Introduction to Solid State Physics

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Lecture –08
Introduction to Hall effect in metals – Part III

We saw that, when you place a metal in a magnetic field and an electric field applied electric field or a voltage that you apply across the material and you apply a magnetic field also to the material. It develops a voltage or a develop an electric field which is along the direction of the flow of current and there is an electric field which is generated transverse to the direction of flow of current.

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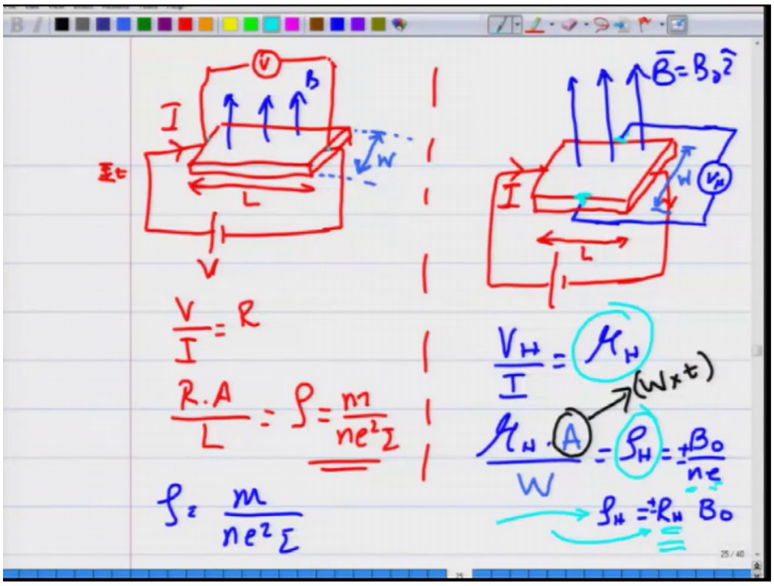

So, if you have the sample which is of length L and with W and you are sending and current from this end you can apply a magnetic field. Then the longitudinal voltage in this experiment is measured from this point to this point between these two points you will measure the longitudinal voltage you are sending in a current I. And as per Ohms law V by I is equal to R, R into area of cross section of the sample divided by the length of the sample is nothing else but the resistivity which is as per your Drude's model is m by n e square tau, were tau is the scattering time in the problem and this is as far as your normal longitudinal conductivity is concerned.

When you are looking at transverse conductivity which is because of the hall effect which I have already explain to you, here you have a sample which is of length L has dimensions which is of length L with W and a thickness t you place the sample in a perpendicular magnetic field and you send the current through the sample which is shown here through this circuit your sending in a current.

Now, in this configuration the hall voltage is measured in the transverse direction, namely between these two points which is perpendicular to the direction of the flow of current. And in this case the voltage that is measured between these two points is the hall voltage, which divided by the current will give you the hall resistance. This hall resistance which I have written as this curly H curly R is to distinguish it from R, because R as you know I have already used this symbol for the hall coefficient that is why I am using a curly R to indicate the hall resistance and this hall resistance into the area of cross section of the sample divided. Now not by the length but by the width of the sample the spacing between the 2 voltage contacts is W.

So, RH into A the hall resistance into area of cross section A divided by the width of the sample will give you your hall resistivity. And this hall resistivity as I have shown you is plus or minus B naught which is the magnetic field that you have applied divided by the density and the charge the magnitude of the charge or your hall resistivity is plus or minus RH which is the hall coefficient into B naught. So, by measuring your so now I have shown you an experiment in which what is the actual experimental geometry in which you can measure the hall coefficient of a material. How do you measure the hall resistivity and measure it as a function of magnetic field you will get information about the hall coefficient.

What the Drude's theory says that this longitudinal resistivity which is along this direction the voltage drop along the direction of the flow of current is independent of the magnetic field, it does not have any dependence on the magnetic field. But actual experiment show this is not true there is a dependence the longitudinal resistance also has a dependence on the magnetic field and this is one of the failures of Drude's model that it is unable to explain the longitudinal that magnetic field dependence of the longitudinal resistivity it goes by the name of Magneto resistance.

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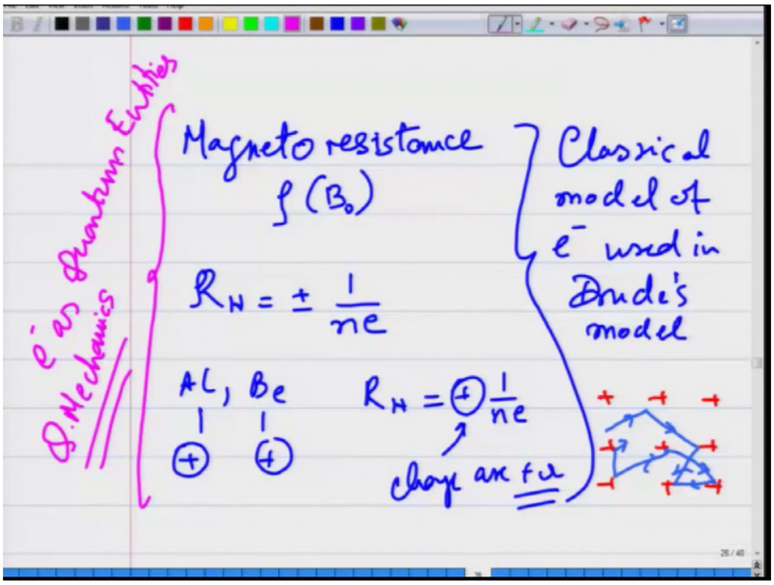

So, Magneto resistance is when the longitudinal resistance depends on the magnetic field, the resistance along the flow of current. Another drawback of the Drude's model is that RH which is equal to one would have expected Drude said that there is only negative charge carriers inside the metal and those are the only possible charge carriers inside the metal. But it turns out that as you saw for aluminium and beryllium these have RH which whose value is positive, the value of RH is positive instead of being negative and RH being positive 1 by n e indicates that the charge carriers are positive. This is completely it cannot understand any of this using the Drude's model how does the charge carrier become positive.

In fact, if you go to even if you do not consider some metals there are certain materials called semiconductors like silicon germanium gallium arsenide, there you will find that depending on how much doping impurities you put into the material the value of RH or the sign of RH changes this from positive to negative and that all of these features none of it can be explained by Drude's model.

So, this is where some of the failures of Drude's model become a parent and none of these can really be explained by the classical model of an electron used in Drude's model. Which considered electrons as really hard billiard ball like structures which were actually colliding from cores of atoms from the ionic course and as they were moving from the resistivity was basically appearing these are the positive ion cores and the idea was this is something which I have repeated earlier is that these electrons are scattering from iron cores and going from one to the other, this is how the scattering is occurring and this is what leads to resistivity.

But it is like a billiard ball which is hitting different points and then moving around this metal and this was the typical model this is the classical picture of the electron. None of these phenomena's which have spoken to you of magneto resistance the change in the sign of the hall carriers the behaviour of semiconducting a materials like silicon germanium gallium arsenide, where the hall coefficient changes sign none of this can be explained by considering a classical picture of an electron.

You have no alternative but to consider electrons as quantum entities and you have to use quantum mechanics to even try and understand some of the basic the simplest of some of these effects which we will do subsequently. So, the classical there are certain limitations of this model of the Drude's model, primarily it considers the electron as a classical object and then use is kinetic theory arguments to define or to understand the behaviour of the electron and while it gives us a very intuitive and a nice way to understand where from does it arise it cannot explain all the details of the behaviour of resistivity of the material.

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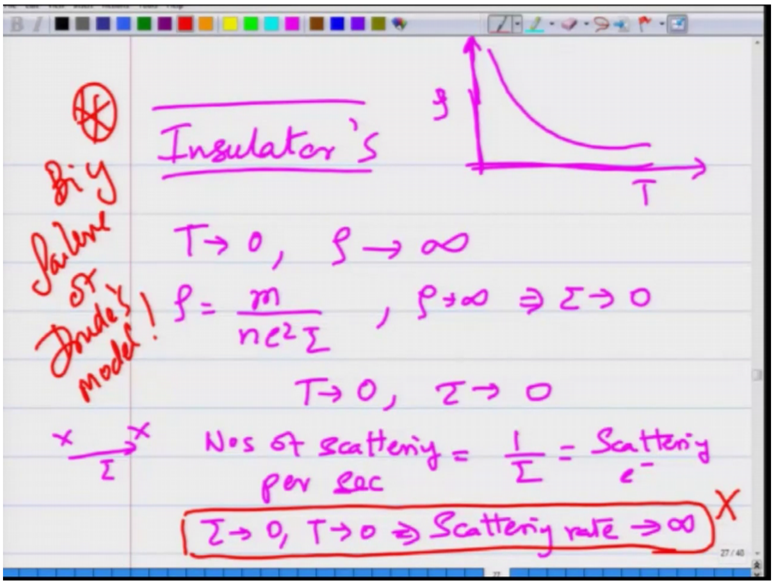

And the biggest failure which comes out from Drude's theory is the behaviour of insulators, this is one of the biggest failures of Drude's theory. Because in insulators if you look at the behaviour of resistivity as a function of temperature, the resistivity actually diverges as you lower the temperature as T tends to 0 the longitudinal resistivity actually tends to infinity it becomes infinitely large in an insulator. In an insulator the resistivity just diverges as you lower the temperature and go towards 0 and what does Drude's theory say, the Drude's theory says that the resistivity is m by n e square tau.

So, if resistivity tends to infinity it implies the tau should be tending to 0 or insulators are materials in which T as T tends to 0 tau tends to 0. If you recall tau is the time interval between 2 successive scattering of the electron it emerges from one and scatters within a time interval tau.

So, the number of scattering per second of the electron is 1 over tau, this is the frequency or the scattering rate this is the scattering rate of the electron and tau tending to 0 as T tends to 0, implies that the scattering rate tends to infinity and this is completely absurd this conclusion which comes out from Drude's theory is completely absurd.

This is not possible and this is one of the major big failures of Drude's model free electron model, it cannot explain the behaviour of insulators. It is just not possible to explain it within this context, there is something missing in this entire picture and although it is very useful and it has it is role to play it gives us an approximate way to understand things, when you go to the details and when you go across different materials there it cannot be explained just by using this classical picture of Drude.

So, we have got an overview of the Drude's model and it can explain certain things and one of the things that it really did well was to explain the equations of motion for an electron in an electric and magnetic field and we had a Hall effect. Now before we go further I would just like to spend a little bit more time to tell you that Hall effect is a very important phenomenon, it is an extremely important phenomena because it has a lot of applications.

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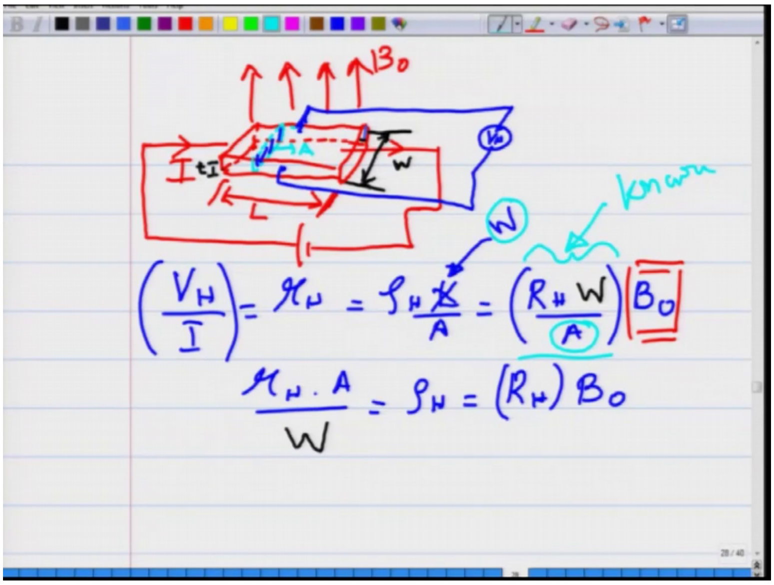

So, when you are using a Hall sensor basically you have a sample whose length is L width is W and a thickness t. You bias it by sending in a current I through the sample and if you take the sample and place it in a magnetic field B naught. Then you will get a transverse voltage VH which develops across the sample the voltage as I had said is perpendicular to the direction of the flow of current. And as you know if I if you take the ratio of the hall voltage to the current that is the hall resistance and this hall resistance is proportional not to rho h into L by A.

But it is rho H which is the Hall resistivity into the width of the sample because, W is the spacing between the hall voltage leads and that is W divided by A which is this area of cross section the cross sectional area of the sample through which the current is flowing perpendicularly through this cross sectional area.

So, it is important for you to actually note that width W is the width of the sample divided by A and that is what appears out here. So, if you measure and you know rho H is proportional to RH into W into B naught. So, RH into W by A all of these are known all of these are known quantities, so these are already pre calibrated and pre found out for your sensor. So, if you measure this ratio you will immediately know if you have a measure of this VH by I you will immediately know by knowing this quantity how much is this quantity earlier you can do early measurements and figure out how much is this quantity for your given material.

Then you know how much is a magnetic field B naught that you have because, you can just invert this and you can get the value of B naught and this is how you can use it as a sensor and some of the applications are what I will speak to you in the next slide, where can it be useful it is useful in various places.

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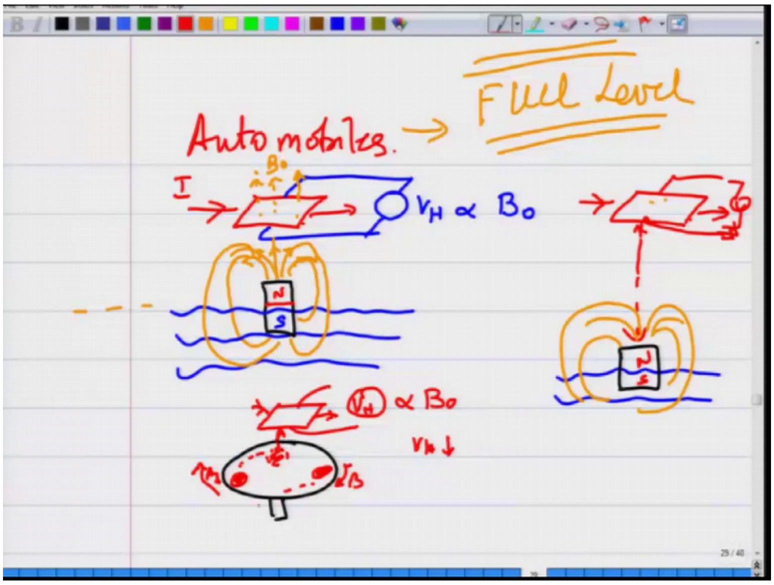

For example in automobiles it is used for measuring the level of the fuel in some cases and how is it measured it is actually a very neat way you have a hall sensor which is placed, a known current is pass through the hall sensor and you are measuring the hall voltage across it VH ok.

Which is going to be proportional to the magnetic field B naught and where from is a magnetic field B naught coming? This is the level of the fuel and on top of the fuel this is your fuel which is present and just above the fuel you are suspending a magnet, you suspend a magnet which has a North Pole and a South Pole.

So, the magnetic fields are generated from this magnet which is floating on top of your this is your magnet and this magnet is this thing and your sample is going to experience your magnetic field which is being generated by the magnet. So, whatever is the magnetic field B naught which this magnet is generating at this particular level is going to measure that magnetic field. What happens if the level of the liquid goes down, if the level of the liquid goes down then the hall sensor is still here, but suppose the level of the liquid now instead of being at this level is say at this level.

The level of the liquid now goes down, as the level of the liquid goes down the magnet also moves down and as the magnet moves down it is much further away from the sample, is much further away from your hall sensor, You are measuring your transverse voltage this is your current I.

But now you can see that the distance between the hall sensors has increased and the magnetic fields which are coming out are going to be much weaker at this point. So, B naught becomes less the hall voltage becomes less and therefore you know you can do a calibration and you can find out that as the hall voltage decreases you know that the fuel is decreasing and by calibrating the system you can find out how much is the decrease in the fluid.

So, you can immediately find out level or fluid level fuel level, you can find the level of fuel inside an automobile by using such techniques. Similarly you can also find the position of a rotating disc using a hall sensor and that is quite simple. Again what you have is a disc which is rotating and let us say we have 2 magnets which are kept here, your hall sensor is placed somewhere at this location.

Now you have input current and you have output voltage VH which you measure which is proportional to the magnetic field and this disc is rotating. As long as these magnets are far away from being away from below the hall sensor the VH will be low, the moment the disc rotates and the magnet comes at this location suppose the magnet come at this location immediately the magnetic field sensed by the hall sensor will be high and it knows that is a magnet in it is location.

So, what is the position of the rotating disc whether it is at position A or position B it will immediately know by the polarity you keep one magnet of one polarity and another magnet of another polarity the hall sensor or the hall voltage will also change sign because it depends on the sign of the magnetic field. And as a result you can know whether your disc has come to this position, whether this position A has come here or whether this position has come at that location you can identify.

So, the position of a rotating disc which is used in industries when the disc rotates and has to stop at some position and you want to know whether automatically want to know where the position and what is the position of the disc you can find it out using hall sensors. So, position of rotating discs can also be found hall sensors are also used for positioning in, of course in aerospace applications as well as in mobile phones there are hall sensors placed and so they have enormous applications and they are being used more and more all across devices that we use.

So, it is a very important area Hall sensor, but as I said we are looking at the physics of these hall sensors Drude model gave us a lot of successes, but there are also some limitations which had spoken. With this we will continue in the next lecture on some other topics.