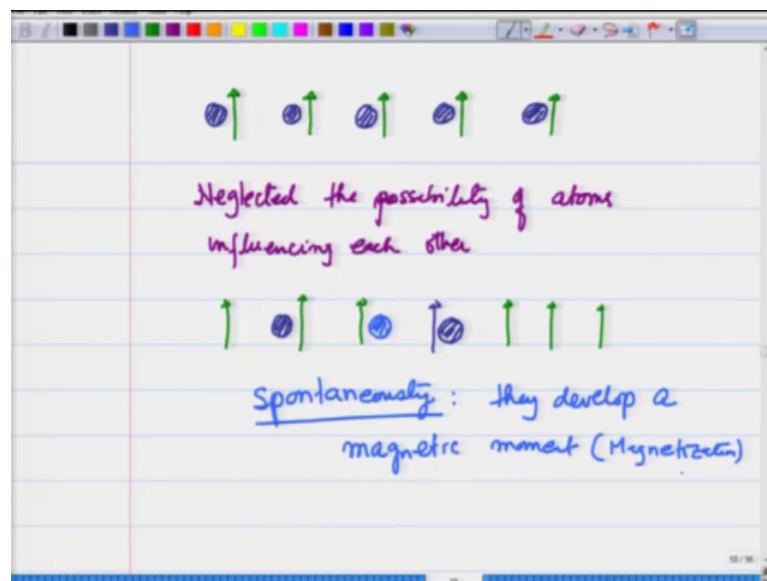


**Introduction to Solid State Physics**  
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**Lecture – 81**  
**Ferromagnetism in solids**

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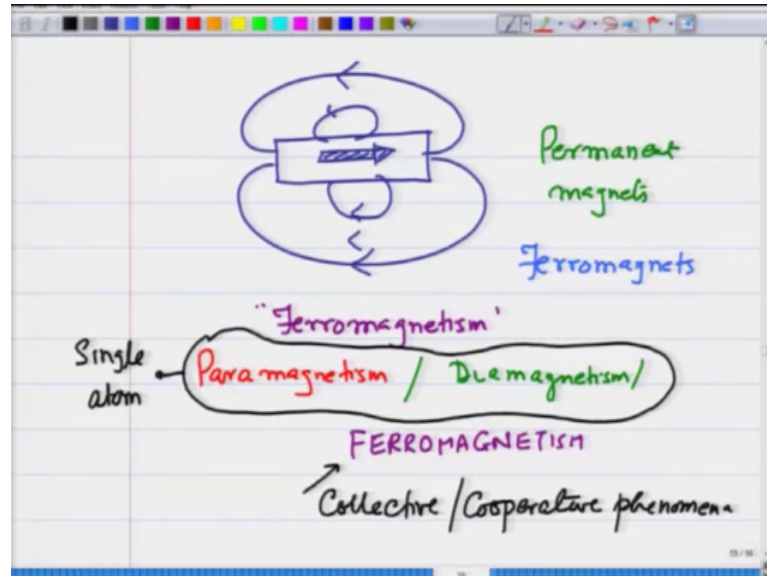


So far we have dealt with phenomena of paramagnetism and diamagnetism in which we considered a solid composed of these atoms which when put in a magnetic field developed a magnetic moment along the field direction and then sum of all these magnetic moments gave me the total magnetization of the system. And along all this what we did was neglected or ignored the possibility of atoms influencing each other, and that means, if one atom developed a magnetic moment, it did not affect the other atom in the system.

But now suppose I had this atom here and it developed a magnetic moment. And it forced the next atom also to have the magnetic moment aligned with it; and that in turn forced the next atom to have the magnetic moment aligned with it and so on. So, they all collectively force each other to have magnetic moments aligned, so that by themselves which I otherwise would call a word called spontaneously that means by themselves

without any external field being applied spontaneously they developed a magnetic moment for the system what I will call the magnetization.

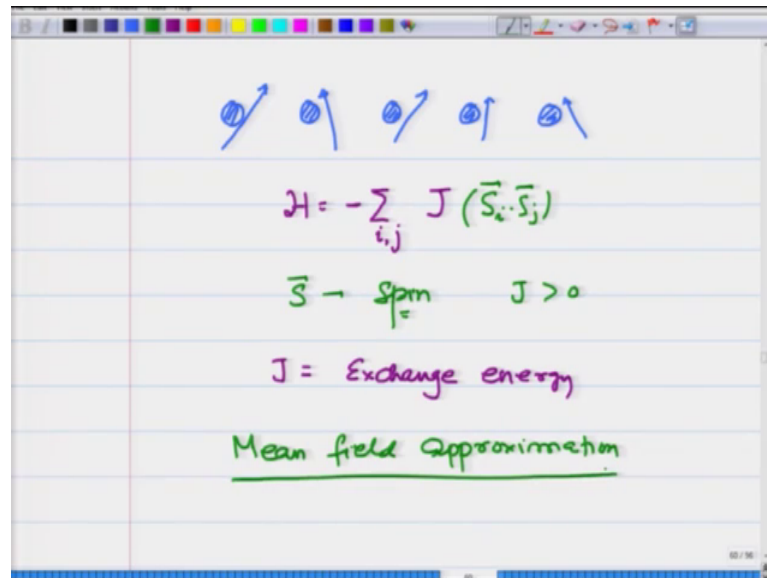
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So, without applying any field, these systems would have a magnetization in a certain direction which in turn would give you these field lines. This goes inside. And these are permanent magnets, which you look you know when you take a magnet from the magnet you can play around with those are those permanent magnets that show this permanent magnetization. And these are known as ferromagnet and the phenomena is called ferromagnetism.

So, now in addition to what we learned earlier, we learned about paramagnetism, then we learned about diamagnetism. And now we have this new thing called ferromagnetism. There is a difference between the two the phenomena that I have written here phenomena of diamagnetism and ferromagnetism is based on single atom, doing its own thing that I added all up. Whereas, the phenomena of ferromagnetism is a collective or cooperative phenomena, where all these atoms cooperate with each other to give a net magnetic moment, and we want to understand how so. I am not going into the properties of ferromagnetism that they show hysteresis, and that they have a saturation magnetization and all that, that you have learnt earlier you know in your B.Sc., in your intermediate. What I am focusing on is how this phenomena arises.

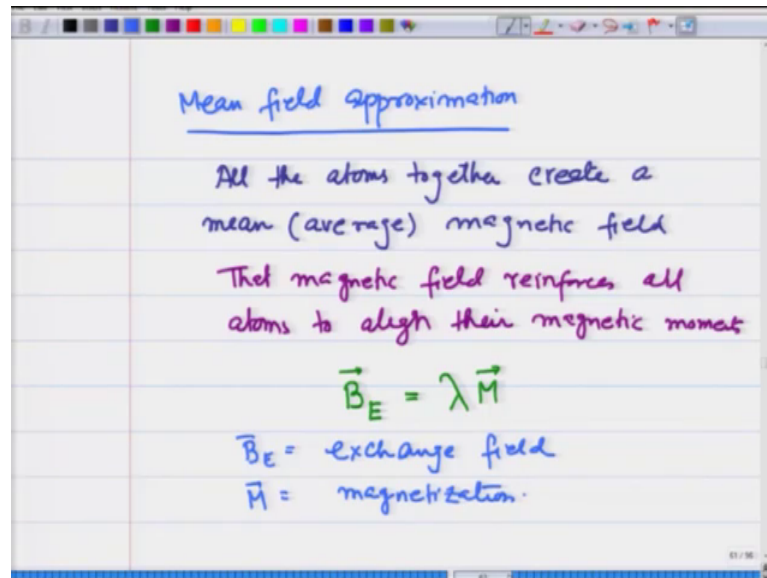
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So, if I have this collection of atoms, and each of them has an angular momentum  $J$ , so that the Hamiltonian is some summation with a minus sign  $i j$  in number  $J S_i \cdot S_j$ , where  $S$  is a spin of the system or it could also be the angular momentum. So, what this would do is and  $J$  is greater than 0.

When two spins or two magnetic moments are aligned, it will lower the energy of the system. And to lower the total energy of the system, all spins may get aligned. This  $J$  is known as the exchange energy. So, what one should be doing is take this Hamiltonian, and solve for the net spin that arises because of this and that is a difficult problem. What we are going to do instead is understand this in some sort of a what is known as mean field approximation.

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What mean field approximation means. It means that all the atoms together create an average or mean magnetic field. So, all these atoms together create a magnetic field. And that magnetic field reinforces all atoms to align their magnetic moments and as they align then they will create even more field and so on. So, in a self-consistent way, they force each other to become aligned, and give a large magnetic moment.

So, this mean field I am going to call  $B_E$  and this is assumed to be proportional to the magnetization of the system. So, I am going to write this  $B_E = \lambda M$ , where  $M$  is the magnetization of the system. So,  $B_E$  is the exchange field, and it is proportional to the magnetization,  $M$  is the magnetization of the system.

So, let us see now what happens. Then I said that this magnetic field reinforces all the atoms to align themselves. So, what this is going to do, because the system is you know the atoms are paramagnetic that basically aligns the magnetic moments proportional to the field.

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$$\vec{M} = \chi_p (\vec{B}_{\text{applied}} + \vec{B}_{\text{exchange}}) / \mu_0$$

Diagram showing atoms with magnetic moments and arrows for  $\vec{B}_{\text{applied}}$  and  $\vec{B}_{\text{exchange}}$ .

$$\vec{M} = \chi_p (\vec{B}_a + \vec{B}_E) / \mu_0$$

$$= \chi_p (\vec{B}_a + \lambda \vec{M}) / \mu_0$$

$$M (\mu_0 - \lambda \chi_p) = \chi_p B_a$$

It is going to give rise to a magnetization  $M$ , which is equal to  $\chi$  paramagnetic  $B$  applied plus  $B$  exchange divided by  $\mu_0$ . So, let us see what happened, I had these atoms sitting here. And I applied a magnetic field  $B$  applied, and then all of these developed a magnetic moment according to the paramagnetic susceptibility. If the system was paramagnetic, that is it, everything will stop here. But this magnetic moment in turn gave rise to additional magnetic field  $B$  exchange. And this  $B$  exchange in turn gave more magnetic moment and this in turn again created more magnetic field and so on.

So that this magnetic moment  $B$  becomes equals  $\chi P$  times  $B$  applied which I am going to write as  $B$  sub  $a$  I am not writing applied fully plus  $B$  exchange divided by  $\mu_0$ . And this  $B$  exchanged is arising because of the magnetic moment in the system is going to be equal to  $B$  applied plus  $\lambda M$  upon  $\mu_0$ , so that this  $\mu$  becomes self consistent. This  $\mu$  gives rise to  $B E$ ;  $B E$  gives rise to  $M$ . And finally, it is so that they become consistent with each other and this gives me  $M \mu_0 - \lambda \chi P$  is equal to  $\chi P B a$ .

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Handwritten derivation on a whiteboard:

$$\vec{M} (\mu_0 - \lambda \chi_p) = \chi_p \vec{B}_a$$

$$\vec{M} = \frac{\chi_p \vec{B}_a}{\mu_0 - \lambda \chi_p}$$

$$= \frac{\chi_p}{\left(1 - \frac{\lambda \chi_p}{\mu_0}\right)} \left(\frac{\vec{B}_a}{\mu_0}\right)$$

Taking  $\chi_p = C/T$

$$\vec{M} = \frac{C}{\left(T - \lambda C/\mu_0\right)} \cdot \frac{\vec{B}_a}{\mu_0}$$

Taking this equation  $M \mu_0 - \lambda \chi_p$  is equal to  $\chi_p B$  applied, I get the magnetic moment  $M$  to be equal to  $\chi_p B_a$  over  $\mu_0 - \lambda \chi_p$  which I can write as  $\chi_p$  over  $1 - \lambda \chi_p / \mu_0$   $B_a$  applied over  $\mu_0$ . Now, taking  $\chi_p$  paramagnetic to be the constant over  $T$  that we had calculated in earlier lectures, I get  $M$  equals  $C$  divided by  $T - \lambda C / \mu_0$   $B_a$  applied over  $\mu_0$ .

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Handwritten derivation on a whiteboard:

$$\vec{M} = \chi_p (\vec{B}_a + \vec{B}_E) / \mu_0$$

And taking  $\vec{B}_E = \lambda \vec{M}$

$$\vec{M} = \frac{C}{\left(T - \lambda C/\mu_0\right)} \cdot \left(\frac{\vec{B}_a}{\mu_0}\right)$$

$\chi_F$

$$\chi = \frac{C}{T - T_W} \quad T_W = \frac{\lambda C}{\mu_0}$$

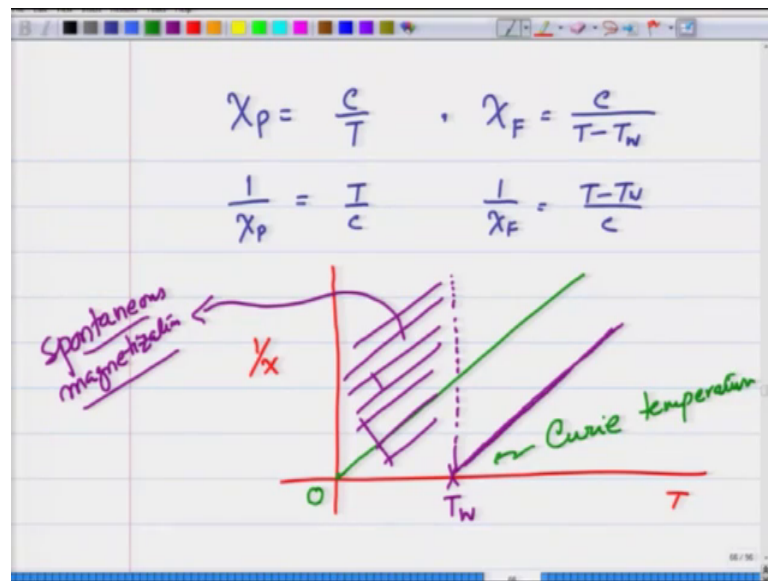
$T_W =$  Weiss temperature  
Weiss  $\rightarrow$  Mean-field theory

So, by taking this equation that  $M$  is equal to  $\chi_p B_a$  applied plus  $B_E$  exchange over  $\mu_0$ , and taking  $B_E$  is equal to  $\lambda$  times  $M$ , I finally get an expression for  $M$  which is

equal to  $C$  divided by  $T$  minus  $\lambda C$  over  $\mu_0$  times  $B$  applied over  $\mu_0$ . And this I am going to call the ferromagnetic susceptibility  $\chi$  ferromagnetic.

So,  $\chi$  in this case becomes  $C$  over  $T$  minus let me call this  $T_W$ , where  $T_W$  is the Weiss temperature. Why Weiss? Weiss was the person who gave the mean field theory. So, the ferromagnetic susceptibility goes as one  $C$  over  $T$  minus  $T_W$ , where  $T_w$  is  $\lambda C$  over  $\mu_0$ .

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So, let us compare  $\chi$  paramagnetic or  $C$  over  $T$   $\chi$  ferromagnetic is  $C$  over  $T$  minus  $T$  Weiss. Generally when it is plotted you plot  $1$  over  $\chi$  P which will be equal to  $T$  over  $C$  and  $1$  over  $\chi$  F which will be  $T$  minus  $T_w$  over  $C$ . So, if I would have plot  $1$  over  $\chi$  versus the temperature paramagnetic susceptibility would go the straight line passing through the origin. On the other hand, the ferromagnetic susceptibility  $1$  over  $\chi$  would be like this and this temperature being  $T_W$ .

Below this Weiss temperature  $\chi$  is not well defined, it is complex. So, you have spontaneous magnetization below this temperature, below this temperature. And above this, it behaves like a paramagnet. This temperature is also called Curie temperature. This is called Curie temperature, because it follows a law like paramagnetic Curie's law beyond the Curie temperature  $T_C$ . So, this is a in a simplified version of how the ferromagnetism arises.

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Ferromagnetism arises: due to exchange interaction between paramagnetic ions. And is a cooperative/collective phenomena.

$$\chi = \frac{C}{T - T_w/c}$$
$$\chi = \frac{C}{(T - T_c)^\gamma} \quad \gamma \sim 1.3$$
$$T_c \sim 1000K$$

So, ferromagnetism arises due to exchange interaction between paramagnetic ions and is a cooperative or collective phenomenon. This is a simplified version where we said that chi of ferromagnets will go as a constant divided by T minus T Weiss T Curie, a very very simplified version. Generally, it is seen that chi goes as C over T minus T this critical Curie-Weiss whatever you want to call it some power gamma, gamma is of the order of 1.3, and T C usually is of the order of a 1000 Kelvin. So, above 1000 Kelvin, these systems will behave like a paramagnet; and below this they have spontaneous magnetization or their ferromagnets.

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Ferromagnetism      ↑ ↑ ↑ ↑

Antiferromagnetism:      ↑ ↓ ↑ ↓ ↓ ↓

Conclude: Ferromagnetism is a collective phenomena arising from interaction (exchange) among atoms.

$$\chi = \frac{C}{(T - T_c)^\gamma} \quad \gamma = 1 \text{ mean field} \\ \gamma \sim 1.3$$



So, with this very, very brief and short introduction to ferromagnetism, I stopped on magnetic phenomena what I have not covered is what is known as also ferrimagnetism that arises when a system has two kinds of magnetic moments, so two kind of atoms. Then I have also not covered anti ferromagnetism, in which the spins align alternatively in one and the opposite direction. This, but as you read through your book (Refer Time: 18:40) book, now you can pick it up very easily. I have given you the basic introduction.

So, to conclude this lecture ferro magnetism is a collective phenomena arising from interaction which is called the exchange interaction among atoms and the susceptibility above the Curie temperature or the Weiss temperature goes as  $T$  or minus  $T C$  raised to some power  $\gamma$  we have derived  $\gamma$ . So, we have said  $\gamma$  mean field is equal to 1; and  $\gamma$  is also seen to be of the order of 1.3 in two systems.

Thank you.