Introduction to Solid State Physics
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Lecture – 71
Calculating carrier density in semiconductors – II

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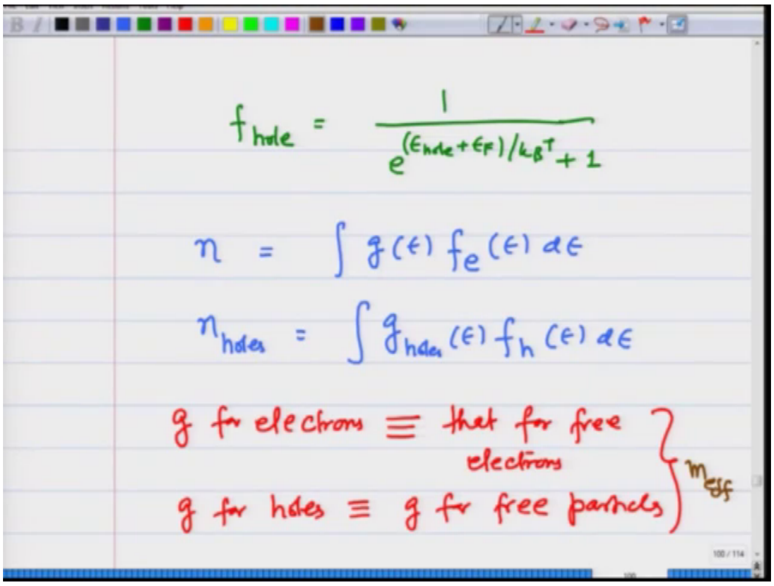

In the previous lecture I derived f hole which is equal to 1 over e raised to epsilon hole plus epsilon F over k B T plus 1 and we had also seen that the number of electrons n is going to be equal to g epsilon this is all about electrons f let me write this e as a function of E d E and number of holes is going to be integration g holes epsilon f h epsilon d E. I had also argued that, g for electrons is identical to that for free electrons. And now I will show that g for holes is also equivalent to g for free particles. The only difference in these two from free particles is going to be that now I have m effective instead of m free.

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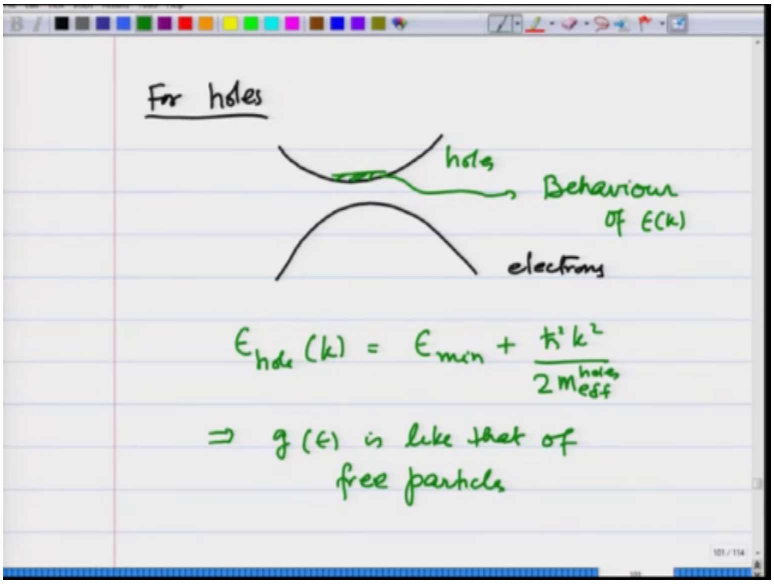

Now for the holes if I have an energy band like this for electrons. We have seen that for holes the e becomes minus e so the band goes like this. This is for the holes and again, near the bottom the behaviour of epsilon k it is like that of free particles. So, epsilon hole as a function of k is like this epsilon min which we can easily take to be 0 plus h cross square k square over 2 m effective of holes.

So, the energy or the dependence of the energy on k is pretty much like free particles and this implies g E is like that of free particles. So, now to calculate the number of electrons and number of holes what we do is fix our integration limit and let us do that now. Before that let me write the formula, g as a function of E is equal to 1 over 2 pi square 2 and let me write this m effective over h cross square raised to 3 by 2 epsilon raised to 1 half.

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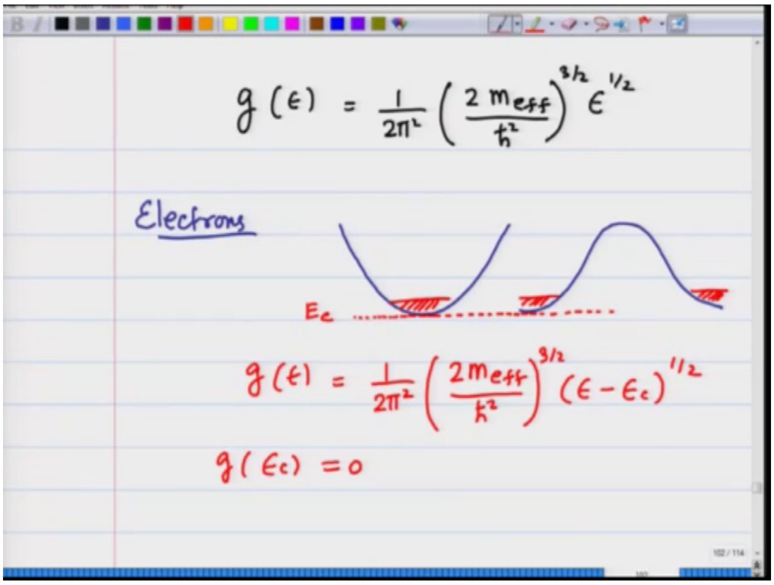

Now, when we are doing electrons and holes we have to put the proper integration limits. So, let us see for electrons what we are going to do? Electrons if there is a band like this or even like this the electrons are filled near the minimum and if this minimum is written as E C or E Conduction then it is easy to see that I am going to have g E is equal to 1 over 2 pi square 2 m effective and this is for electrons divided by h cross square raised to 3 by 2 E minus E C raised to 1 half. So, g E C is equal to 0 and it rises, to some energy which is not very high as long as the electrons are up to the level where the energy dependence is k square this is what the g E is going to be. How about for the holes?

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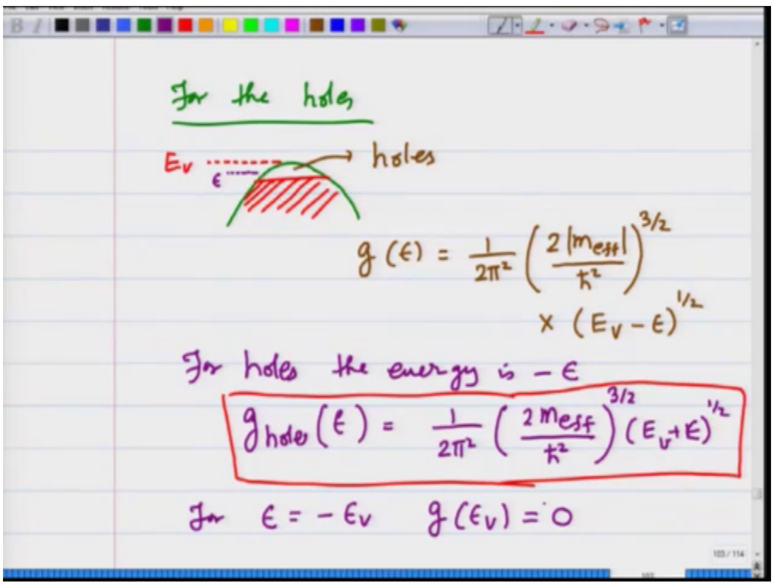

Again let me make a band like this, the electrons are filled up to certain point and this empty region these are the holes. So, the distribution of holes is exactly the same as that of electrons. So, I am going to have g of E is equal to 1 over 2 pi square 2 mod m effective. Let me write it mod m effective because electron effective mass here is negative over h cross square raised to 3 by 2 times E V minus E raised to 1 half where E V is the top of the valence band, this is E V. So, I want E V minus E to be a positive number and this is E, E is right here.

Now for holes, the energy is minus that of the electron. So, g holes for energy of hole equal to epsilon I am going to have 1 over 2 pi square 2. Now I can write m effective without the mod sign because hole effective mass becomes positive automatically h cross square raised to 3 by 2 E V minus minus I will have plus E over raised to 1 half. So, this is the density of states in terms of the hole energy, E V is a given number. Notice that for epsilon hole equals minus E V g E V is equal to 0. So, this is all consistent. How about the limits?

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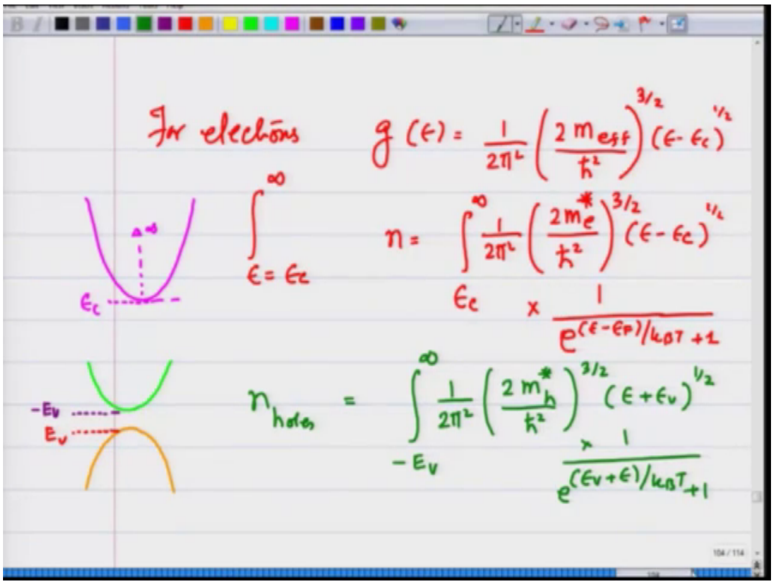

Now, for electrons g E was equal to 1 over 2 pi square 2 m effective in the conduction band over h cross square raised to 3 by 2 E minus E C raised to 1 half. So, integration limit is going to be from E equals E C to infinity. So, this is a band and we are starting E from E C and all the way up to infinity. So, the number of electrons n is therefore going to be integration E C to infinity 1 over 2 pi square 2 m e by that we will understand that this is electron mass but effective mass. To indicate it is effective let me also put a star on top. 3 by 2 E minus E C raised to 1 half times the probability which is going to be 1 over e raised to E minus E F over k B T plus 1. This is the number of electrons.

Similarly, number of holes n holes is going to be let me again determine the limits. Here was the electronic band and here is a band for the holes. If this was the top of the valence band was E V the bottom of the hole band is going to be minus E V. So, I am going to have number of holes is equal to minus E V to infinity 1 over 2 pi square 2 m hole star to show it is effective mass divided by h cross square raised to 3 by 2 E plus E V raised to 1 half times 1 over E raised to epsilon v plus E over k B T plus 1. This is the integration that we got to vary out. So, we have now determined the formula that I should be applying to calculate the number of holes and number of electrons.

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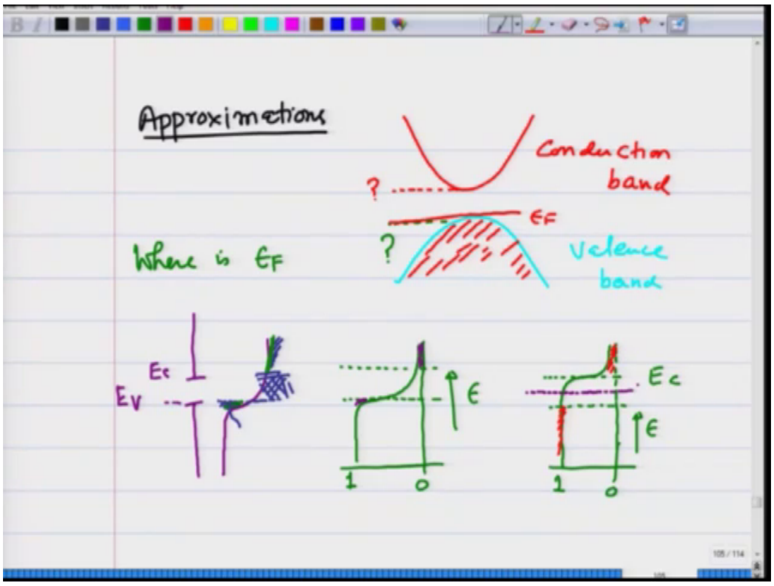

Now, some approximations; so, when I have these conduction band and the valence band, where is epsilon F or the Fermi energy? Is it at the top here question mark, is it at the bottom here question mark or is it somewhere else? So, let us see, at 0 temperature all these states the valence band is completely filled. And therefore, E F can be at the top of the conduction band there is a possibility. On the other hand as the temperature is increased you will see, now let me make this slightly differently. Here is the gap and here is the valence band. This is E V this is E C on top. The Fermi Fermi-Dirac distribution would change like this.

Roughly if E F was on top of the valence band all the electrons from here would go out and go into the conduction band. There is no electrons in between region all the electrons will be here. Whatever the number of electrons that are getting out of here should be going into the conduction band, but you notice that the probability that has reduced out here is larger than the probability of occupation in the conduction band, because if the Fermi level is here let me make it again more clearly here is the gap the Fermi Fermi-Dirac distribution would go like this.

Where this is 0, 1 and this is E. So, the number of electron that I have gone out of here will be larger than the number of electrons that have reached in the conduction band. On the other hand look at the other possibility. If the Fermi level was at E C then, the Fermi Fermi-Dirac distribution as temperature rises would be like this. Where this is 1, this is 0 and this is E. In this case hardly any electrons are going out of the valence band, but many electrons are here in the conduction band in both the cases, case in the middle and on the right there is a disbalance. Where as I know in an intrinsic semiconductor the number of electrons that go out from the valance band should go only to the conduction band and s o, they should be equal.

So, I am going to have E f somewhere in the middle. When you are looking at these slides please go over this argument yourself and convince yourself that it will be somewhere in the middle. If it is on top of the valence band or at the bottom of the conduction band there is there will be this is going to be a disbalance in the number of electrons that leave the valance band and those number of electrons that reach the conduction band.

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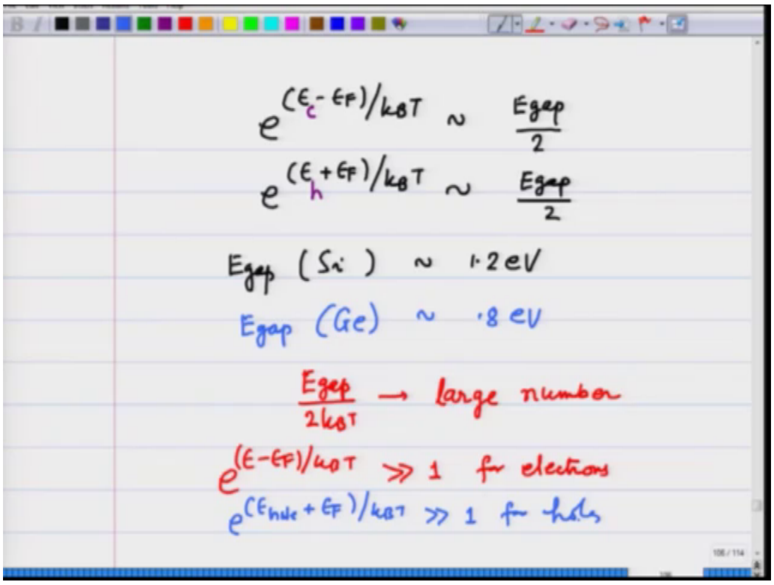

Now, if it is somewhere in the middle let us look at these numbers. Now, e raised to E minus epsilon F over k B T is going to be of the order of E gap divided by 2. Similarly for the holes, E plus epsilon F over k B T is going to be of the order of E gap over 2. I should be little careful and write E C here and E hole here. And E gap for silicon is about 1.2 electron volts. E gap germanium is about 0.8 E V.

So, you can see that E gap divided by 2 k B T is going to be a large number. If that is the case, then e raised to epsilon minus epsilon F over k B T is going to be much larger than 1 for electrons, and e raised to epsilon hole plus epsilon F over k B T is also going to be much larger than 1 for holes and therefore I can neglect that one. And I can write f e as e raised to minus epsilon minus epsilon F over k B T and f hole is equal to e raised to minus E plus epsilon F over k B T. It is understood that here E refers to epsilon hole, which is minus epsilon missing electron.

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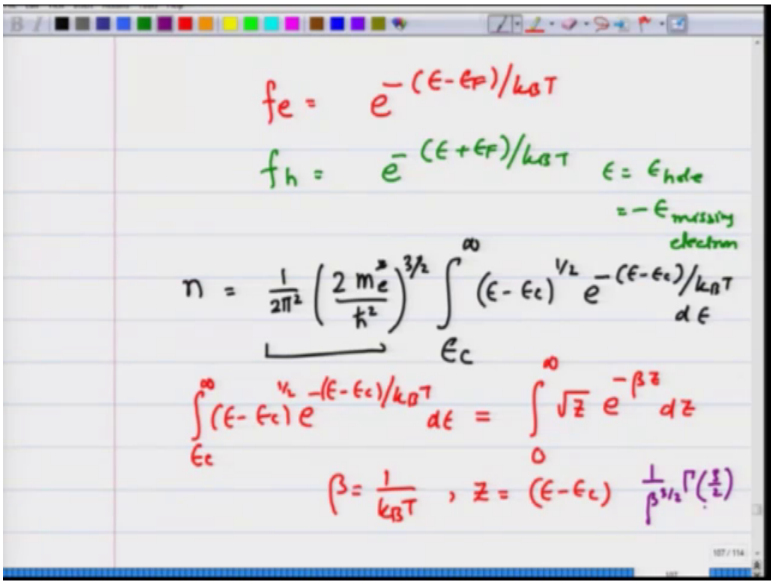

Once I have done this the rest of the job is very easy. So, number of electrons is going to be equal to 1 over 2 pi square 2 m electron star over h cross square raised to 3 by 2 E C to infinity, E minus E C raised to 1 half e raised to minus E minus E C over k B T d E. The first term is a constant.

It remains out and the second term E C to infinity E minus E C e raised to minus E minus E C over k B T d E Can be written as this is raised to 1 half can be written as integration 0 to infinity square root of Z e raised to minus beta Z d Z where beta is 1 over k B T and Z equals epsilon minus epsilon c. This integral is very straightforward it is 1 over beta raised to 3 by 2 gamma function of 3 by 2. This is this function.

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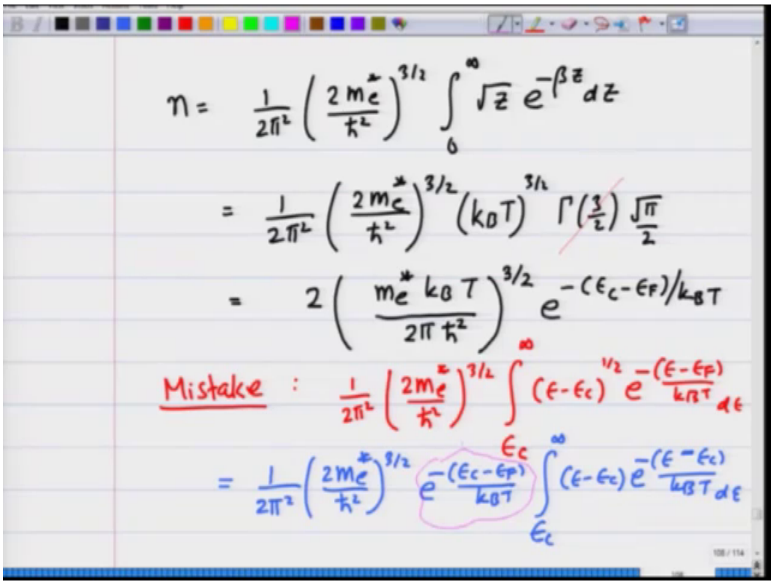

So, let me rewrite this again. So, number of electrons is equal to 1 over 2 pi square 2 m e star over h cross square raised to 3 by 2 integration 0 to infinity square root of Z e raised to minus beta z d Z and this is equal to 1 over 2 pi square 2 m e star over h cross square 3 by 2. 1 over beta raised to 3 by 2 which is k B T raised to 3 by 2 gamma 3 by 2 and gamma 3 by 2 is nothing but I cut this and instead write a square root of pi by 2. And therefore this whole thing can be written as 2 m e star k B T over 2 pi h cross square raised to 3 by 2 and there is a factor outside minus E C minus E F over k B T.

So, what is the mistake I did? If you are careful you would have seen it the integral was 1 over 2 pi square 2 m e star over h cross square raised to 3 by 2 integration E C to infinity E minus E C raised to 1 half e raised to minus E minus E F over k B T d E. And this can be written as 1 over 2 pi square 2 m e star over h cross square raised to 3 by 2 e raised to minus E C minus E F over k B T integration E C to infinity E minus E C e raised to minus E minus E C over k B T d E. Rest of the calculation goes through because of this mistake I had not gotten this term into the picture which I have now taken care off.

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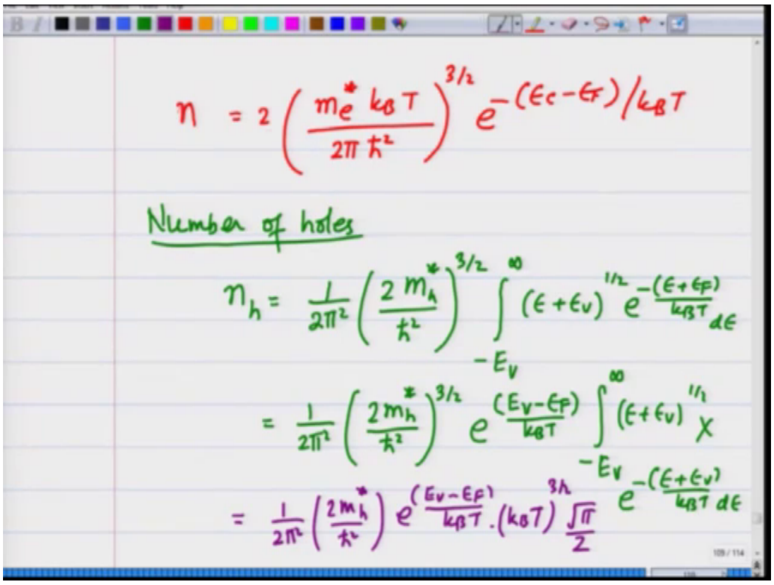

So, number of electrons in the conduction band is 2 m e star k B T over 2 pi h cross square raised to 3 by 2 e raised to minus E C minus E F over k B T. How about the number of holes? The number of holes n h is going to be 1 over 2 pi square 2 m h star over h cross square raised to 3 by 2, where m h star tells you the effective mass of the hole integration minus E V to infinity epsilon plus E V raised to 1 half e raise to minus E plus epsilon F over k B T d E. Which now I am going to write as equal to 1 over 2 pi square 2 m h star over h cross square raised to 3 by 2 e raised to E V minus epsilon F over k B T integration minus E V to infinity epsilon plus epsilon v raised to 1 half times e raised to minus e plus E V over k B T d e.

This is exactly the same integral as we encountered in the case of electrons and therefore I can write this as, equal to 1 over 2 pi square 2 m h star over h cross square e raised to E V minus E F over k B T times k B T raised to 3 by 2 square root of pi by 2 that is the answer that we get for this. So, the number of holes also comes out to be exactly similar to that of number of electrons. And I get n h is equal to m h star k B T over 2 pi h cross square raised to 3 by 2 e raised to E V minus E F over k B T. This is the number of holes.

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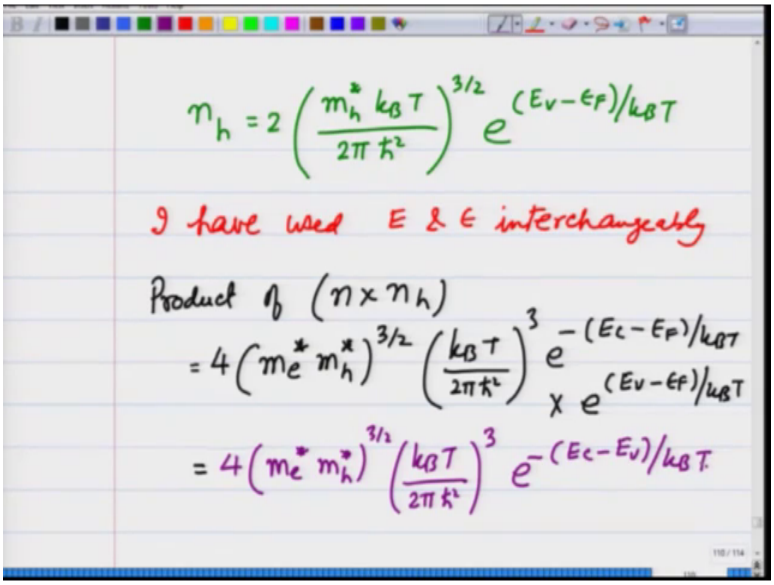

Now, what a caution I have used E and epsilon interchangeably. So, that I hope I was going along you understood. Now the product of n the number of electrons and number of holes is therefore going to be equal to m e star m h star raised to 3 by 2 k B T over 2 pi h cross square raised to 3, e raised to minus E C minus E f over k B T times e raised to E V minus E F over k B T. And this comes out to be m e star m h star raised to 3 by 2 k B T over 2 pi h cross square raised to 3 e raised to minus E C minus E V raised to divided by k B T. So, let us write it again because I want to spend some time on this formula.

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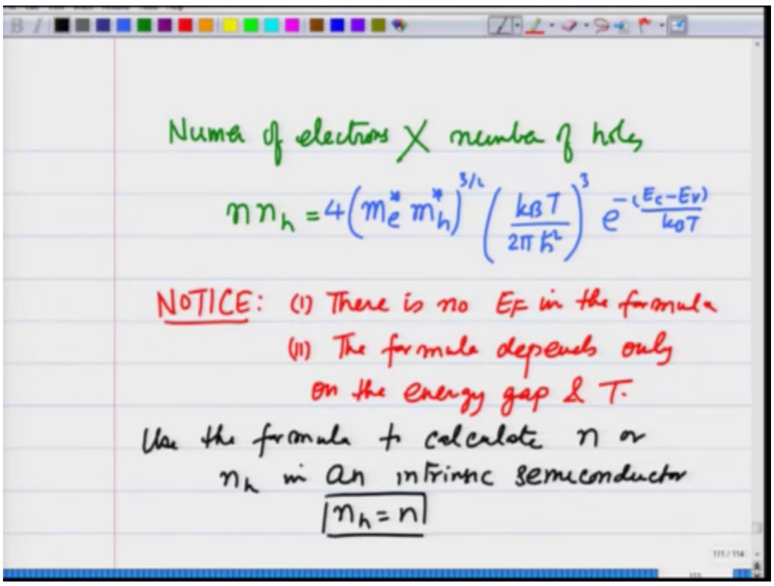

So, we get number of electrons times number of holes n times n h is equal to m e star m h star raised to 3 by 2 k B T over 2 pi h cross square raised to 3 e raised to minus E C minus E V over k B T. Notice 1; there is no E F in the formula number 2; the formula depends only on the energy gap and temperature T. Therefore this formula is going to be in general valid. No matter where the Fermi level is and this is very important. This is important because I can apply it even when impurities are there and Fermi level get shifted, we come to that later. But right now I can use this formula, use the formula to calculate n or n h in an intrinsic semiconductor. Using the fact that n h equals n in an intrinsic semiconductor and I also then use it to calculate the Fermi energy. So, let us do that now.

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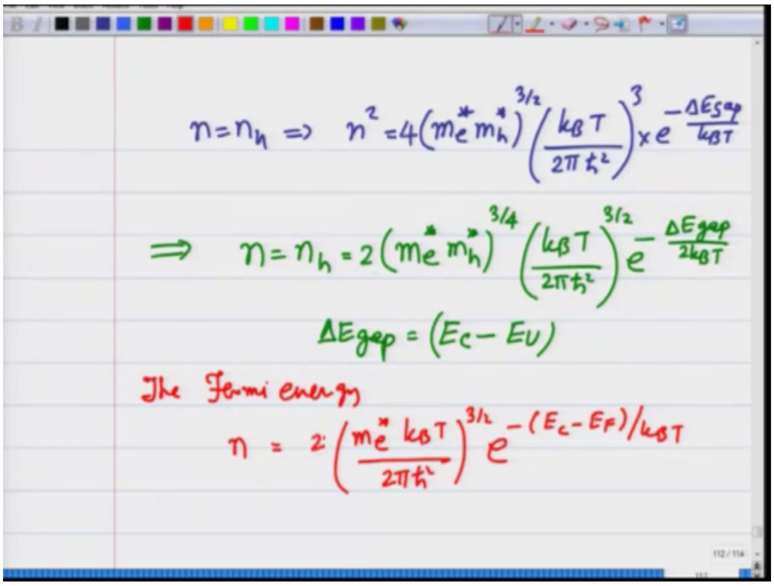

So, I have n equals n h and therefore this implies n square is going to be equal to m e star m h star raised to 3 by 2 k B T over 2 pi h cross square cubed times e raised to minus delta E let me write it as gap over k B T. And therefore n equals n h equals m e star m h star raised to 3 by 4 k B T over 2 pi h cross square raised to 3 by 2 e raised to minus delta E gap over 2 k B T where delta E gap is equal to E C minus E V. So, this gives me the number of electrons or number of holes. How about the Fermi energy? Recall that n was equal to 2 m e star k B T over 2 pi h cross square raised to 3 by 2 e raised to minus E C minus E F over k B T and it is similarly was given in terms of E F minus E V. You can use these to calculate E gap and I leave that as an exercise.

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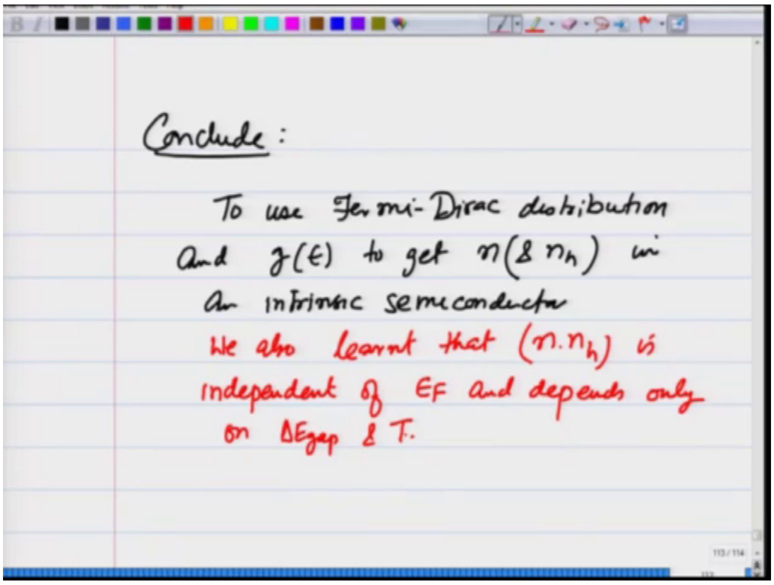

So, what we have done in this lecture is use Fermi-Dirac distribution and g E to get n and n h is the same thing in an intrinsic semiconductor that is point number 1. Point number 2 we also learnt that n times n h is independent of E F and depends only on delta E gap and temperature T. In the next lecture we will use these to calculate the number of electrons and holes in a doped semiconductor.

Thank you.