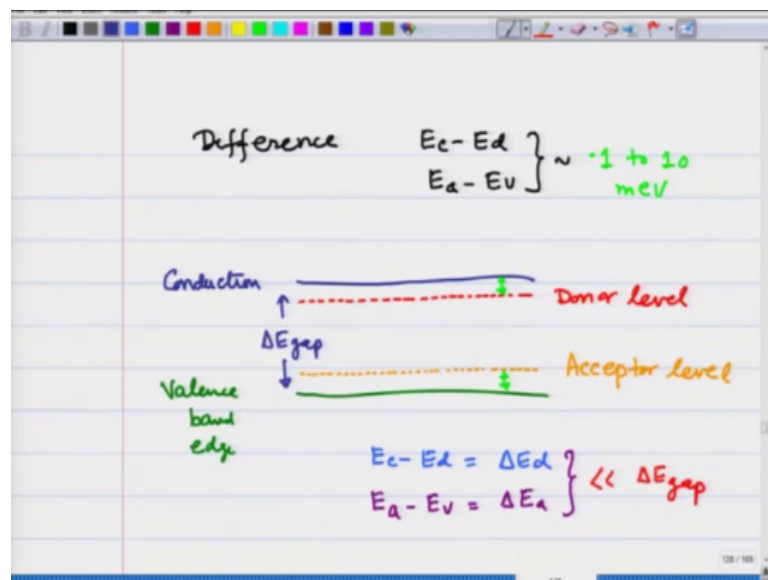


Introduction to Solid State Physics
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Lecture – 73
Charge carrier density in n-type and p-type semiconductors

In the previous lecture, we dealt with acceptor and donor levels in a semiconductor and showed that the difference $E_c - E_d$ or $E_a - E_v$ is of the order of 0.1 to 10 meV.

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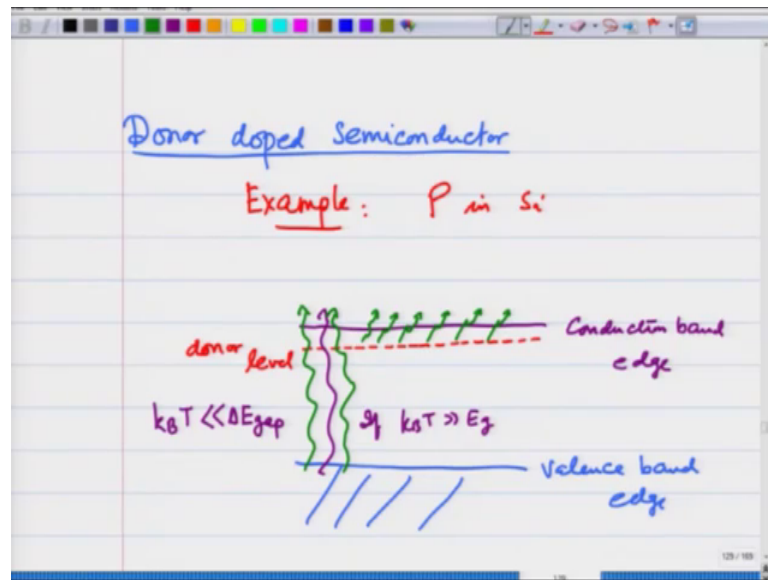


So, let me just show that in picture and I will make the band straight now. So, here is the conduction band edge and here is the valence band edge and the donor level is going to be somewhere here.

And the acceptor level is going to be somewhere here. Of course, this exaggerated, they are even closer. So, this is I am going to call acceptor level and I am going to call this donor level. They are much smaller these gaps out here or out here are much smaller than the intrinsic band gap. So, let me just write that $E_c - E_d$ which I will call ΔE_d or $E_a - E_v$ which I am going to call ΔE_a . They are much much less than ΔE_{gap} which is out here.

So, at room temperature which is of the order of 25 milli electron volts, main contribution to the charge carriers will come either from donor level or acceptor level if both are not present. So, let us now understand what the behavior of these charge carriers is going to be.

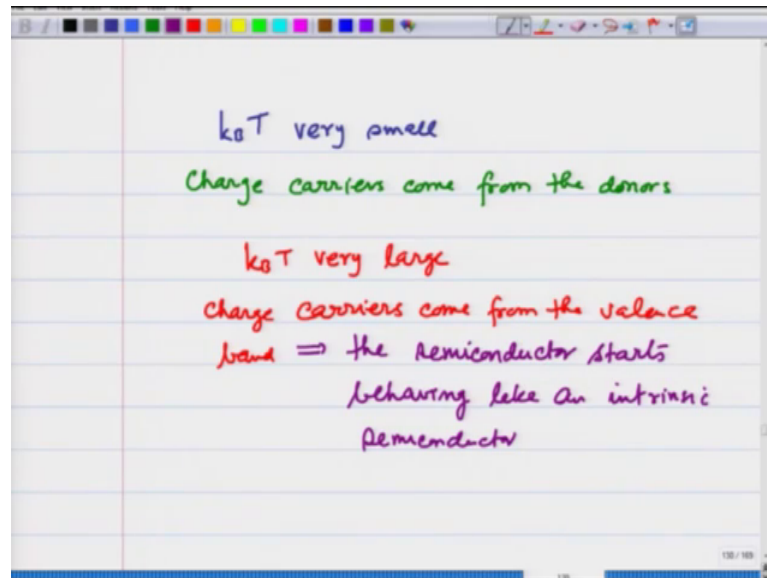
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So, let us first understand donor doped semiconductor and the example is going to be phosphorous in silicon. It is just for the example sake. So, if I were to see the levels here is the valence band edge. So, lower side is odd valence band, on top is conduction band edge and here is the donor level, right underneath it. If the temperature is small, so, let us say $k_B T$ is much smaller than E_{gap} the only contribution to electrons in the conduction band will then come from the donor level.

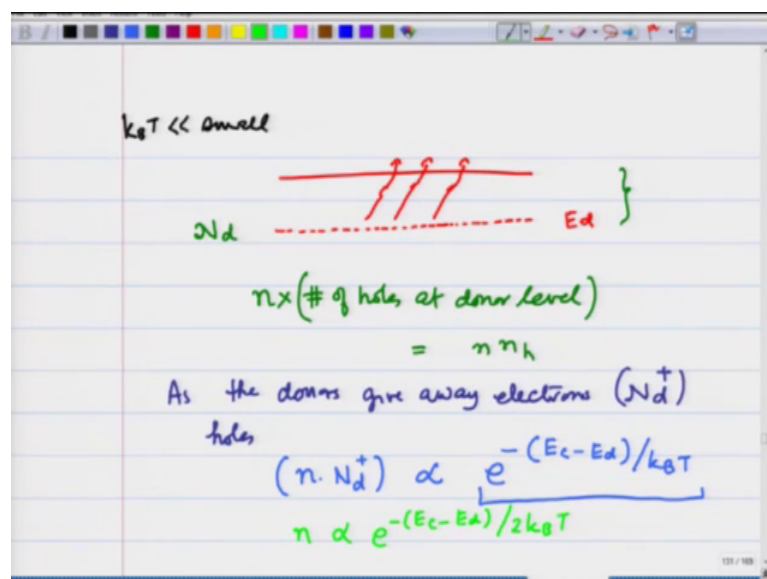
If T becomes very large right on the other hand if $k_B T$ is much larger than E_{gap} then electrons will start coming from the lower level also. And in fact, as I remarked earlier the concentration of donor levels is very small and therefore, at large temperatures is the electrons from the valence band that come to conduction band that will dominate the charge carriers. So, let us identify these regimes.

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$k_B T$ very small and remember this smallness and largeness is with respect to the band gap or the donor level with respect to the conduction band then charge carriers come from the donors and this behavior is going to be different from when I have $k_B T$ very large, in that case charge carriers come from the valence band. And therefore, the semiconductor starts behaving like an intrinsic semiconductor. So, let us see how the behavior changes roughly with respect to the temperature.

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When $k_B T$ is very small, then here is the conduction band, here is the donor level mainly carriers are coming from here. So, like we did for intrinsic semiconductors mainly things are coming from the donor levels and let us say the concentration of donor levels is N_d .

Then what I am going to have is that the number of electrons in the conduction band times the number of holes at donor level is going to be equal to that $n p$ and I am going to repeat this calculation taking only these two levels. If I do that what I should have is as the donors give away electrons I am left with N_d plus the donors that have ionized holes. So, I am going to have the number of electrons times the number of holes proportional to E raise to minus now the energy gap that I am going to have is E_c minus E_d over $k_B T$.

So, the temperature dependence will come mainly from this term and exactly like we did for the intrinsic semiconductors and then is going to be proportional to E raise to minus E_c minus E_d divided by $2 k_B T$. So, this is the temperature dependence when $k_B T$ is very small and main contribution to the charge carriers in the conduction band comes from the donor levels alone.

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The image shows a digital whiteboard with the following handwritten content:

$$n = 2 \left(\frac{m_e^* k_B T}{2\pi\hbar^2} \right)^{3/2} e^{-(E_c - E_F)/k_B T}$$

$$n N_d^+ = f(T)$$

N_d^+ = Number of ionized atoms
= Number of holes

$$N_d^+ = N_d e^{-(E_F - E_d)/k_B T}$$

At low temperatures $n \sim N_d^+$

$$n \approx \left[2 \left(\frac{m_e^* k_B T}{2\pi\hbar^2} \right)^{3/2} \right]^{1/2} e^{-\frac{(E_c - E_d)}{2k_B T}}$$

If you want to get the prefix also then I am going to have n equals $2 m_e^* k_B T$ over $2\pi\hbar^2$ cross square raise to 3 by 2 e raise to minus E_c minus E_F over $k_B T$. This is the number of electrons and we are going to have n times N_d plus equals a function of

temperature alone by the law of mass action where N_d plus is equal to the number of ionized atoms which is equal to the number of holes.

Now, I am going to calculate later that we are going to have N_d plus is equal to N_d where N_d is the number of donor atoms E raised to minus E_F minus E_{donor} divided by $k_B T$. Now, using the law of mass action and multiplying $n N_d$ plus we can get the number of electrons in the conduction band. Notice that at low temperatures all the electrons are coming from the donor levels. So, I can say that n is roughly equal to N_d plus and then using all this I am going to get n equals $2 m_e$ star $k_B T$ over $2 \pi \hbar$ cross square raise to 3 by 2 this whole thing raise to one half E raise to minus E_c minus E_d divided by $2 k_B T$.

And I am going to put a an approximate sign here because we have made certain assumptions along the way. Let us now fill in the details of this calculation.

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The image shows a whiteboard with handwritten notes and diagrams. At the top, it says "(k_BT) very small". Below this is a diagram of energy levels: a dashed blue line labeled "donor level" is below a solid red line labeled "Conduction band". The Fermi level E_F is indicated by a red horizontal line between the donor level and the conduction band. Green arrows point upwards from the donor level towards the conduction band. Below the diagram, the text "(1) Where is the Fermi level" is written. The equation
$$n = 2 \left(\frac{k_B T m_e^*}{2\pi \hbar^2} \right)^{3/2} e^{-\frac{(E_c - E_F)}{k_B T}}$$
 is written in the center. At the bottom, it says "By law of mass action $n \cdot$ (holes in the donor level) = $f(T)$ ".

So, let us work on finding out the number of carriers when $k_B T$ is very small and that is when I say smallest relative to the band gap of the intrinsic semiconductor. So, in that case what we had argued is that this is the conduction band edge this is where it starts from and I have the donor level right here and the electrons are going to come mainly from here.

So, first question I ask is where is the Fermi level? And you can see that the Fermi level again arguing like we did for the intrinsic semiconductors is going to be somewhere in the middle of the donor levels and the conduction band edge. And the number of electrons is going to be equal to; remember the way we did the calculation for the intrinsic semiconductors, number of electrons is going to be proportional to you know $2 k_B T m_e^* / 2 \pi \hbar^2$ raised to $3/2$ $e^{-E_c - E_F / k_B T}$.

Now, this is the number of electrons in the conduction band; E_F I do not know yet and by law of mass action that we discussed earlier I am going to have n times the holes in the donor level is equal to a function of temperature. So, using this I can find the number of electrons n in the conduction band due to ionization from the donor level.

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The image shows a whiteboard with handwritten notes. At the top, it says "Number of holes in the donor level = N_d^+ = Number of donor atoms that are ionized". Below this, there is an energy level diagram with three horizontal lines: a solid line at the top labeled E_c , a dashed line in the middle labeled E_f , and a solid line at the bottom labeled E_d . To the left of the diagram is the label N_d^+ . To the right, there are two formulas: $f(E_d) = \frac{1}{e^{(E_d - E_f) / k_B T} + 1}$ and $f_h(E_d) = 1 - \frac{1}{e^{(E_d - E_f) / k_B T} + 1} = \frac{e^{(E_d - E_f) / k_B T}}{1 + e^{(E_d - E_f) / k_B T}}$.

Now, let us see the number of holes in the donor level is going to be equal to N_d plus by N_d plus I mean the number of donor atoms that are ionized. So, to calculate N_d plus I am again going to use the same trick I use for the intrinsic semiconductors and let us see what we did there. Here is my conduction band edge, here is the Fermi level E_F which I have already argued is somewhere in the middle of E_d and E_c .

Now, the probability of occupation of E_d is going to be equal to $1 / (e^{(E_d - E_f) / k_B T} + 1)$. So, probability of having a hole f_h here if not being occupied at E_d is going to be equal to $1 - 1 / (e^{(E_d - E_f) / k_B T} + 1)$.

T plus 1. So, this is equal to $e^{(E_d - E_F)/k_B T}$ over $1 + e^{(E_d - E_F)/k_B T}$.

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The image shows a whiteboard with the following handwritten content:

$$f_h = \frac{e^{(E_d - E_F)/k_B T}}{e^{(E_d - E_F)/k_B T} + 1}$$

$(E_d - E_F) < 0$

$\frac{E_d - E_F}{k_B T} \rightarrow$ large negative number

$$f_h = e^{(E_d - E_F)/k_B T}$$

Number of holes
 $N_d^+ = N_d e^{(E_d - E_F)/k_B T}$

So, I get f_h which is $e^{(E_d - E_F)/k_B T}$ over $1 + e^{(E_d - E_F)/k_B T}$. Now, we already said $k_B T$ is very small and $E_d - E_F$ is negative and $(E_d - E_F)/k_B T$ is going to be large negative number. So, I can write f_h is equal to $e^{(E_d - E_F)/k_B T}$ roughly. And therefore, the number of holes at the donor level N_d^+ is going to be equal to N_d times $e^{(E_d - E_F)/k_B T}$.

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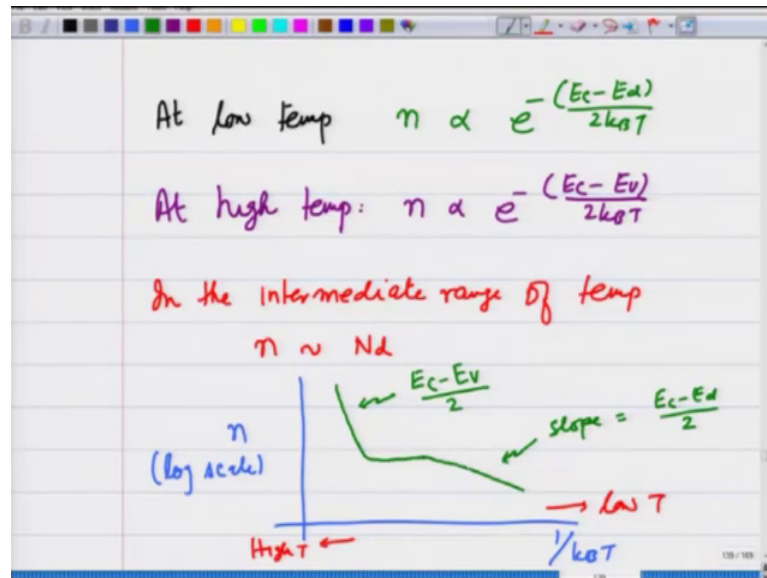
$$\begin{aligned}
 n \cdot N_d^+ &= 2 \left(\frac{m_e^* k_B T}{2\pi \hbar^2} \right)^{3/2} e^{-(E_c - E_F)/k_B T} \\
 &\quad \times N_d \cdot e^{(E_d - E_F)/k_B T} \\
 &= 2 N_d \left(\frac{m_e^* k_B T}{2\pi \hbar^2} \right)^{3/2} e^{-\frac{(E_c - E_d)}{k_B T}} \\
 n &\approx \left[2 N_d \left(\frac{m_e^* k_B T}{2\pi \hbar^2} \right)^{3/2} \right]^{1/2} e^{-\frac{(E_c - E_d)}{2k_B T}} \\
 \text{Intrinsic Semiconductors} &\sim e^{-\frac{(E_c - E_v)}{2k_B T}}
 \end{aligned}$$

This gives me N_d plus is equal to $2 m_e^* k_B T$ over $2\pi \hbar^2$ cross square 3 by 2 times e raise to minus E_c minus E_F over $k_B T$ times N_d times e raise to E_d minus E_F over $k_B T$ which is nothing, but $2 N_d m_e^* k_B T$ over $2\pi \hbar^2$ cross square raise to 3 by 2 e raise to minus E_c minus E_d divided by $k_B T$.

Since the particles are coming mainly the electrons are coming mainly from the donor levels I can therefore, then say that n is therefore, is proportional to or roughly equal to $2 N_d m_e^* k_B T$ over $2\pi \hbar^2$ cross square raise to 3 by 2 this whole thing raise to 1 by 2 e raise to minus E_c minus E_d over $2 k_B T$. You see this calculation exactly like what we did for intrinsic semiconductors, except now I get this factor of N_d here rather than there it was the expression for the hole because all the holes are coming from the donor level.

And this is the proportionality term for the temperature which in the case of intrinsic semiconductors was e raise to minus E_c minus E_v over to $k_B T$. So, that E_v has been now replaced by E_{donor} .

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So, at low temperatures n goes as e raised to minus E_c minus E_d divided by $2k_B T$ and at high temperatures, I had already argued that now that high temperatures the domination is going to be from the carriers coming from valence band and therefore, n is going to be proportional to e raised to minus E_c minus E_v over $2k_B T$.

And in the intermediate range of temperature the n would be roughly equal to N_d because all the donors would have been ionized. So, if I were to plot the carriers on a log scale versus 1 over $k_B T$. So, that this is high temperature this is low T , I am going to get a curve which is slope E_c minus E_v by 2 in the beginning then there will be a saturation because intermediate range all the ions are ionized and then there will be a less slope which will be proportional to E_c minus E_d . So, here slope is E_c minus E_d divided by 2 . Here the slope will be E_c minus E_v divided by 2 .

So, you see how over a range of temperature, the carriers n the conduction band are varying at high temperature they the change is very fast depending on the band gap of the intrinsic semiconductor, at low temperatures it is slow; it is E_c minus E_d divided by 2 and in between the range remains constant and this is how the conductivity of the system is also going to be affected. And you can see from the kind of range I gave you earlier for E_c minus E_d at room temperature it is the carriers coming from the donor or acceptor level that are going to be dominating the conductivity and things those properties transport properties.

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Acceptor Levels : $e^{-\frac{E_a - E_v}{2k_B T}} = n_h$

Conductivity of doped semiconductors

Exactly the same calculation would also be acceptable for acceptor levels. In acceptor levels I am going to get $E_a - E_v$ over $k_B T$ as a proportionality constant for number of holes, so, that I will leave for you. So, the conductivity of doped semiconductors is going to give you two different slopes depending on what temperature range are you at.

So, we will derive in the next lecture the expressions for the conductivity and also other transport property like hall coefficient for a semiconductor and show their usefulness that will complete our lectures on semiconductors.

Thank you.