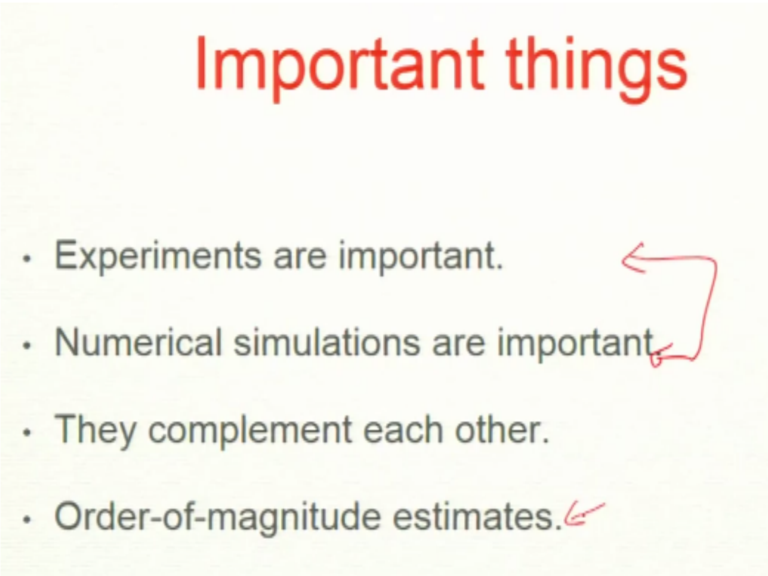
So first principle theories are important to construct this model, K-epsilon model is directly from Kolmogorov theory, this one, but it helps you to model the flow, you don't need to start from very fine mesh and you don't need to really solve all these case. So this helps you in efficient numerical schemes, so which is practical for weather simulations, if you really want to do this called direct simulation where you don't want to make a model.

Then it will be grid which will bigger than the best ever computer up today, okay, so we can estimate bit later, but is really really impossible to do direct simulation of the earth. If it is somewhat meaningless you know we don't really, it is so complicated, the terrain is complicated, the ocean has lot of stuff. So people make large scale simulation throughout. The grid is not millimeter, grid is kilometers, so grid size is kilometer, not millimeter. For direct simulation, of course we need millimeter, but weather it is kilometer.

So we need to of course work with this various parameterization. So typically these are not first principle calculation, so this is the major difference, so they are physicist's approach and engineering approach, okay, but of course, both are important and so please do not misunderstand that I am trying to put down one or the other and it is very important to crosstalk in these in fact fluid mechanics is a field which is very practical, so people really crosstalk.

So any fluid mechanics meeting if you go, you will find physicist, engineers, atmospheric physicist, meteorologist, engineers of course various types mechanical engineer, aeronautical engineer, chemical engineer, then there will be astrophysicist will be there, and mathematicians, there is some applied mathematicians, so this is really a mix of various sorts of people, okay. So, I kind of gave the picture of what different people do, no, mathematicians they will look for properties of Navier-Stokes.

They will not be too worried about real plane and so on, so idealized system, boundary condition idealized. Physicists will make a model you know which is like periodic box, but still viscosity is important, so get some data. Engineer will make real object, no, and try the model flow around plane. Chemical engineers very important, no, why because mixing, so all these chemical process mixing is very important for them, so chemical engineering this turbulence is very critical for lot of this process mixing in chemicals, okay. **(Refer Slide Time: 35:56)**



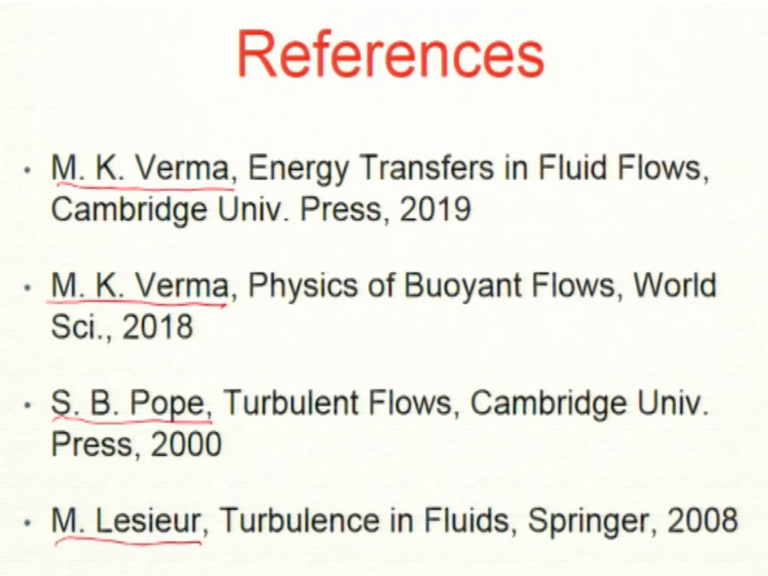
So important things. So from these examples, we can see the experiments are important. Numerical stimulations are important, also order of magnitude, so many of these will be just order of magnitude estimates, okay. Since we really don't have exact solutions, so we make some order of magnitude estimates and many times they work, but approximate, and so these 2 experiments of simulation, they complement each other. So what do you mean by that?

So this example of liquid metals, no, what is liquid metal like molten iron is an example of liquid metal, no, you seen this movie terminator, so terminator is a liquid metal you know. So this is robot which is made of liquid metals. So liquid metal experiments we can reach very high range somewhere, you can apply magnetic field, so we can do many interesting aspects with that.

So in fact, car plates, you know before they surface of car, so it is molten and they apply some magnitude, some they do various processes, but unfortunately so if I want to see what is turbulence inside, we can't see inside a metal, it is opaque. So, it is not like water, water we can see inside, air, water. So then, we are basically lost. So people do ultrasound, okay, so this ultrasound is done for this, so this is for this option, we can't inside, so uses ultrasound, but simulation become very handy for this, so I can see inside, okay.

I can get velocity field anywhere in the floor. The problem in simulation is we can't reach very high Reynolds number, okay, so which you can get in experiments, experiments can go up to 10^8, 10^9 Reynolds number. Simulation is a struggling at 10^5, okay. So these are plus points and negative points. So in fact, they are really complementary and the people do both. Convection the lots of stuff experiments this can't do, but we can do, the simulation, but we can't reach Rayleigh number or Reynolds number which experiments can reach.

So, these are complementary data we get from these two important aspects of fluid mechanics and I would also say that numerical simulation is more like an experiment is, V3 design experiment, so basically I am not able to get this from experiments, real experiments, so then we will say let us do simulation, so and then based on the data, we make models, but try to make first principle models, okay, and is the set order, this is an important aspect and we should learn this point. **(Refer Slide Time: 38:58)**



So, these are some of the references. So this book is not in print, but I will give you the pdf right now. This is in print, so we will use some of these instability calculations , I will do in midway of the course, third week or fourth week and there are quite a few other books, but I think these 2, also we will use some of it , if I don’t use it, but you can have a look, this book by Pope and Lesieuar. I think that is it for today. Thank you.