

**Introduction to Solid State Physics**  
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**Lecture – 78**  
**Examples of performing paramagnetic susceptibility calculations**

We have so far considered paramagnetic systems and I have shown you how atoms develop a magnetic moment when they are put in a magnetic field. This happens because they carry orbital angular momentum, the spin angular momentum in their electrons. And under the L S coupling we also calculated the Lande g factor and what is the magnetic moment that they will develop. In this lecture I want to give you a feel for the numbers, kind of numbers that arise when we deal with these systems.

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Bohr Magnetron:

If we calculate the magnetic moment of an electron in  $n=1$  state of the Bohr model of H atom, that is equal to one Bohr magneton.

$$\mu_B = \underbrace{\left(\frac{q}{b}\right)}_{\text{Current}} \underbrace{h^2 \pi}_{\text{Area}} = \frac{e \omega}{2\pi} \times \pi h^2 = (e \omega h^2 / 2)$$
$$\frac{e r^2 \omega}{2} = \frac{e}{2 m_e} (m_e r^2 \omega) = \left(\frac{e \hbar}{2 m_e}\right) = \mu_B$$

So, first concept that we used was Bohr magneton. So, Bohr magneton is defined as, if we calculate magnetic moment of an electron in  $n$  equals 1 state of the Bohr model of hydrogen atom that is equal to 1 Bohr magneton.

So, this is a number because after you solve the Schrodinger equation you know that there is no angular momentum or no magnetic moment for the  $n$  equals 1 state, but this gives us a number. So, let us see if I calculate  $\mu_B$ , this will be equal to the charge per unit time going around in the orbit which is equivalent to current times  $r$  square times  $\pi$ .

Where this is the now, area of the orbit and the first term is the current. Which I can write as,  $t$  is the time period. So, this  $I$  can write as  $e$  times  $\omega$  over  $2\pi$  times  $\pi r^2$ . And this  $\pi$  cancels. I get this equal to  $e \omega r^2$  by 2. And I can immediately relate it to the angular momentum of the system.

So, I am going to write  $e r^2 \omega$  by 2 as,  $e$  over 2 mass of the electron times mass of the electron  $r^2 \omega$   $m r^2 \omega$  is the angular momentum. So, I can write this as  $e$  and in the first Bohr orbit that angular momentum is  $h$  cross divided by  $2 m e$  and that is a Bohr magneton.

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The image shows a digital whiteboard with handwritten text and equations. The top section is titled "Bohr Magneton:" and shows the derivation of the Bohr magneton  $\mu_B$  as  $\frac{e\hbar}{2m_e}$ . It then substitutes values:  $1.6 \times 10^{-19} \times 1.05 \times 10^{-34}$  over  $2 \times 9.1 \times 10^{-31}$ , resulting in  $\approx 9.3 \times 10^{-24} \text{ JT}^{-1}$ . The bottom section is titled "Paramagnetic susceptibility" and shows the formula  $\chi_e = \frac{\mu_0 \mu_B^2 g^2 J(J+1)}{3k_B T} \times N_{Av}$ .

So, Bohr magneton is  $\mu_B$  which is equal to  $e \hbar$  cross over 2 times mass of the electron. And let us calculate its value. This is going to be 1.6 times 10 raised to minus 19  $\hbar$  cross is 1.05 times 10 raised to minus 34 divided by 2 times 9.1 times 10 raised to minus 31. And this comes out to be very close to 9.3 times 10 raised to minus 24. The units are Joule per Tesla.

So, this is the kind of magnetic moment that an electron in an atom can be expected to have. Of the order of 10 raised to minus 23 to 10 raise to minus 4 Joules per Tesla and this is a unit then we use. Next, we calculated the paramagnetic susceptibility  $\chi_e$  which is given as  $\mu_0$ , where  $\mu_0$  is the magnetic permeability of vacuum times  $\mu_B$  Bohr magneton square times  $g^2$ , where  $g$  is that Lande  $g$  factor,  $J(J+1)$  divided by  $3 k_B T$ .

So, let us find the order of magnitude of magnetic susceptibility of this is per atom and then we multiply this by the Avogadro number to get it for per mole.

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The image shows a handwritten derivation of the magnetic susceptibility per atom,  $\chi_e$ . The derivation is as follows:

$$\chi_e = \frac{N_a \mu_0 \mu_B^2 g^2 J(J+1)}{3 k_B T}$$

$$\sim \frac{6 \times 10^{23} \times 4\pi \times 10^{-7} \times 10^{-46} \times 10^2}{3 \times 1.38 \times 10^{-23} \times 300}$$

$$\sim \frac{100 \times 10^{-30}}{200 \times 10^{-23}}$$

$$\sim 10^{-7}$$

$$\chi_e = \frac{M}{H} = \underline{\underline{\text{dimensionless}}}$$

So,  $\chi_e$  is  $N_a \mu_0 \mu_B^2 g^2 J(J+1)$  divided by  $3 k_B T$ . And let us put in numbers and Avogadro is of the order of 6 times 10 raised to 23 times  $\mu_0$  is 4 pi times 10 raised to minus 7  $\mu_B$ , we just calculated is of the order of 10 raised to minus 23. So, I am going to put it as 10 raised to minus 46  $g^2 J(J+1)$  roughly put it of the order of 10 divided by 3 k B is 1.38 times 10 raised to minus 23.

Let us take the temperature of the, room temperature 300. So, this comes out to be, let us see. Now, 3 times 1.38, that is roughly 5. So, we can cancel this, this and this as 2. Then, 6 times 2 is 12, times 48, times 4 is 48, times pi. So, that is roughly of the order of 100. So, I can write this as 100 times 10 raised to minus 23. So, minus 7, 10 raised to minus 30 divided by 300 times 10 raised to minus 23. So, this comes out to be of the order of 10 raised to minus 7. So, I can say this of the order of 10 raised to minus 7 to 10 raised to minus 5. And this is a dimensionless number because  $\chi_e$  is magnetic moment divided by H. And I leave it for you to check that this is dimensionless.

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The image shows a whiteboard with handwritten mathematical equations. At the top, the magnetic susceptibility  $\chi_e$  is given as  $\chi_e = \frac{\mu_0 \mu_B^2 g^2 J(J+1)}{3k_B T}$ . This is then simplified to  $\chi_e = \frac{\mu_0 \mu_{B \text{ effective}}^2}{3k_B T}$ . Below this, the effective Bohr magneton is defined as  $\mu_{B \text{ effective}} = \text{effective magneton Number} \times \mu_B$ . Finally, the effective magneton number is defined as  $\text{effective magneton Number} = g \sqrt{J(J+1)}$ .

$$\chi_e = \frac{\mu_0 \mu_B^2 g^2 J(J+1)}{3k_B T}$$
$$= \frac{\mu_0 \mu_{B \text{ effective}}^2}{3k_B T}$$

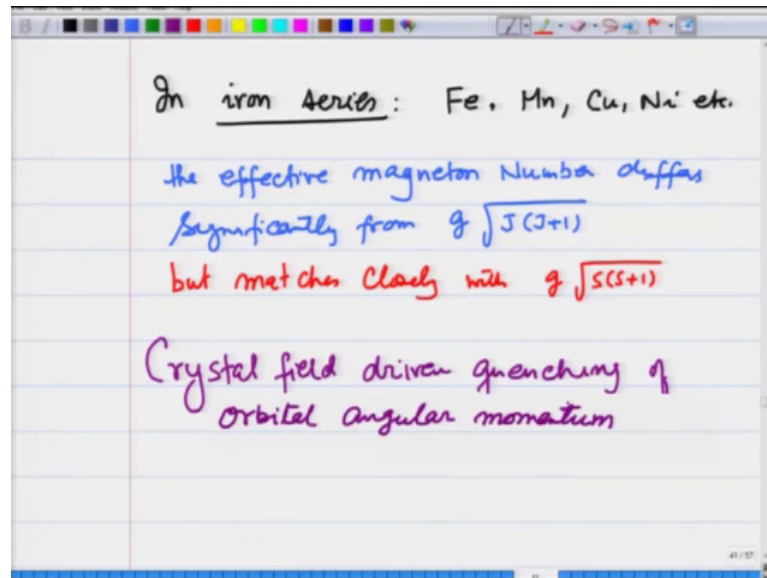
$\mu_{B \text{ effective}} = \text{effective magneton Number} \times \mu_B$

$\text{effective magneton Number} = g \sqrt{J(J+1)}$

So, this is the kind of susceptibility we are talking about. Now, when I write this  $\chi_e$  as per atom, per atom we write this as  $\mu_0 \mu_B^2 g^2 J(J+1)$  divided by  $3k_B T$ . Then I can write this as  $\mu_0 \mu_{B \text{ effective}}^2$  divided by  $3k_B T$ .

Now,  $\mu_{B \text{ effective}}$  is effective Bohr magneton. I can write this as effective magneton number times  $\mu_B$ . So, that we also talk about these atoms in terms of effective magneton number, which is defined as a square root of  $J(J+1)$  times  $g$ . So, this is a number that one kind of compares in experiments.

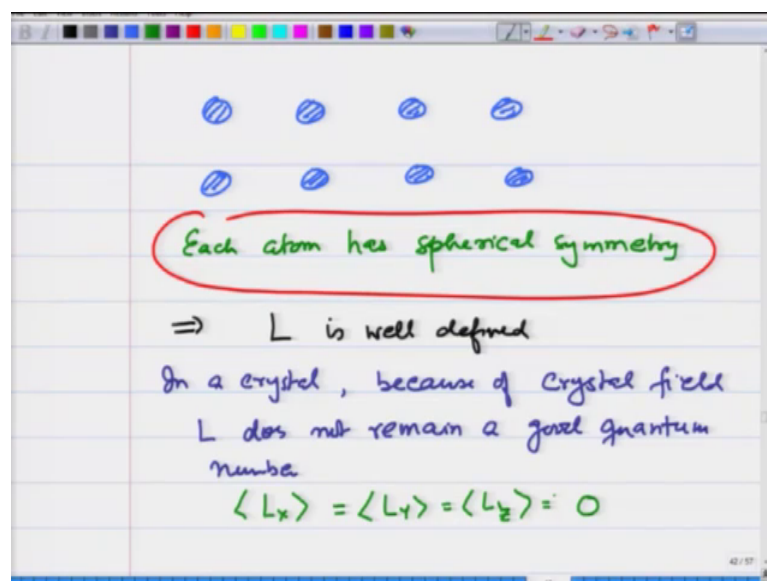
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Now, it is found that in iron series which include systems like iron, manganese, copper, nickel etcetera. The effective magneton number differs significantly from  $g\sqrt{J(J+1)}$ . But, matches closely with  $g\sqrt{S(S+1)}$ .

$g$  for  $S$  is going to be anyway too. Why does that happen? And this introduces an idea called crystal field driven quenching of orbital angular momentum. What does that mean? Let us understand that.

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When I have these atoms in the crystal, what I have assumed in calculating the magnetic moment is that, each atom has spherical symmetry. This is the assumption. And what it means is that  $L$  is well defined.

However, in a crystal the crystal field may be so strong that spherical symmetry is broken right? So, in a crystal because of crystal field and that arises basically because our atoms are affecting the field of each one of them.  $L$  does not remain a good quantum number. That means, it is not conserved and what it means is that the expectation value of  $L_x$  and expectation value of  $L_y$  and expectation value of  $L_z$  is all 0. And therefore, there is no angular momentum.

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The image shows a digital whiteboard with handwritten notes in green and purple ink. The notes are as follows:

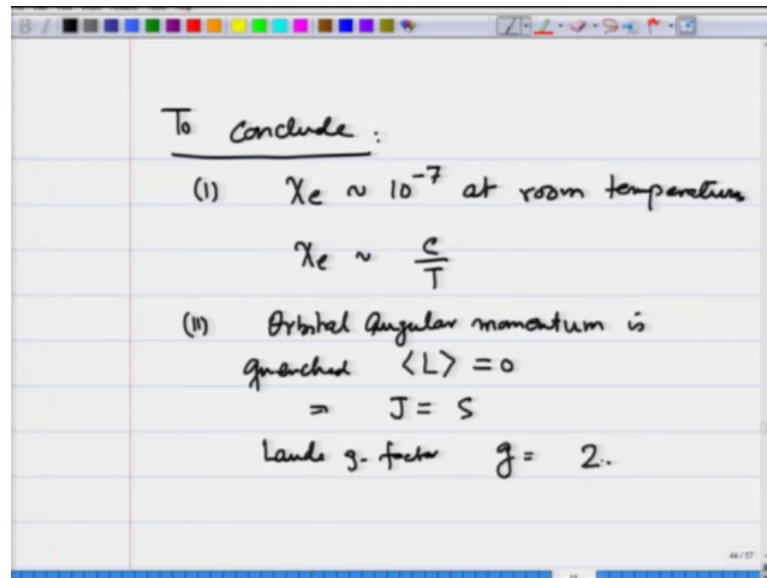
$$g \quad L=0$$
$$\Rightarrow J = L + S = S$$

This gives effective magnetic Number

$$= g \sqrt{S(S+1)}$$
$$= 2 \sqrt{S(S+1)}$$

If  $L$  is 0, this means  $J$  which is  $L$  plus  $S$  is going to be equal to  $S$  alone. And this gives the effective magnetic number as equal to  $g$  square root of  $s$   $s$  plus 1 which will be nothing, but 2 square root of  $S$   $s$  plus 1. And in iron series it is indeed seen to be this.

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So, to conclude this lecture, we have seen number 1, that  $\chi_e$  is of the order of  $10^{-7}$  at room temperatures. Of course when temperature goes down,  $\chi_e$  is going to go higher because  $\chi_e$  varies as  $C/T$ . And number 2, orbital angular momentum is quenched. This is the technical terms. And that means, basically  $L$  expectation value becomes 0, which is a mathematical statement of saying that  $L$  is no longer a good quantum number. It cannot be defined and this means that  $J$  is equal to  $S$  and the Lande  $g$  factor in these systems is corresponding to  $S$  and  $g$  therefore, comes out to be 2.

So, with this I conclude our introduction to paramagnetic substances. In the next two lectures I will be introducing you to diamagnetic systems and ferromagnetic systems.

Thank you.