Introduction to Solid State Physics
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Lecture – 75
Paramagnetism in solids - I
Magnetic moment and Lande g factor for atoms

This week we are going to learn about another solid state phenomenon called magnetism.

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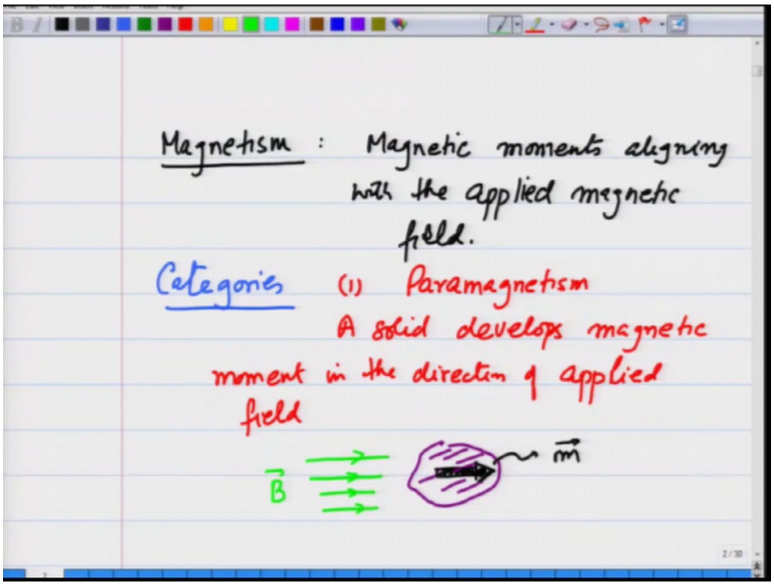

Magnetism in solids or any system arises from magnetic moments aligning with the applied magnetic field. It can also arise spontaneously which is known as Ferro magnetism. So, let me just write the categories of magnetism that we have. One paramagnetism and this is a phenomena where a system a solid develops magnetic moment in the direction of applied field. So, if I take a substance a sample and put it in an applied magnetic field say in the direction going to the right, then it will develop a magnetic moment in the same direction as the applied field. So, in here this is the magnetic moment developed in the sample and this is the applied field.

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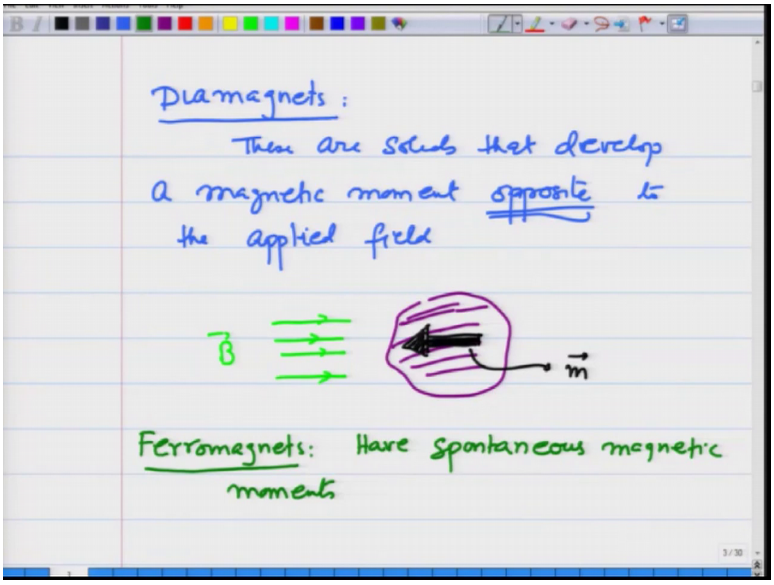

Another category of systems are known as diamagnets and these are systems that develop a magnetic moment opposite to the applied field. So, if I have a sample of the solid and I apply a magnetic field going to the right, this fellow would develop a magnetic moment in the opposite direction. So, this is the developed magnetic moment. This kind of you can think as a manifestation of the Faraday's law where a system develops currents that oppose the applied magnetic field.

And then we have Ferro magnets that have spontaneous magnetic moments. What does that mean? That means, even without applying a magnetic field these have magnetic moments. The magnets that you play around with are actually Ferro magnets. Then they are related systems to Ferro magnets called antiferro magnets ferri magnets that we may talk about, but mainly I want to focus on these 3 systems in these lectures.

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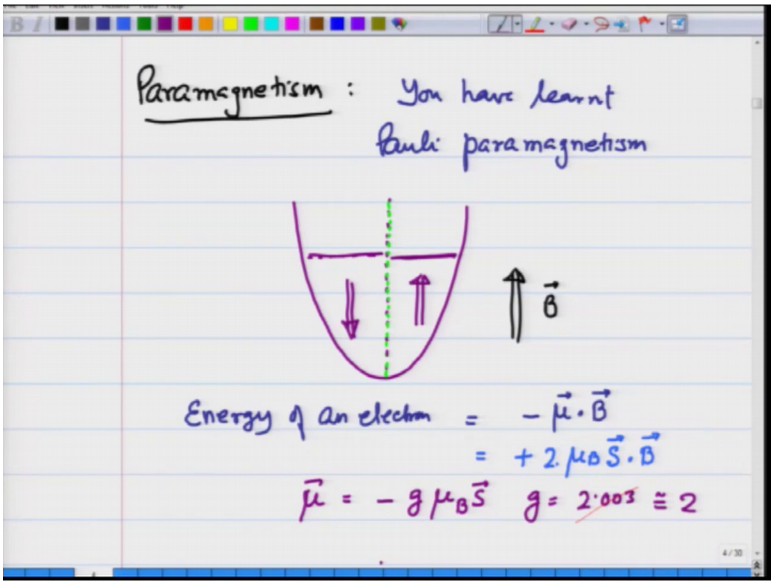

So, I want to start with para magnetism. And you are already familiar with this. You have learned earlier when professor Banerjee was taking lectures, in the context of free electrons you have learned about Pauli magnetism. What was Pauli paramagnetism?

It was if I have a system of electrons free electrons half of them have spin in one direction and the other half have spin in the opposite direction. And the line in the middle is the dividing line if you like. When I apply a magnetic field say going up then half of these electrons gain energy and half of them were lower energy. How is the energy of an electron is given? Energy of an electron is given by the interaction of it is magnetic moment with the field and the magnetic moment of the electron arises due to it is spin.

There is no orbital magnetic moment in this case because electrons are moving around. So, this is given as minus the magnetic moment dot B. And in the case of electron the magnetic moment is given as 2, what is known as the gyromagnetic ratio times the Bohr magneton times the spin of the electron dot B. The plus sign comes, this sign is plus because the magnetic moment mu of an electron due to the spin is minus g mu Bohr magneton times S where g is the value of actually 2.003, but we take it to be roughly 2. So, this is the origin.

So, what happens is those which are in the same direction as a spin have higher energy. And those which are in the opposite direction to the spin have lower energy. And that number therefore, the one with the lower energy increases. They spin in the opposite direction to the field means that the magnetic moment is in the same direction as the applied field and therefore, we develop the system develops due to these free electrons a magnetic moment in the direction of the applied field. So, that was known as the Pauli para magnetism.

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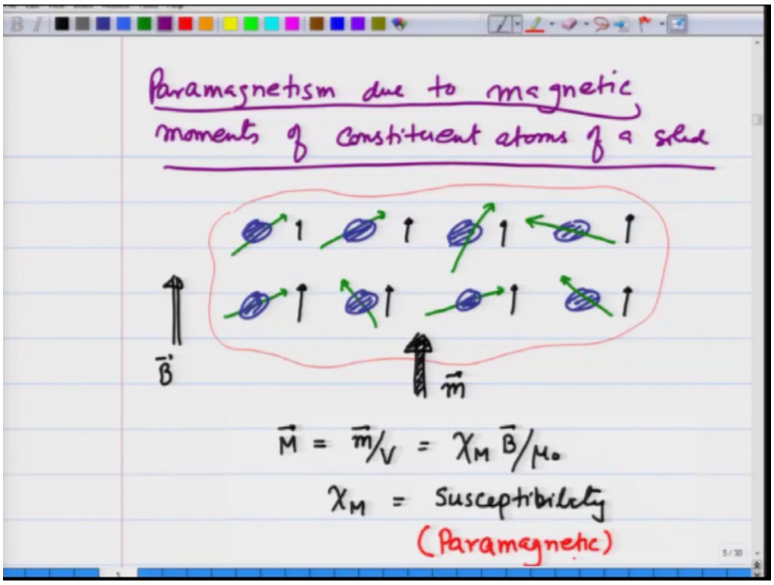

What we are talking about is para magnetism due to magnetic moments of constituent atoms of a solid. Slightly different context in that when I have the solid which is made up of these atoms each of these has a magnetic moment. Due to orbital motion of the electron as well as a spin and when I apply a field B these magnetic moments start aligning with the applied field. And therefore, this entire sample which I am showing by this encircled line develops a net magnetic moment. Due to this applied field B and therefore, it is a paramagnet the magnetization m which is magnetic moment per unit volume is given as the name susceptibility.

So, this is defined as chi magnetic B divided by mu 0 where mu 0 is the vacuum permeability. Now, in this case this chi M is the paramagnetic susceptibility. So, you want to understand how it arises. How the magnetic moments of a system aligned with B and what kind of magnetic moments do they give rise to and that will give us an expression for chi E.

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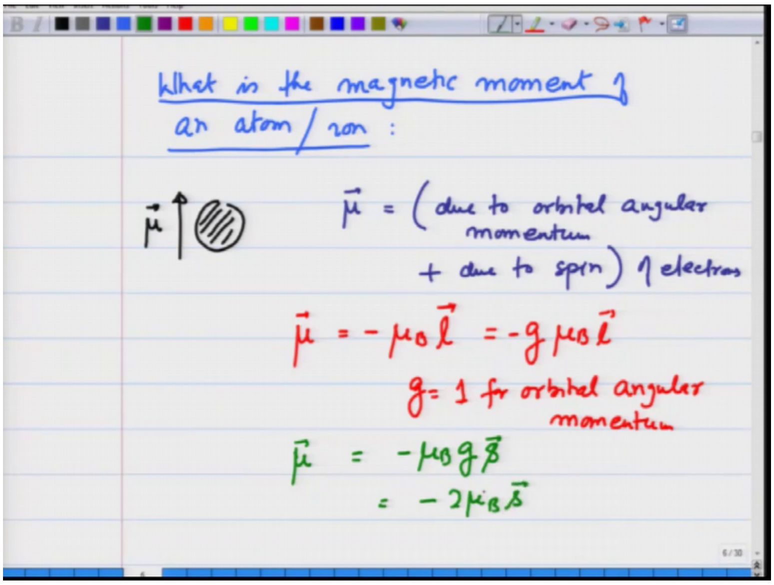

To do that I have to understand first what is the magnetic moment of an atom or ion because in a solid I may even have those atoms as ions because the other electrons may be just moving around. So, if I take a system of these atoms and each atom has some magnetic moment mu, it will arise due to 2 different kinds of angular momentum of the electron.

Mu will be one due to orbital angular momentum plus due to spin both of electrons. Now, if you recall your atomic physics or go back to your 12th grade where we had defined mu due to angular momentum comes out to be mu B times the angular momentum of the electron and there is a minus sign in front.

If I write it in the form that I was writing earlier I will write it as g mu B l with a minus sign in front where g is equal to 1 for orbital angular momentum. And mu due to spin is going to be minus mu B g due to a spin times s small which is minus 2 mu B due to spin.

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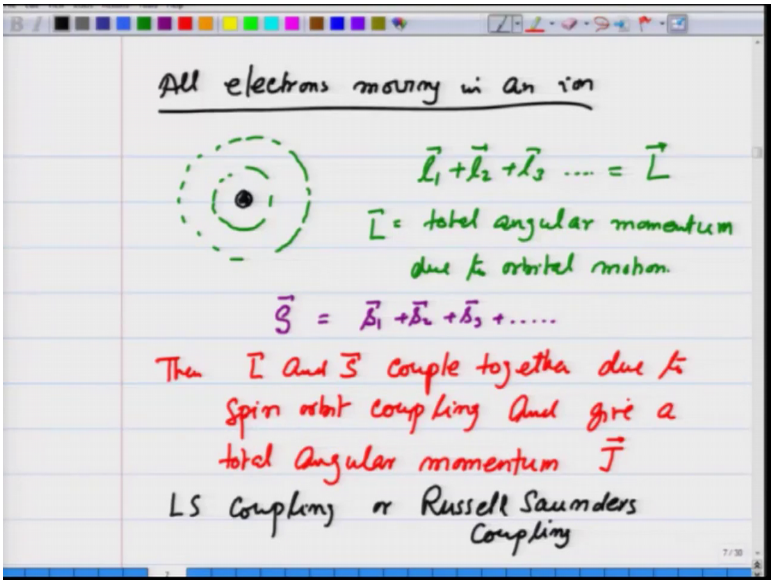

So, all electrons moving in an iron, what do they do? So, let us see here is a nucleus here is the electrons am making a very simplistic picture moving in different orbits. Then the orbital angular movement add up so am going to have the net angular momentum to be l 1 plus l 2 plus l 3 and all this adds up to a total angular momentum L. So, L is the total angular momentum due to orbital motion.

Similarly, all the spins are going to add up s 1 plus s 2 plus s 3 and so on. And then L and S couple then L and S coupled together due to spin orbit coupling. And give a total angular momentum J. This is known as LS coupling or Russell Saunders coupling. When I work in this coupling it means that the spin orbit coupling, the spin orbit interaction is much larger than the applied field. So, electrons first coupled together to form this total J and then the effect of the magnetic field on this total J it is on that total J through it is magnetic moment.

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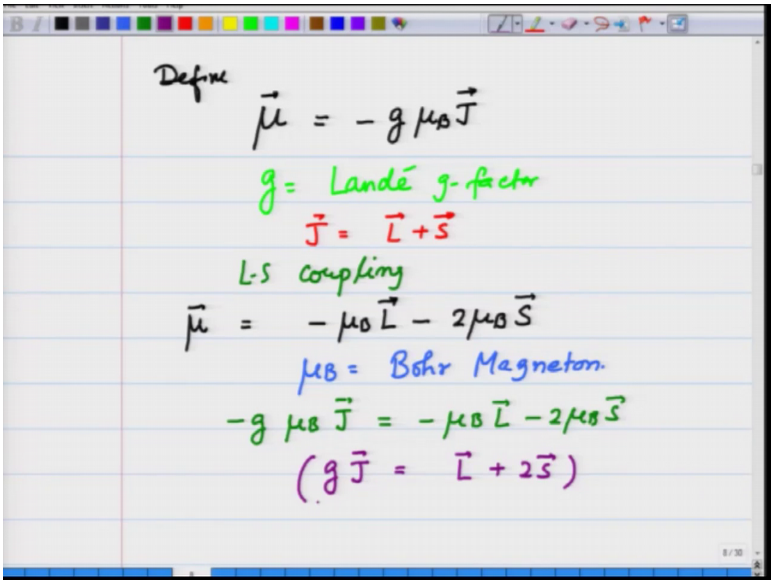

So now we are going to define mu. Due to this total angular momentum as equal to minus g mu B J, exactly like we were doing for the orbital angular momentum or the spin angular momentum g. Now we are going to call the Lande g factor or simply the g factor. It gives you the proportionality constant between the total angular momentum and the magnetic moment.

Remember it was 1 for orbital angular momentum and it is 2 for a spin angular momentum. So, it is going to be something in between. What is J? J is equal to L plus S. So, we will now get an expression for g what is that effective lambda g factor that gives you the proportionality constant between the total angular momentum and the magnetic moment and we are; obviously, working under the LS coupling. So, within that now we have amble derive this Lande g factor. So, to obtain this let me write mu is equal to mu is arising due to orbital angular momentum and spin angular momentum.

So, it is going to be minus mu B times total L orbital angular momentum minus 2 mu B s. I should have specified earlier I think I did not mu B is the Bohr magneton. Now this mu is also equal to minus g mu B J. So, therefore, we get minus mu B L minus 2 mu B S. And therefore, I get g times J is equal to l plus 2 S and from this am going to get an expression for g.

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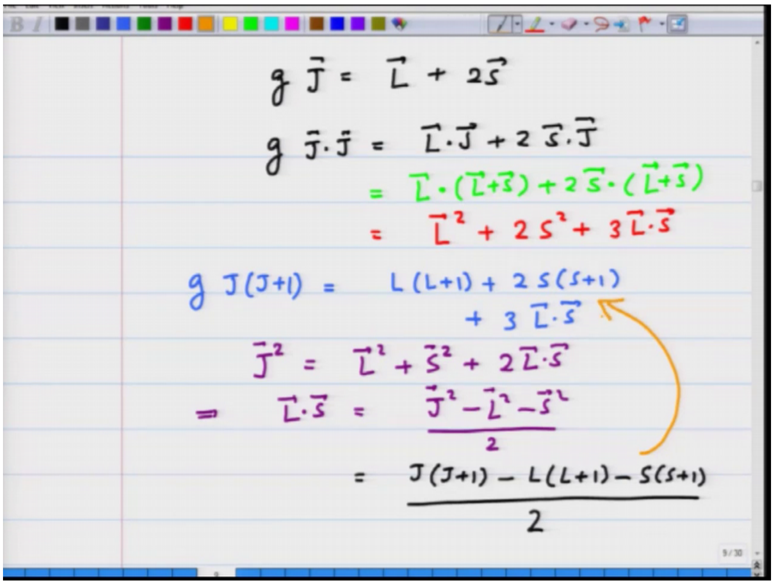

Let me go to the next page we have g J is equal to L plus 2 S. If I take the dot product with J I get g J dot J is equal to L dot J plus 2 S dot J which I can write as L dot L plus S plus 2 S dot L plus S which I can then write as l square plus 2 S square plus 3 L dot S. If I write their expectation values, I get g J J plus 1 this is the expectation value for the total angular momentum is going to be L L plus 1 plus 2 S S plus 1 plus 3 L dot S.

Let me now calculate the value of L dot S in terms of L and J and S. Number J square vector is going to be equal to L square vector plus S square vector plus 2 L dot S and therefore, L dot S comes out to be J square minus L square minus S square divided by 2. Let me put vectors on top, which then is equal to J J plus 1 minus L L plus 1 minus S S plus 1 this whole thing divided by 2, this is the value of L dot S. And I am going to substitute this in this expression on top.

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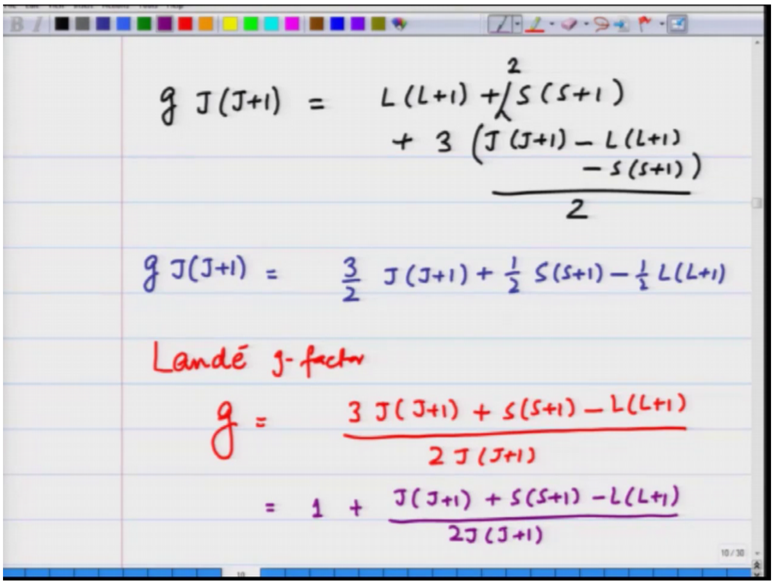

To get g J J plus 1 is equal to L L plus 1 plus S S plus 1 plus 3 times J J plus 1 minus L L plus 1 minus S S plus 1 this whole thing divided by 2. Here is a 2 here also this gives g J J plus 1 is equal to 3 by 2 J J plus 1 plus 1 by 2 S S plus 1 minus 1 by 2 L L plus 1. And therefore, the lambda g factor g is equal to 3 J J plus 1 minus or plus S S plus 1 minus L L plus 1 divided by 2 J J plus 1. This is also written some times as equal to 1 plus J J plus 1 plus S S plus 1 minus L L plus 1 divided by 2 J J plus 1 whichever form you find suitable remember that.

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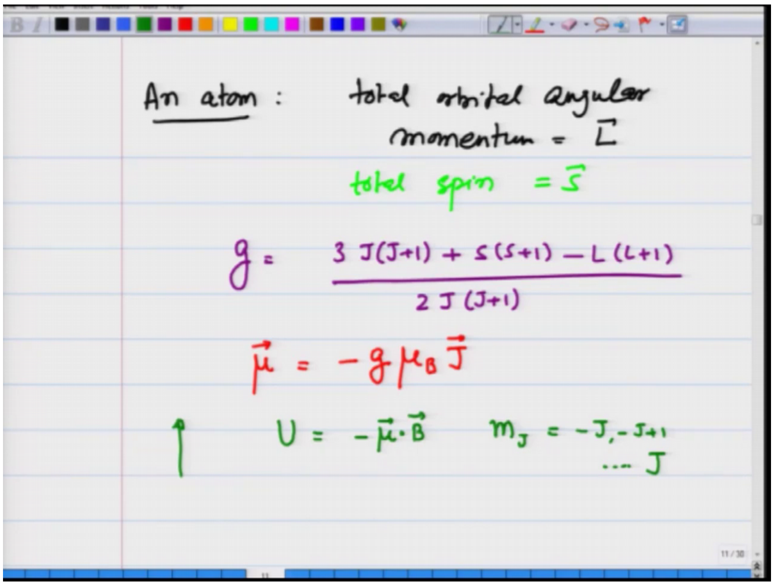

So, what we found now, if I have an atom that has a total orbital angular momentum equals L. Total spent equals S then taking into consideration the magnetic moments due to L and S, I get the Lande g factor g equals 3 J J plus 1 plus S S plus 1 minus L L plus 1 divided by 2 J J plus 1.

Therefore, the magnetic moment of the system is given as minus this g that we have calculated mu B where mu B is the Bohr magneton times J. And when I apply a magnetic field this each mu is going to have an energy U equals minus mu dot B. And therefore, each m J value of J which varies from minus J minus J plus 1 all the way up to J is going to have a different energy these levels will split.

And they will be populated differently at a given temperature and that gives rise to a magnetic moment. So, in this lecture this is what I have derived that how the magnetic moment of an atom is calculated and now I am going to apply this to calculate the paramagnetic susceptibility of a collection of these atoms in a solid.