Introduction to Solid State Physics
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Lecture – 09
Understanding Thermal conductivity of a metal using Drude's Model Part-I

We will now look at another property which is that of the thermal conductivity of the material. In some of my earlier lectures we had looked at a model to explain the electrical conductivity of a metal. Now, we will look at the thermal conductivity of a metal.

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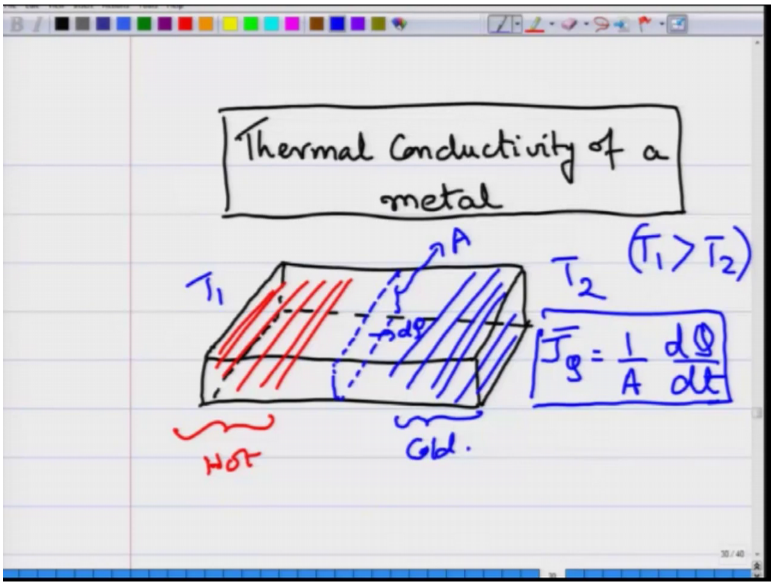

So, what is thermal conductivity? Thermal conductivity is related to having a metal which has a hot and cold end, let us take a piece of metal and you have a hot end and a cold end. And let us say one end of it is hot, this is hot end and you have another end which is cold.

And what you will do is that as you. So, this is maintained at some temperature T 1 and this is maintained at a temperature T 2, where T 1 is greater than T 2. And then what you do is that, you will measure for any cross sectional area in the sample, this is your cross sectional area how much is the net amount of thermal energy which is crossing this particular area, say A is the area of cross sectional area of the sample you will measure, the net thermal current which is the amount of thermal energy crossing per unit time per unit cross sectional area because you have one end which is at high temperature the other end is at low temperature.

So, now, there is a flow of energy from one end thermal energy from one end to the other. And this is almost like electric current. In electric current, what you have is there are flow of charges, and you measure something which is the current density which is the amount of current which is passing perpendicular to a cross sectional area, per your cross sectional area that is the current density per unit cross sectional area of the sample is the current density which is related to the flow of charge. But now, we will look at the flow of energy from one end of the sample to a another where the two ends are maintained at two different temperatures and so you will define a current density associated with it.

And this is the heat current density which is say J Q which is 1 by A, area of cross section dQ by dt. The current density is flowing in a certain particular direction in whichever direction there is a difference in temperature, the current will flow in from the hotter end to the colder end. So, this is the heat current the thermal current which is generated. Q is the amount of heat, dQ is the amount of heat that is flowing perpendicular to this cross-sectional area in this direction for this particular cross-sectional area in time dt. And so, you will measure how much is the heat current density.

Now, while this is a measure of the heat flow the thermal current density which is flowing perpendicular to this cross sectional area of the sample, you also know that this current density is related to the temperature difference, and the gradient in the temperature difference across the sample.

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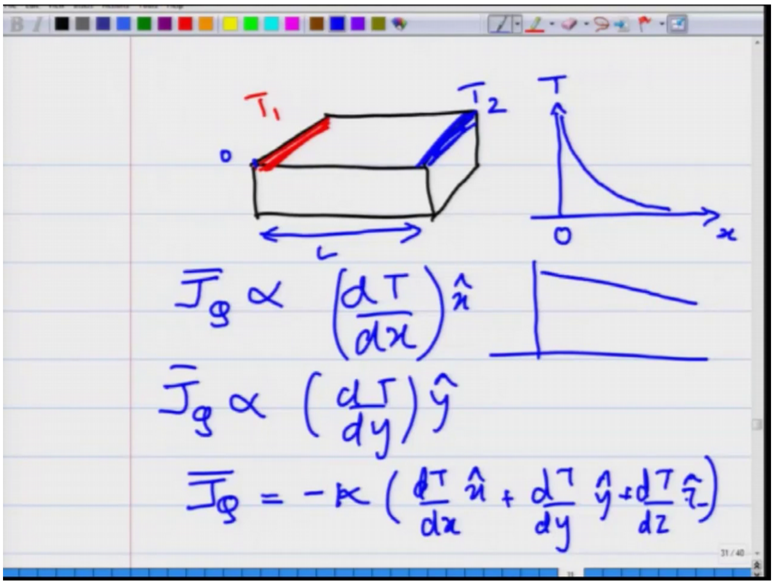

Namely, if you have this sample and this is your hot end and this is your cold end, ok. This is that temperature T 1 which is higher and this is at temperature T 2. Then, the amount of heat current that is going to flow will depend on the gradient, namely this temperature difference is maintained over how much distance L, ok. So, the amount of heat current which will flow will be proportional to the gradient.

How much difference in temperature is being maintained over a certain distance across the sample, it will depend on that. For example, if the temperature gradient across the sample if let us say if I am looking at the temperature profile across the sample. And if this temperature profile falls very rapidly, then you will have a large amount of heat flow. There is going to be a large amount of heat flow, because there is going to be a very large gradient, the temperature is going to change very rapidly over short distances so there is going to be a lot of heat flow a large current will be generated, ok.

And if you have a very slow gradient, that the gradient is changing very slowly across the sample then of course, the heat current will be much less if the difference in temperatures is not too large over this distance then effectively again you will know that there is not going to be much heat flow. So, if these two are almost the same with very little temperature difference over the distance of the sample then again the heat flow will be small. So, the heat current not only depends on the temperature difference it is also depending on the distance over which this temperature difference is maintained. So, you measure it by the gradient. So, this is the gradient in the x direction.

Similarly, if you maintain a current gradient in the y direction you will have. So, this is the current which you have in the x direction because of a gradient in the x direction. For a sample you can also have a gradient in the y direction dT by dy in the y direction and in the z direction also you can have. So, typically you will write in general your thermal heat current will be minus kappa which is the thermal conductivity into dT by dx x cap plus dT by dy y cap plus dT by dz z cap. And therefore, the thermal heat current will be minus of kappa times the gradient in temperature, the grad of the temperature profile across the sample.

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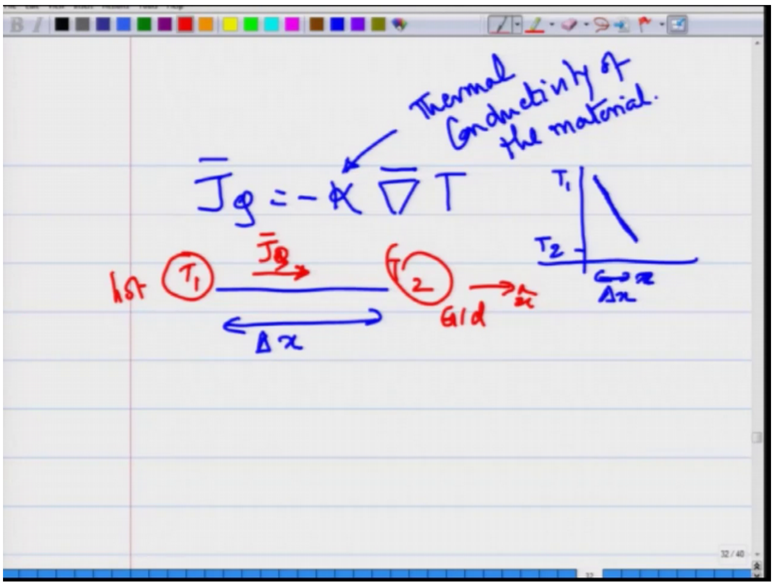

This is going to be a general expression for your thermal current density, where this kappa is the thermal conductivity of the material, and we want to study the behavior of this thermal conductivity.

And one more point is that the negative sign is because the current you want just how electric current is defined, the direction of the current and the flow of electrons are in the opposite direction. If you look at the gradient the gradient if you have T 1 to T 2 this is T 1, T 2 over a distance x delta x, if this is the from the hot end to the cold end then the gradient is negative, but you have a current in a certain direction which is in the plus x direction. So, along the sample this is delta x, and this is T 1 and this is T 2. So, you have a gradient this is the hot end and this is the cold end. So, your dT by dx is actually negative, but your current flow is in the plus x direction if this is your x cap direction your current is still in the x, your J of q is in this direction. So, you will have the negative sign to a count for the negative gradient. So, although the gradient is negative you have a current which is in this direction and to maintain that you will have the negative sign, coming into this picture.

So, with this structure let us try and calculate the thermal conductivity of the material, and for that purpose we again take use of the ideas from Drude's theory on the Drude model of the metal in order to calculate the thermal conductivity. It in fact, gives us some very good insight into what goes into calculating the thermal conductivity of the metal. So, we begin our calculations in the following way.

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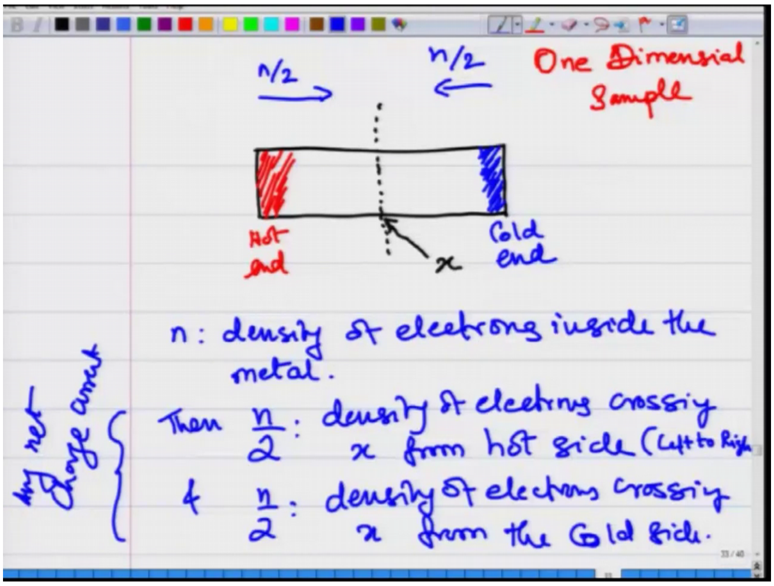

That let us take, first we take a one-dimensional sample and we do our calculations we do it for one-dimensional sample. So, for this one-dimensional sample we first take this sample and we have a hot end which is what I am shading in red and you have a cold end which is what I am shading in blue, ok.

And let us take some point, say I am measuring x from this direction this is my 0 and I start measuring and going in this direction since its one-dimensional I only have the x coordinate. So, let us say I have this is my point x, and here I want to find out how much is the net thermal current which is flowing across this point x. Now, what will contribute to the thermal current? What are going to be the charge carriers which are going to carry the energy? Thermal current is nothing else, but energy. So, what are going to be the charge carriers inside the metal? It again has to be electrons. Again, it is the electrons which are going to carry the energy or the momentum as they cross this point x.

And for the thermal current we will assume that electrons are coming from the hot end side as well as the cold end side. So, this is the hot end and this is the cold end. Electrons are going to come from hot end as well as the cold end, there are electrons which are coming from both sides, and we will consider if n is the density of electrons inside the metal density of electrons inside the metal then n by 2 is the density of electrons crossing x from hot side and n by 2 is the density of electrons crossing x from the cold side. So, you have n by 2 electrons, the equal density of electrons which are coming from the hot end as well as coming from the cold end. So, one set of electrons have higher energy the other set of electrons have lower energy, but there is the same density of electrons which are crossing the point x.

Now, this is important because the electrons are only carrying thermal energy and we do not want any net charge or net current electric current to flow along this direction, along across x. And that is why we maintain that n by 2 is the number of hot electrons which are flowing from left to right.

These are the electrons which are moving from left to right and we have the same number of electrons n by 2 electrons, n by 2 number of electrons are moving from right to left from the cold end to the hot end. So, that across x you do not have any net charge current. There is equal and opposite set of electrons the velocity of electrons are equal and opposite across this point x. So, there is going to be no net charge current. The charge current is going to cancel out there will only be some heat current that you will see. So, n by 2 is the number of electrons which are moving from the hot end to the cold end as they are crossing x and similarly n by 2 number of electrons at any instant of time are crossing the point x from cold end to the hot end.

Now, with this in mind let us again go back to our diagram and do the calculation.

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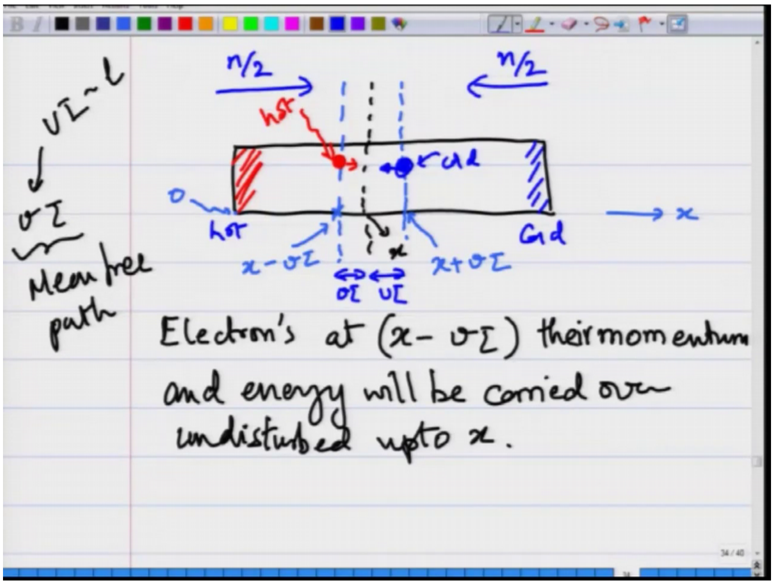

So, you have your hot end and you have your cold end, you have your x that and there are n by 2 number of particles and n by 2 number of particles, n by 2 number of particles moving from the hot towards the cold end as they are crossing x the point x they are crossing this point x. And similarly, n by 2 number of particles from the cold end to the hot end they are moving and that many number of particles are crossing the point x.

Now, let us look at how much is the net flow of energy across this x. And for this, let us consider your coordinate axis this starts from 0 at this point this is x and it is increasing in this direction. So, we can think of our x axis as if it is along this direction, let us consider a point which is marked here this point let us consider at x minus v times tau, where v is the mean velocity of the electrons and tau is the scattering time.

Let us consider this point which is at a distance x minus v time tau. This is on this towards the hotter side this point is towards the hotter side. So, the electrons which are going to come from this end, from x minus v times tau will be the hot electrons, this is the hot electrons and they will come from the hotter end carrying higher energy and they will reach the point x.

Why have we considered x minus v times tau? We have considered this distance x minus v times tau because the electrons the electrons at x minus v times tau the momentum and energy, their momentum and energy will be carried over undisturbed up to x because what is v times tau; v times tau if you recall is the mean free path over this distance v times tau as per Drude's model there is going to be no collisions experienced by the electron because tau is the time interval between two successive scatterings v is the average velocity. So, this is the mean free path of the electron.

And so, if an electron which is a hot electron at this end is coming with some energy it will continue to reach this point x with its same energy and momentum without changing because it does not undergo a scattering. Similarly, if you consider a point around x which is that x plus v times tau, the electron which is coming here is now a cold electron. The electron which is going to approach this point x is going to be a cold electron which is at x plus v times tau and whatever will be its momentum and energy, will be continued as it moves up to x because between this distance both of these are within mean free path v times tau and v times tau.

So, from either ends the hot electron and the cold electron, if you are within v times tau of this distance the hot electron whatever is this energy will be continued and it will move up to x. Similarly, the cold electron which is present on x plus v times tau you have a flux of cold electrons which are in this direction they will move in this and contribute to the net heat flow in this direction, ok. So, you have electrons which are present on either ends of x minus v times tau and x plus v times tau and let us see what is the net heat current which they generate. So, we look at it in the next lecture.