**Introduction to Solid State Physics**
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**Lecture – 76**
**Paramagnetism in solids - II**
**Temperature dependence of paramagnetic susceptibility and Curie's Law**

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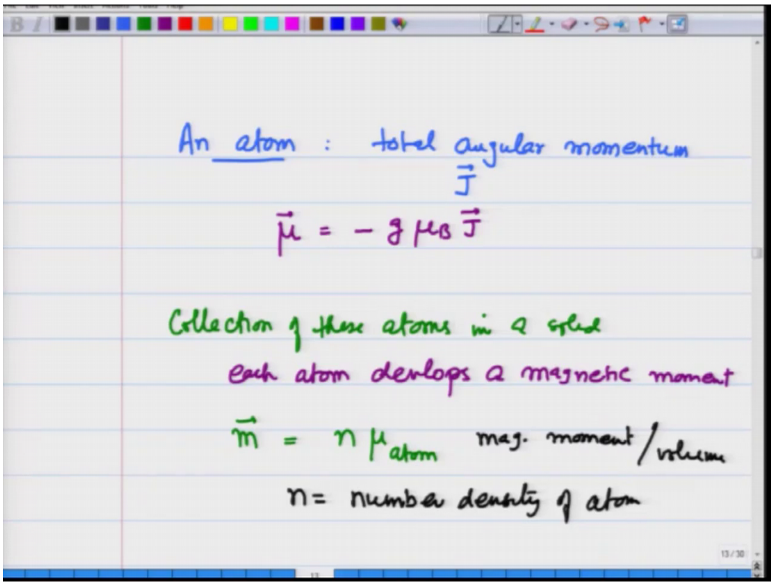


In the previous lecture, I derived the magnetic moment and the Lande g factor for an atom. And what we concluded there is that an atom has a total angular momentum J, it will then have the corresponding magnetic moment mu which is equal to minus g mu B times J.

Now, we have this collection of these atoms in a solid. So, we have a collection of these atoms in a solid. Then if I apply a magnetic field, each atom develops a magnetic moment and assuming that these atoms or these magnetic moments do not interact with each other. So, each atom is giving its own magnetic moment. The total magnetic moment m of the system is going to be equal to n times mu atom, where n is the number density of atoms. So, this m should be called the magnetic moment per unit volume or the magnetization. So, what I want to now calculate is how this mu atom is given as a function of temperature.

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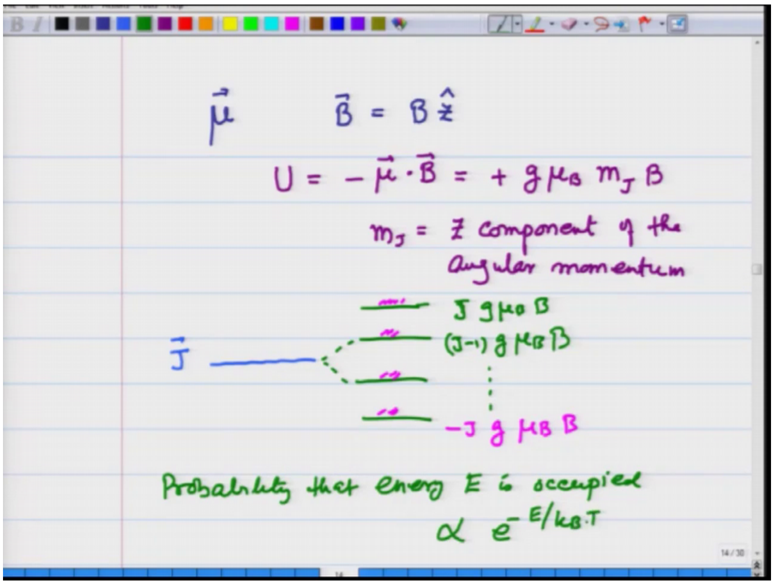


Now, when this system the magnetic moment is put in a magnetic field let us say field B is in the z direction so, am going to write this as B z. I choose z direction because that is easy to calculate things with. Then the energy of each atom is going to be minus mu dot B and this is going to be equal to plus g mu B m J times B, where m J is the z component of the angular momentum. So, I am going to have this atom which had initially a level J, with J angular momentum is going to split, is going to have all these different levels.

Now, you can see that if m J is positive, energy is positive. So, with m J positive I am going to have the level go up so, let us say this is J g mu B B, next one is going to be J minus 1 g mu B B and so on and the lowest level is going to be minus J g mu B B. These are the energy levels into which this will split and at a temperature T, the atoms are going to be distributed among these levels. However, the number of atoms in each level is going to be different. That is because the probability that energy E is occupied by an atom is going to be proportional to e raised to minus E over k Boltzmann times T.

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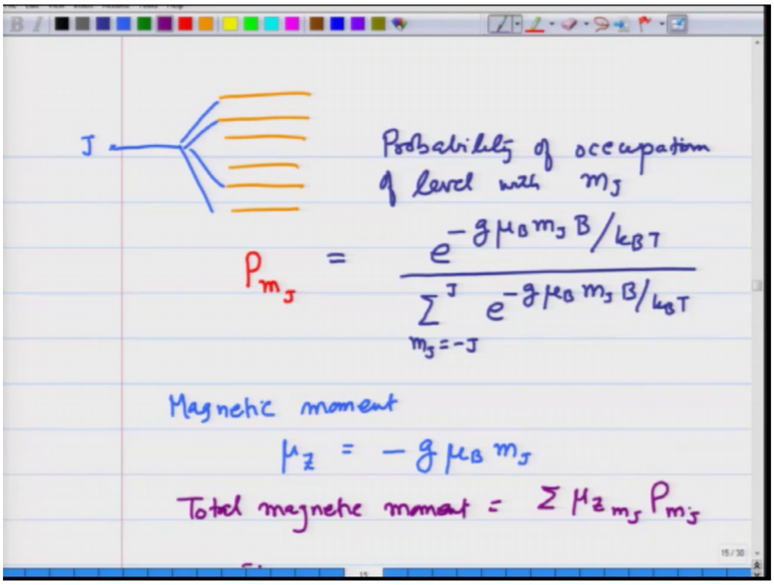


Therefore, if you put it all together, here is this atom with level given by J and this has split into different levels right let me show these levels by different colors. Each level has a different probability of occupation. So, probability of occupation of level with z component m J is going to be equal to e raised to minus g mu B m J B over k B T divided by, I have to normalize the probability sum over m J varying from minus J to J e raised to minus g mu B m J B over k B T. And let me label this as probability that level m J is occupied.

Now, the magnetic moment mu in the z direction for this level is going to be minus g mu B m J. So, if I multiply each of this magnetic moment by the corresponding probability and add it up I am going to get the total magnetic moment. So, total magnetic moment is going to be summation of mu z for m J so, times the probability that this m J level is occupied.

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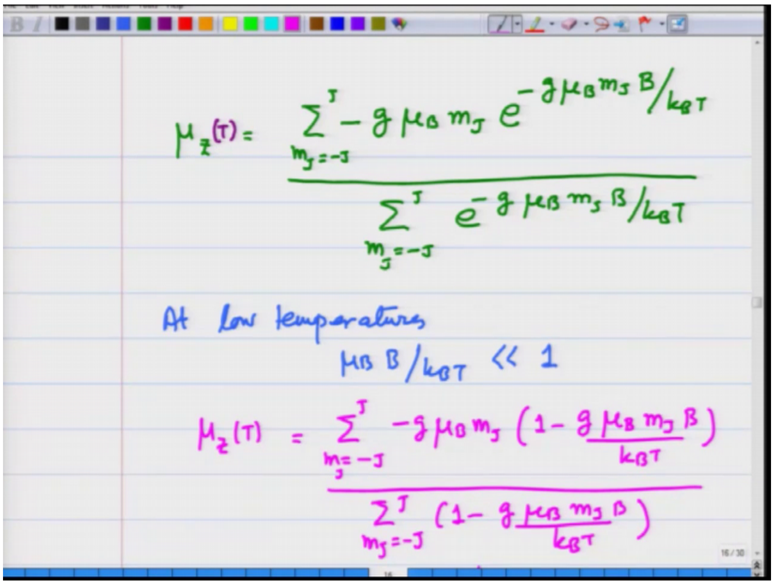


So, let us do that. So, mu z is going to be minus g mu B m J e raised to minus g mu B m J times the applied field divided by k B T summed over m J equals minus J to J divided by summation m J equals minus J to J e raised to minus g mu B m J B over k B T. This is the net magnetic moment and if you like, I can call this as a function of temperature and as a function of applied field B. Now, what we will find is that at low temperatures and we are going to specify what these low temperatures are later in the next lecture when I do certain examples.

Let me take this mu B times B over k B T to be much less than 1. Then I can expand this exponential term and write mu z as a function of temperature as equal to summation m where m J varying from minus J to J minus g mu B m J times 1 minus g mu B m J B over k B T divided by summation m J equals minus J to J 1 minus g mu B m J B over k B T. Under this approximation this is what mu z is going to B. Let me write this again on the next page.

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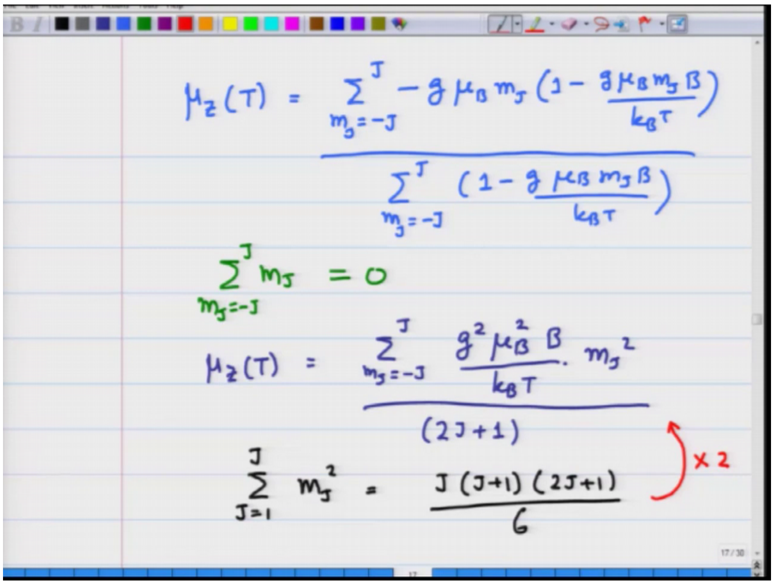


So, I have mu z as a function of temperature is equal to summation m J equals minus J to J minus g mu B m J 1 minus J mu B m J B over k B T divided by summation m J equals minus J to J 1 minus g mu B m J B over k B T. Let us look at the first term in the numerator. Summation m J m J equals minus J to J is equal to 0 because they are equal number of negative and positive terms. And therefore, I can write mu z T equals summation m J equals minus J to J g square mu B square B over k B T times m J square divided by the second term in the denominator will be 0 and I get 2 J plus 1.

Now, use the series sum formula for summation m J square J equals 1 to J. Is going to be equal to J J plus 1 times 2 J plus 1 divided by 6 and I will substitute that in the expression above. Now this is going here multiplied by 2 because I have m J going from minus J to minus 1 and m J going from 1 to J and 0 anyway does not contribute.

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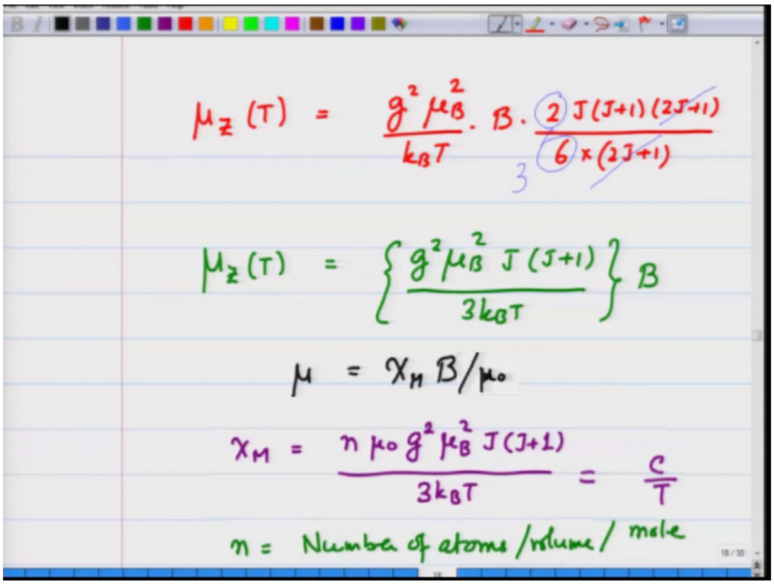


So, I get mu z T as equal to g square mu B square over k B T times B times 2 times J J plus 1 2 J plus 1 divided by 6 times 2 J plus 1. And I cancel this term and this 2 goes with 6 and gives me 3 and therefore, I end up getting mu z T as equal to g square mu B square J J plus 1 divided by 3 k B T times B. And if I compare this with the expression for susceptibility, mu is equal to chi magnetic times B divided by mu 0. I get chi m equals n mu 0 g square mu B square J J plus 1 divided by 3 k B T. Where n is the number of atoms per unit volume or per mole depending on how you want to write, what you want to calculate chi m for. And this equals C by C where C is a constant.

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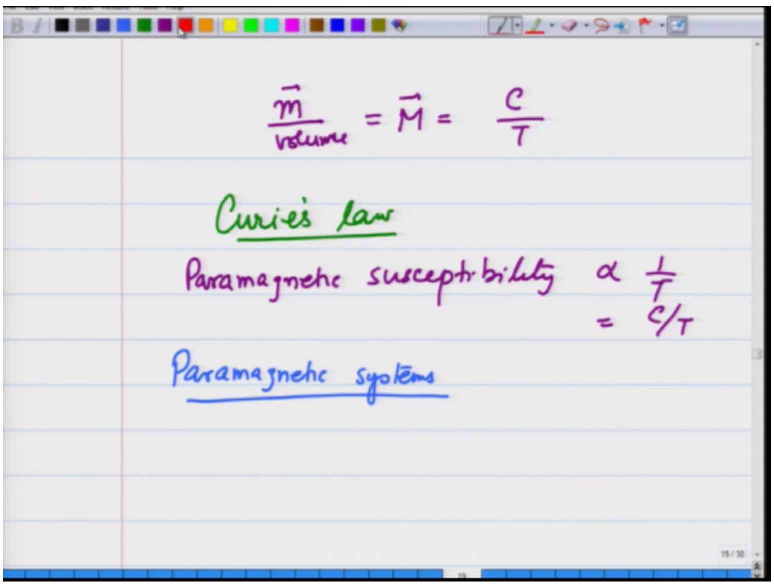

So, what I found through this calculation is that, when I put this collection of atoms in a solid and the atomic magnetic moments are not interacting with each other; I get the magnetic moment m per unit volume, which is also known as the magnetization is equal to some function C divided by T, it is inversely proportional to T. This is known as Curie's law. What does that law say? It says that, paramagnetic susceptibility is proportional to 1 over T is goes as some constant divided by T. As temperature goes up the paramagnetic susceptibility goes down because the levels start filling roughly equally at very high temperature.

So, this is the introduction to paramagnetic systems. Systems where what we have done is, we have taken an atom see how the levels are split under Russell Saunders or L s coupling when a field is applied. And then populated them according to the Boltzmann's law and using that we found the paramagnetic susceptibility.

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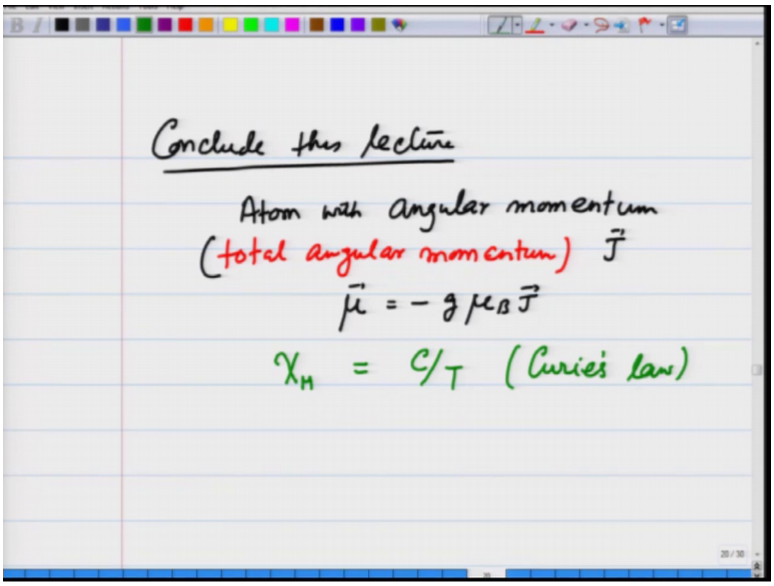


So, to conclude this lecture, what we have found is that atom with angular momentum and when I say angular momentum, I mean the total angular momentum. So, let me specify total angular momentum J has magnetic moment mu equals minus g mu B J. And when I apply magnetic field these levels due to different z component of J split and populate differently and that leads to a paramagnetic susceptibility chi m equals sum C over T, which is known as Curie's law. How these Js and L and s are calculated will be the subject of next lecture.