Introduction to Solid State Physics

Prof. Manoj K. Harbola
Prof. Satyajit Banerjee
Department of Physics
Indian Institute of Technology Kanpur

Lecture – 07
Introduction to Hall effect in metals – Part II

So last time we had begun of with looking at the phenomena of Hall effect, where we were looking at the conduction of electrons through the metal in the presence of a magnetic field as well as an electric field and we so what happens when you have a magnetic field.

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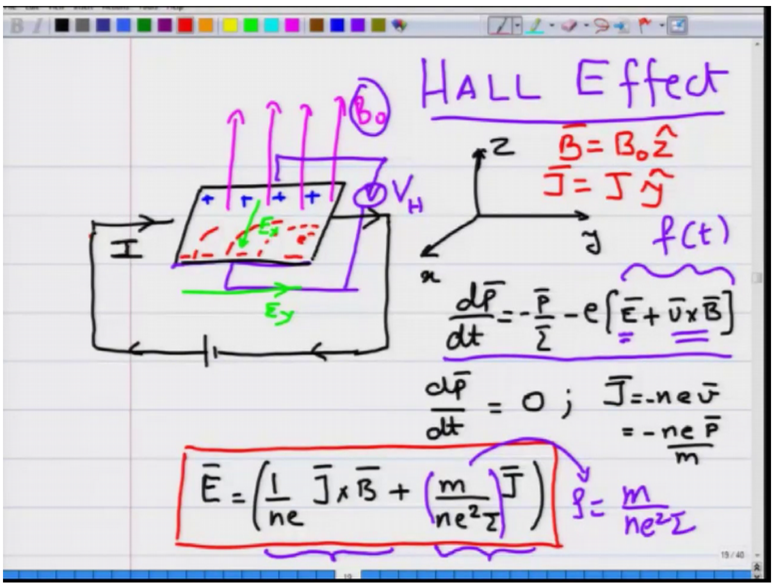

So, if you have a current I which is flowing through the material you are sending it through a thin material you are sending a current I and by convention the electrons flow in the opposite direction, because of the presence of this magnetic field which we have applied perpendicular to the plane of the material, the electrons experience a Lorentz force and they start curling they start moving the Lorentz force is pushing them towards this edge of the sample.

As they start collecting on this edge this edge becomes negatively charged and the other end becomes positively charged. So, there is a potential difference generated and because of this potential difference there is an electric field which is along this direction and you will measure a voltage this voltage that you measure is the Hall voltage which is transverse to the direction of the flow of current. The current is flowing from the left to the right direction and the field or the electric field or the voltage that you are measuring is perpendicular to that direction and this is the Hall effect phenomena.

So, using your we wanted to understand this phenomena using the Drude's model and how do we incorporate this affect into the Drude's equation. So, this was if you recall the Drude's equation that dp by dt rate of change of momentum of the particle has a drag term which is P tau which comes, because of collisions experienced by the electron and this is the force which the electron experiences. The force comes from 2 parts one is the electric field which acts on the electrons and the other is the magnetic field which is nothing else but the Lorentz force.

Now, the drag term balances the force which is acting on the electron and the system acquires the steady state, where there is no rate of change of momentum and this is the steady state of the system. Your current density will be given by J is equal to minus n e into v which is the velocity of the electron or minus n e divided by m into P which is the momentum. And if you incorporate all of this and rearrange the terms you can have electric field on one side and on the other side now you have 2 different terms, one is a term which is the J cross B term and there is another term which you see and it looks very familiar. If B is equal to 0 then you only have the term m by ne square times J, the electric field is m by ne square times J if you have only an applied electric field and no magnetic field.

What this term is if you recall this is nothing else, but the resistivity which you have derived from the Drude's model row is equal to m divided by n e square tau. So, this second term which I show you here is the Drude's resistivity and the first term is basically coming because effect of the magnetic field. So, let us look at this equation a little bit more as we go along and what is the consequences of this equation.

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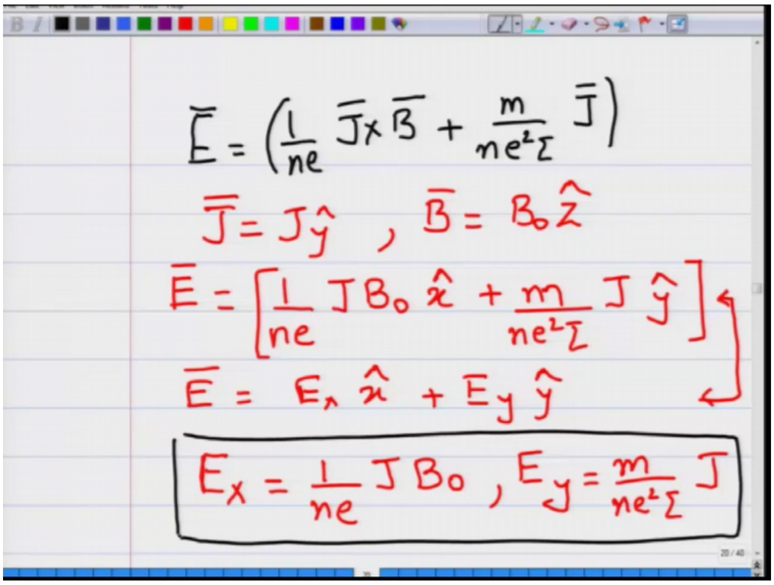

So, the electric field is 1 by n e J cross B plus m by n e square tau times J, this is your electric field which is present in the presence of an applied electric field as well as a applied magnetic field. Now if you recall from the previous figure the magnetic field that we had applied B was B naught z cap as per this configuration of as per their choice of axis B is B naught z cap and the current density J is J which is running along the y cap direction. So, J is along the y cap direction and B is along the z cap direction.

So therefore, your electric field can be rewritten if you put all of this in you will get a term 1 by n e J times B naught, where J is the magnitude of the current density J times B naught which is along the x cap direction plus m by n e square tau J in the y cap direction, this is your net electric field and you can now consider your electric field has 2 components one is along the x cap direction and the other is along the y cap direction.

You can see that when you apply the magnetic field now the electric field has not only a y component, but it has also an x component the moment you apply a magnetic field and now you can by comparison of these 2 equations, if you compare these 2 equations. You will show that the electric field E x is one over n e J times B naught and the electric field along the y direction is m by n e square tau J which is the current along the y direction. So, let us go back to our figure.

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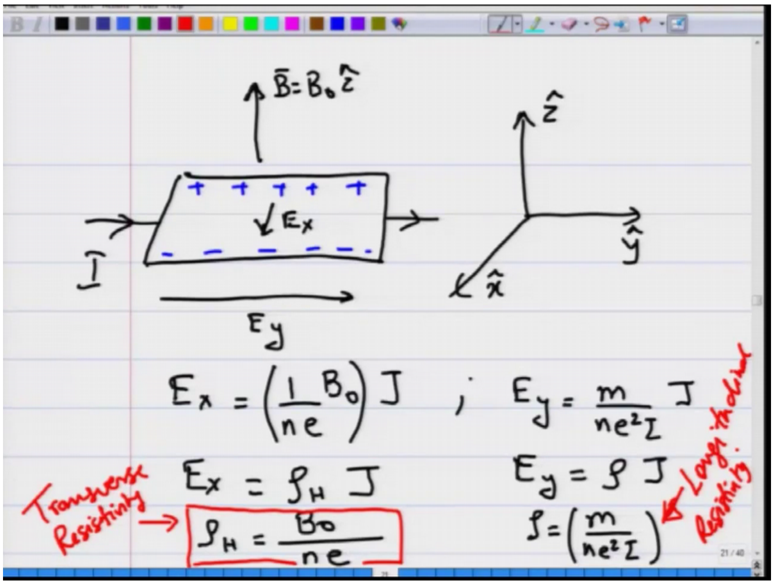

If we go back to our figure this is our sample was this is what our sample was, we had because of the Hall effect we had positive charges on this end and negative charges which were created on this end. The current was flowing along this direction the current I was flowing along this directions, this was our coordinates x y and z the magnetic field was pointing along the z direction.

So, we had a magnetic field applied along the z direction and we know that there is an electric field along the y direction and there is an electric field along the x direction and we have just now calculate it using the Drude's model, the electric field along the x direction is 1 over n e B naught into J and the electric field which is along the y direction is the usual m by n e square tau times J. This electric field which is along the y direction is nothing else but your Drude's resistivity where row is m by n e square tau. What you expect is that along the direction of the current there is a resistivity and that resistivity is nothing else but m by ne square tau.

But in the direction perpendicular to the direction of the current you have another resistivity which is coming into the picture and that is the hall resistivity. This is acting in a direction which is perpendicular to the direction of the flow of current and this hall resistivity is proportional to the magnetic field and inversely proportional to the density of electrons and the charge of the electrons. You get a very important result that the hall resistance or the hall resistivity which is acting along the direction perpendicular to the direction of flow of current is proportional to the applied magnetic field and is inversely proportional to the density of electrons and the charge of the electron.

The other is the longitudinal conductivity this is called the longitudinal conductivity, this is called the transverse conductivity, the longitudinal conductivity is the Drude's conductivity which is along the direction of flow of current and the transverse conductivity which is perpendicular to the flow of current is the hall conductivity the transverse resistivity and this is the longitudinal resistivity.

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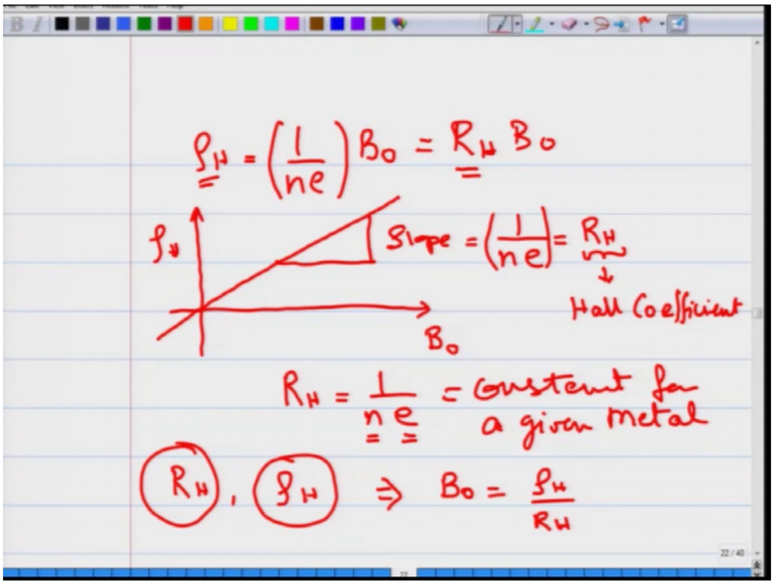

Let us discuss the transverse resistivity a bit more which is the hall resistivity, the hall resistivity is 1 over ne into B naught. If you plot the hall resistivity as a function of magnetic field this expression shows that this hall resistivity is going to be a straight line, where the slope of this straight line the slope of the straight line is going to be 1 over ne this quantity 1 over n e is given a term RH and this is called as the Hall coefficient. It is the Hall coefficient RH is inversely proportional to the density of electrons and it depends on the charge of the charge carrier which is moving through the material.

And other very important aspect is that if you measure the hall or the longitudinal resistivity, then immediately if you know the one by any of this material if you know and you have measure the hall coefficient of the material, then immediately by measuring since RH you have already determined it from a different experiment. If you in a certain experiment when you place this material in a magnetic field which is unknown, if you measure the hall resistivity and you already know what is the RH of the material because this is constant for a given material this is constant for a given metal.

Then what you immediately know is from this relation that the magnetic field in your experiment is just the rho H divided by RH, this is RH into B naught. So, if you measure this and if you already know what is RH of the material, you can immediately find out what is the unknown magnetic field you can take this material put it in a region where there is some magnetic field whose value you do not know. If you measure RH in that value of unknown magnetic field, if you measure rho H the resistivity the hall resistivity you will immediately get a measure of B naught and so it becomes a very powerful tool to actually measure magnetic fields and that is why it is used as hall effect is used as a sensor more as we go along about this.

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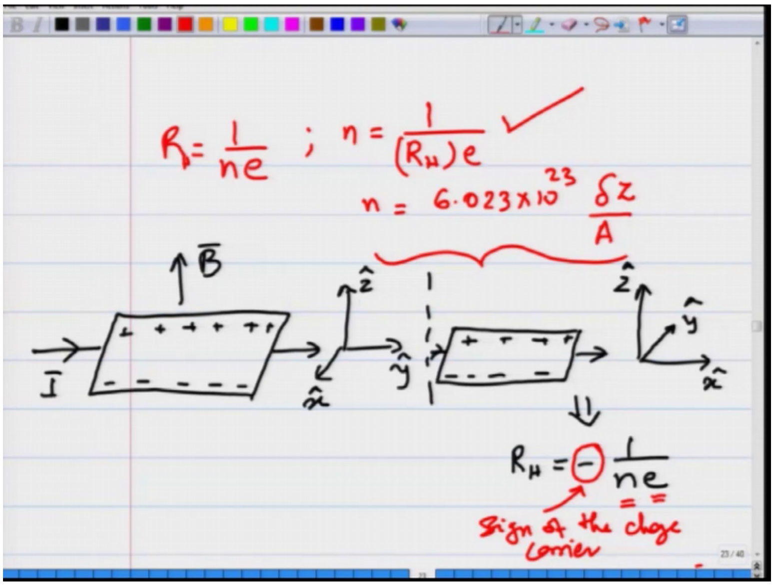

Now, if you see RH that we got is 1 over ne is the charge of the electron n is the density. So, you know the charge of the electron, so therefore by measuring RH you can immediately get an idea of how much is the density of electron. The density of charge carriers in a material is nothing else but RH into e. So, it becomes a very useful tool to also measure the density of electrons in a given metal, how many charge carriers that their per unit cc of the material.

That is a very important parameter we had already obtain the number which if you recall is 6.023 into 10 raise to power of 23 the density of the material divided by the atomic mass into Z, where Z is the number of charge carriers of the valence electrons how many electrons are given out by each atom that gives you a measure of an estimate of the density.

But if you actually want to know how much is the density of charge carriers present inside the material experimentally this is the way to determine it, you measure the RH by measuring the hall resistance or the hall resistivity as a function of magnetic field and then you can determine n because you know e.

Another slide technical point is that it would have been nice if it not only here you have a charge of an electron which is e, but it would have been nice if they would have been a negative sign because then it would mean it is sensitive to also the sign of the electron. Now to keep things consistent so that you have a negative sign for an electron we can change our coordinate system.

So, if you recall we worked out the above problem, this particular problem we worked out in this particular configuration that current I is the magnetic field B we worked it out in this configuration where x y and z. Instead of working out this problem with this coordinate axis if I had chosen this particular coordinate axis current is still in this direction electrons are still going to flow there is still going to be a hall effect.

But instead of this choice of coordinate axis if we change the choice of coordinate axis slightly nothing changes the physics remains exactly identical, but I choose that this axis is the x axis that means a current is flowing along the x direction and y is along this direction. So, from this choice I move to this choice and now I will get an electric field which is the longitudinal electric field which is along the x and the transverse electric field will be along the minus y direction and here for the electrons which are moving inside the material and undergoing the deflection, if I calculate RH it will be minus 1 over ne.

And this is very important because the negative sign actually makes a lot of difference, the hall coefficient is not only dependent on the density of charge carriers the magnitude of the charge carriers how much charge is present on the charge carriers, but it also depends on the sign of the charge carrier.

In fact, this is very nice the one of the successes of the Drude's theory partial successes of the Drude theory is that it can show the hall effect very nicely. It shows the Hall effect very clearly, but if you explore a little further then there are some things which do not seem right. So, people measure the hall coefficient for different set of materials and when they measure the Hall coefficient for different set of materials this is what they found.

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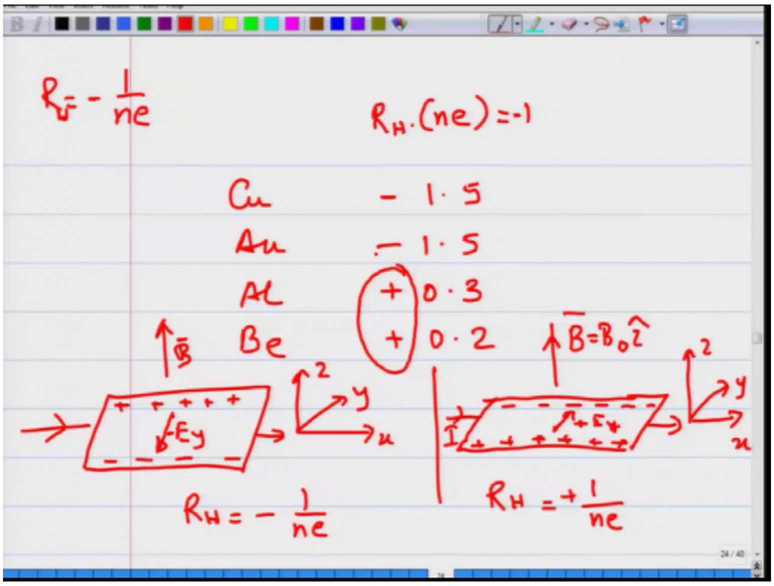

So, I will write down the Hall coefficient for a popular set of materials. So, copper so if I RH into density of electrons into the charge of the electron this should be equal to 1, as per what I have shown because RH is minus 1 by n e it should be minus 1 ok, copper gives you minus 1.5 silver also gives you a value of minus 1.5 so the sign is all right.

But when you go to aluminium suddenly there is a sign change it goes to plus and if you take beryllium it goes to 0 plus 0.2. So, something very strange has happened from minus sign the Hall coefficient has changed sign. If you recall the negative sign was coming in our system because, you had negative charge carriers if you recall our choice of axis we go to the new choice of axis which gives us the negative sign xyz you have your current flowing like this, electrons are deflecting in this magnetic field B and this in this case RH is minus 1 by n e.

But for the moment just for the sake of argument if instead of negative charge carriers which are electrons, if you consider positive charge carriers what would happen. If you consider positive charge carriers with exactly the same choice of axis x y and z current is in this direction magnetic field is in this direction. Now instead of negative charge carriers we are going to have positive charge carriers, so positive charge carriers will drift in this direction from left to right unlike when you had electrons this is the current directions. So, in electrons the electrons are coming from the other side if you have positive charge carriers they are in the same direction.

What will happen here is that on this end you will have positive charge carriers and the other end will be negative and now you see that in this case for negative charge carriers the electric field was in the minus y direction, now here in the case of positive charge carriers the electric field becomes in the plus y direction.

Here if you calculate your RH exactly in the same way you will get a plus 1 by n e. So, it looks very surprising that aluminium and beryllium these types of metals get a positive sign, which indicates as if there are positive charge carriers inside the metal.