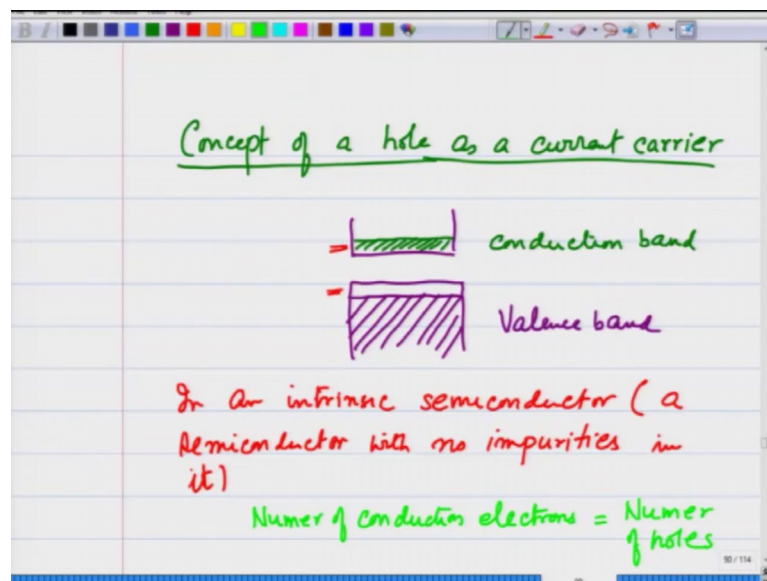**Introduction to Solid State Physics**
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Indian Institute of Technology, Kanpur

Lecture - 70
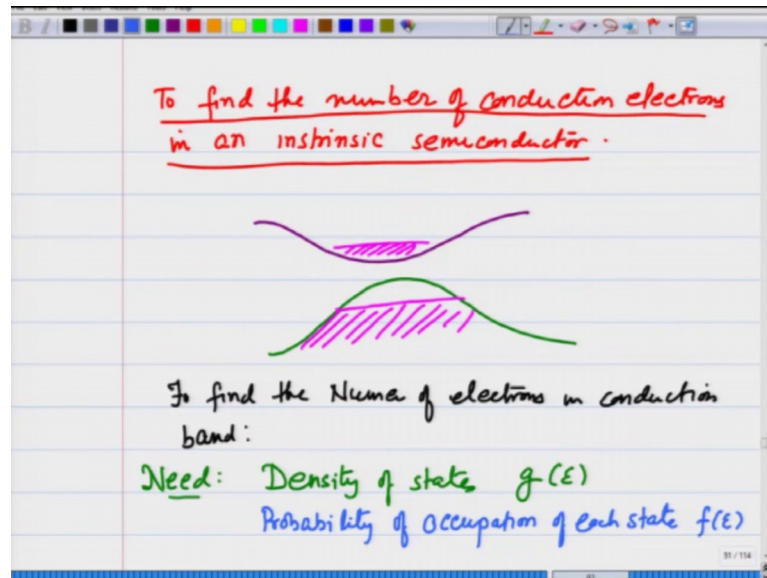**Calculating carrier density in semiconductors - I**
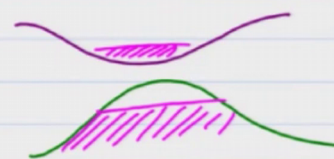
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In the previous two lectures I have discussed the concept of a hole as a current carrier. This becomes important in a semiconductor because in a semiconductor what I have is two bands which are partially filled. The lower band which I call the valence band is filled quite a lot and partially empty so, this is what I am going to call valence band and the upper band which we call the conduction band has a few electrons. So, this is conduction band. As you can see very clearly if, it is a pure semiconductor then the number of electrons a number of missing electrons is the same.

So, in an intrinsic semiconductor and by that we mean a semiconductor with no impurities which we will talk about later in it. So, in an intrinsic semiconductor; a semiconductor with no impurities in it we can clearly see that the number of conduction electrons is equal to number of holes. And holes are in the valence band and the conduction electrons are in the conduction band. So, we want to use this to now get the number of holes and number of electrons that can carry the current.

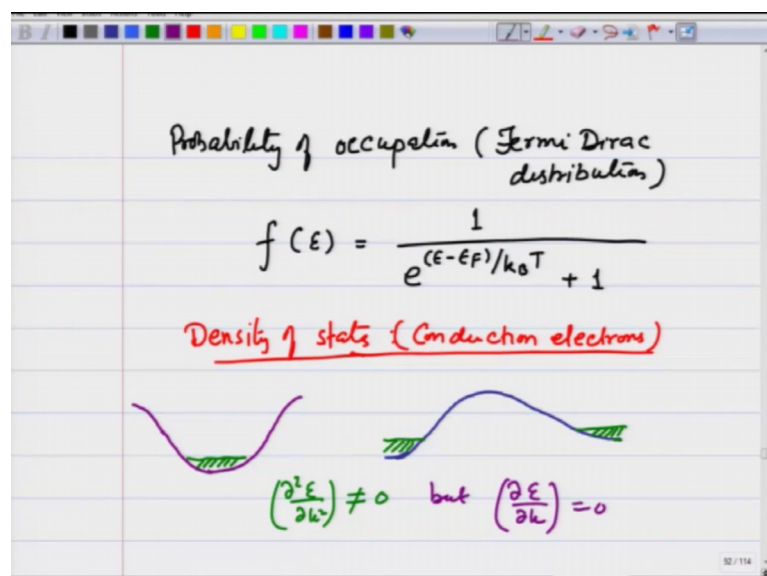So, our aim now is to find the number of conduction electrons in an intrinsic semiconductor. This is our aim. And what ingredients do I need for it? So, let us look at the band picture. I have this valence band, so I have this valence band and the conduction band and some electrons are here at the bottom and in the valence band some holes are there. So, to find the number of electrons in conduction band, I need density of states which I am going to call g E. So, number of states per unit energy range and I need the probability of occupation of each state which I am going to call f E and both these are known. You have seen this earlier in connection with the free electrons.
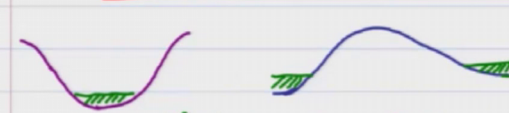
Let us first talk about the probability. This is given by Fermi Dirac distribution formula. So, f the probability or the occupation probability at energy E is equal to 1 over e raised to epsilon minus epsilon Fermi divided by k B T plus 1. The density of states for the conduction electrons, let me write this; is going to be like free electron density of states. Let me show you how. So, I have this band the conduction band which is filled only near the bottom.

You may think I made it in a particular way that is why it looks like this even if the band was like this the other way then it will be filled again at the bottom and near the bottom d 2 E by d k square is not 0 but d E by d k is 0 because we are talking about the minimum of the energy. And therefore, if I were to expand the energy let us call it now conduction band as a function of k it will be E at k 0 where the minimum is and we have seen either it is at the Brillouin zone boundary or at k equals 0 plus d E d k k 0 times delta k plus 1 half d 2 E by d k square at k 0 delta k square.

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The second term is 0 because we are talking about the minimum of the band and therefore, I have E conduction as a function of k is equal to E k 0 plus 1 half 1 over m effective at k equals k 0 k minus k 0 square. If we take k 0 to be 0; so that means, I have shifted my; the way I measure k to the minimum. If k 0 is at Brillouin zone boundary then I will take that point to be 0. So, that delta k itself becomes the measure of k then I can write E conduction is understood I will write C for it as k equals E k 0 which I will

take to be 0. I will write that again is 1 over 2 m effective times k square. There should also be h cross square here because of the definition of m effective.

So, I will write out here h cross square and I have taken in this E k 0 to be 0. I shifted the reference of where I measure the energy from. So, if I take the point where the band is a minimum and measure the energy from there I can write the energy as h cross square k square over 2 m effective.

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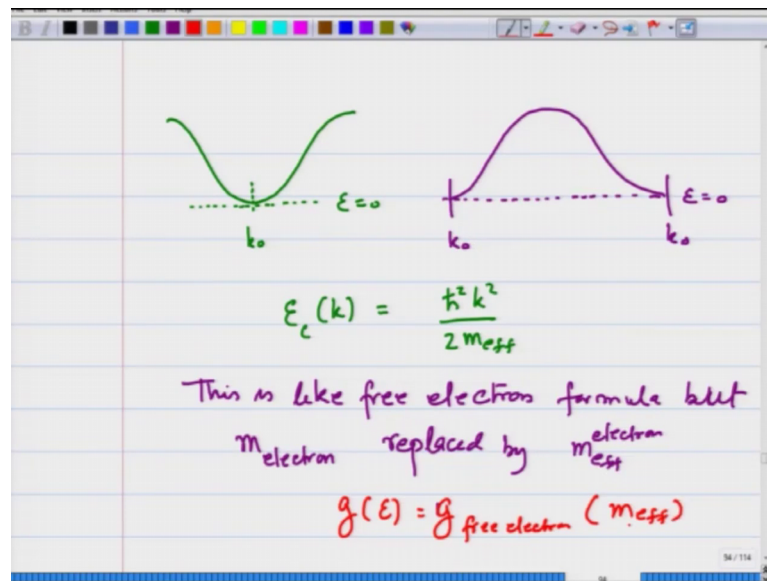
This is the band this is my k 0 and right at the minimum is E equals 0. If the band is the other way here is my k 0 which is the same as k 0 on the other side and this is my equal 0. The picture then that emerges is that E conduction as a function of k is equal to h cross square k square over 2 m effective. So, this is free electron formula as a function of k, but m electron replaced by m effective of the electron. And therefore, g E is going to be like g free electron, but with m effective rest of the thing is the same. And the total number of is going to be equal to 0 to infinity g E, f E, d E. This is going to be the number of conduction electron as a function of temperature which we will calculate. Taking the form of g E for free electrons and taking f E.

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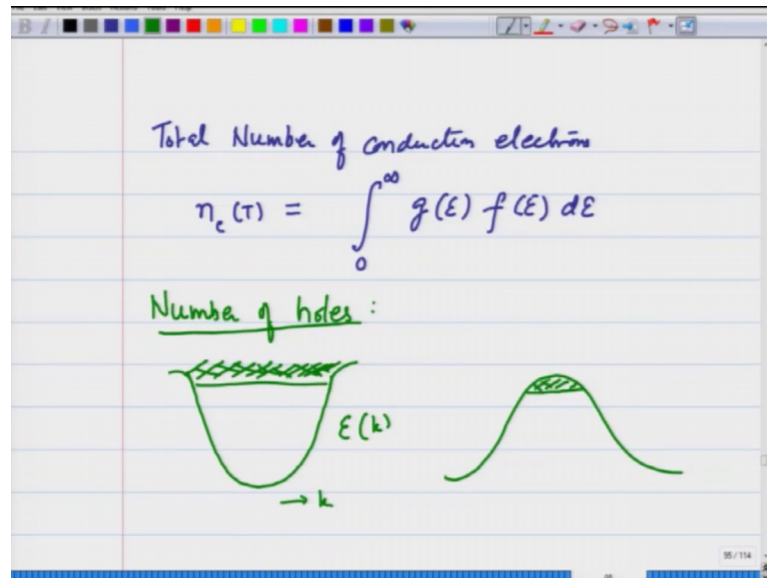
How about the number of holes? Number of holes now I am going to take the band for the hole as the band for the hole and here is now I am going to take band for the hole as we obtained earlier and I have a E k for the hole and this is a k. The band again could be like this or like this. In this case the holes will be from the top and I should have made the other one carefully and in this case the bands would be right the holes will be right from the top. So, this time the shaded region is the holes.

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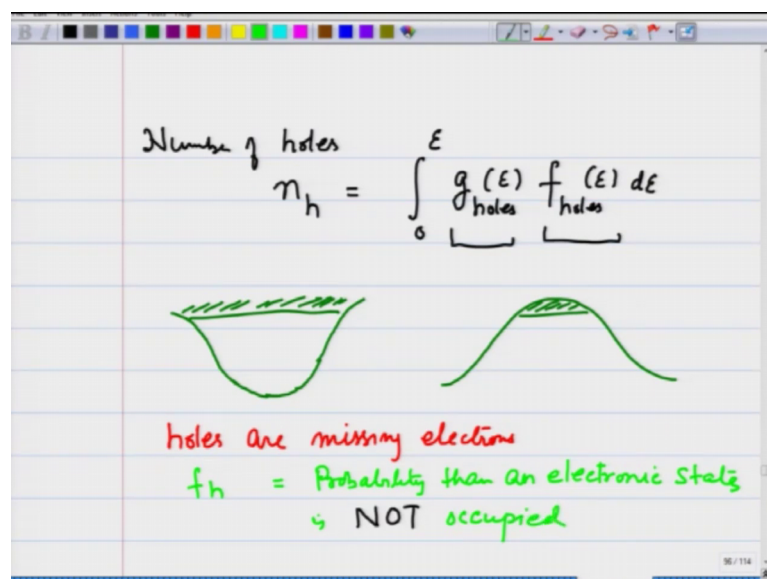
Now, the number of holes which I will write as n h will also be equal to 0 to E, we are now E refers to the energy of the holes, the density of states for the holes as a function of E times the occupation probability of the holes as a function of E d E. So, what we now need to calculate is g holes and f of holes. Now again if you look at the bands for the holes; holes are nothing but missing electrons; these are the holes. In this case the holes are coming from the top of the band. Holes are missing electrons. So, f h is going to be the probability that an electronic state is let me write this figure not occupied.

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$$f_h(\varepsilon) = 1 - f_{elect}(-\varepsilon)$$

$$= 1 - \frac{1}{e^{(-\varepsilon - \varepsilon_F)/k_B T} + 1}$$

$$= \frac{e^{(-\varepsilon - \varepsilon_F)/k_B T}}{1 + e^{-(\varepsilon + \varepsilon_F)k_B T}}$$

$$= \frac{1}{e^{(\varepsilon + \varepsilon_F)/k_B T} + 1}$$

So, f h E is going to be 1 minus f electron minus E. Minus E is the energy for the electron and this is going to be 1 minus 1 over e raised to minus E minus E F upon k T plus 1 k B T which I can write as e raised to minus E minus epsilon F over k B T divided by 1 plus e raised to minus epsilon plus epsilon F over k B T which is equal to 1 over e raised to epsilon plus epsilon F over k B T plus 1. So, we have calculated the occupation probability for hole as a function of energy of the hole as equal to 1 over e raised to epsilon hole plus epsilon [FL]; epsilon F does not change is the property of the system k B T plus 1.

And to calculate the number of holes,- we again do integration g hole epsilon hole f hole epsilon hole d E hole. So, now, from now on I will drop this e hole from here and just write this as integration g epsilon f hole epsilon d E and now what we have to do is calculate g E or the density of states for the hole also and I will do that in the next lecture.