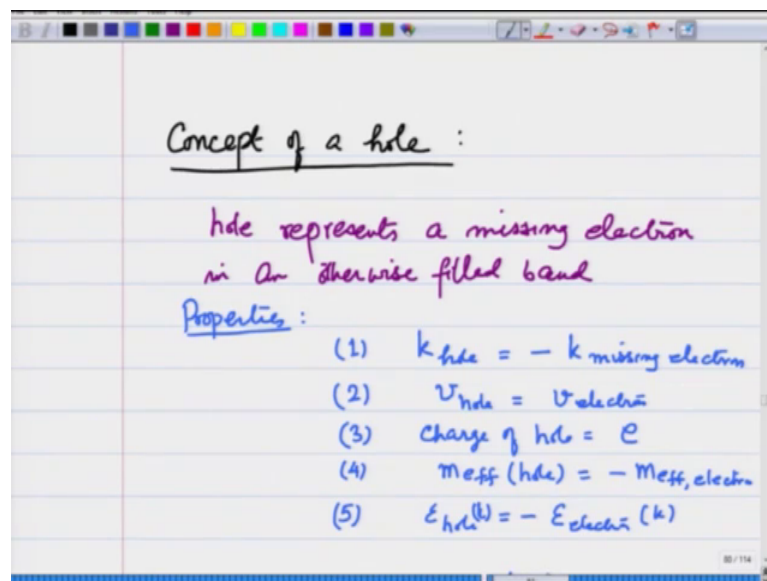


**Introduction to Solid State Physics**  
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**Lecture - 69**  
**Concept of hole as a current in semiconductors – II**

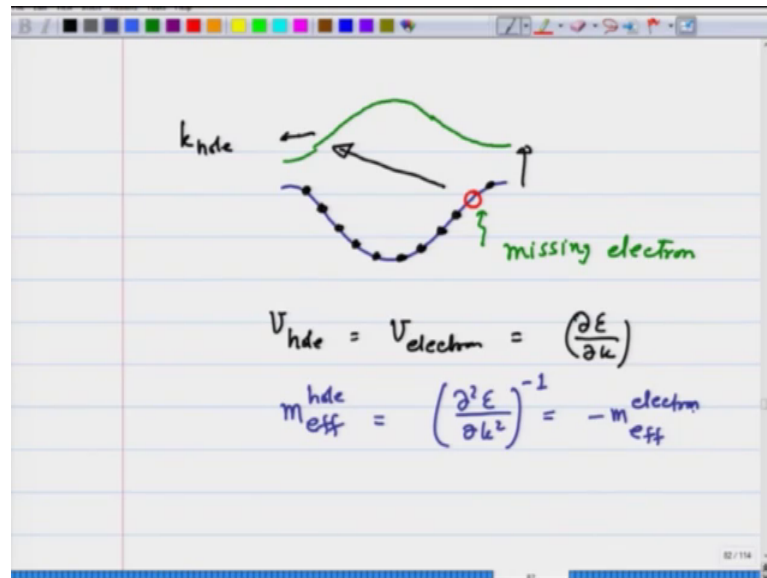
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In the previous lecture, I explained to you the concept of a hole. And let me just tell you what I did. What I started with was that a hole represents a missing electron in an otherwise filled band and therefore, it is easier to talk in terms of hole than talking about the rest of the electrons. And to talk about it we had to do certain proper bookkeeping and the properties for the hole that we wrote for number 1: that  $k$  for the hole is minus  $k$  for the; I will not keep writing missing electrons for the time being I will write missing electron, but from now on I will just write electron.

2: velocity the hole is same as velocity of electron. 3: charge of hole is equal to positive  $e$ ,  $e$  is a positive number I write the charge the electron as minus  $e$  and 4: that  $m$  effective of the hole is equal to minus an effective of electron and the 5th property that I wrote and I was writing it in random is that  $E$  hole is equal to minus  $E$  electron of course I am talking about hole being made by missing electron from  $k$ .

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So, the picture that we drew after this was that if, there is a band from which an electron is moved is otherwise filled so, all these filled dots are showing you the filled states and this empty red one is showing you the hole or the missing electron. Then in the band for the hole is going to be like this, and this is the missing electron, then the  $k$  for the hole goes the other way. This is going to be  $k_{hole}$  which is minus the  $k$  of the missing electron and the energy also goes the other way. So, the band gets inverted. This is a picture that we drew and all the other conclusions for example, when I said that  $v_{hole}$  is equal to  $v_{electron}$ . This was by using the definition that  $\Delta E$  over  $\Delta k$  is the definition for the velocity, when we say velocity we mean the group velocity and the science for  $E$  and  $k$  both change. When we talk about the hole and therefore,  $v$  comes out to be the same.

Similarly, when we talked about  $m_{effective}$  of the hole this was using the definition of  $d^2 E$  over  $d k^2$  square inverse and in this case since I am taking  $d k^2$  square the sign of change of  $k$  does not matter but  $E$  sign changes and therefore, this comes out to be minus  $m_{effective}$  of the electron. In this lecture I want to look at the same problem from a slightly different perspective; the perspective of that of a current. So that you get a clearer picture of what a hole is like and you get a feel for it. Again as I said earlier this is another way of bookkeeping. How the other electrons are doing instead of talking of all the other electrons I am talking about the missing electron or the hole.

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Recall:  $k_{\text{hole}} = -k_{\text{electron}}$

$\sum_{\text{filled band}} k = 0$

$\Rightarrow \sum_{2N-1 \text{ states}} k = -k_{\text{missing electron}} = k_{\text{hole}}$

Possibility  ~~$\sum_{\text{band filled}} v_{\text{electron}} = 0$~~

~~$\sum_{2N-1} v_{\text{electron}} = -v_{\text{missing elec}} = v_{\text{hole}}$~~

So, recall that when I obtain  $k_{\text{hole}} = -k_{\text{electron}}$  I use the fact that summation over  $k$  for a filled band is equal to 0. And therefore, this implies that summation over  $2N - 1$  states which are filled  $k$  is equal to minus  $k_{\text{missing electron}}$ . And this is what I am going to call  $k_{\text{hole}}$ . This is how I got  $k_{\text{hole}} = -k_{\text{electron}}$ . I am going to use the same trick to look at the hole again.

Now, one possibility is that I say summation of  $v_{\text{electron}}$  over the hole band is 0 and therefore, I could get summation  $2N - 1$   $v_{\text{electron}}$  is equal to minus  $v_{\text{missing electron}}$  and this is  $v_{\text{hole}}$ . But this does not sit properly with what we derived earlier. So, this is incorrect I am going to cut it.

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Current of  $(2N-1)$  electrons is to be represented by a hole

Filled band:  $\text{Current} = -e \sum_{\text{filled band}} v_{\text{electron}} = 0$

$\Rightarrow -e \sum_{2N-1 \text{ states}} v_{\text{electron}} = e v_{\text{electron}}$

From  $\frac{dh}{dt} = F \} \Rightarrow \text{charge (hole)} = e$

Let us look at what error are we making? Recall that finally, why am I talking about the holes is that I want to represent the current by the rest of the electrons. So, for the current of  $2N - 1$  electrons is to be represented by a hole. That is my whole idea the moment I talk about the current I necessarily have to bring in the charge of the hole, and therefore it is not the velocity that I will add up to 0, it is the current that I will add up to 0. So, let us see if I have a filled band due to symmetry in this band the current which is equal to summation  $v$  electrons times minus  $e$  and I am going to write this over the filled band is equal to 0. And therefore, this implies that minus  $e$  summation  $2N - 1$  states of  $v$  electron is going to be  $e v$  electron. So, the current by these filled states is the same as by this missing electron out here.

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$$\vec{j} = -e \sum_{2N-1 \text{ states}} v_{\text{electron}} = e v_{\text{electron}} = e_{\text{hole}} \times v_{\text{hole}}$$
$$\Rightarrow v_{\text{hole}} = v_{\text{electron}}$$

Point to be noted:  ~~$\sum v_{\text{electron}} = 0$~~

$-e \sum_{\text{filled band}} v_{\text{electron}} = 0$

Now recall that from the dynamics, from the dynamics equation  $d\mathbf{k}$  by  $dt$  equals force and that force necessarily was electromagnetic we got that charge and the hole was equal to positive  $e$ . So, let us collect all these together and write that  $\mathbf{j}$  due to that missing electron is equal to minus  $e$  summation  $2N-1$  states  $v_{\text{electron}}$  which is equal to  $e v_{\text{that missing electron}}$  and this is then equal to  $e_{\text{hole}} \times v_{\text{hole}}$  by definition. And this immediately implies that  $v_{\text{hole}}$  is equal to  $v_{\text{electron}}$ . Same result as we derived earlier by taking the derivative of the energy with respect to the wave vector  $\mathbf{k}$ . This time we have taken the current.

So, again the point to be noted, is that we did not do summation  $v_{\text{electrons}}$  is equal to 0. We did not do this. I am going to cut it, we did not do this. Instead what we did was we said some summation minus  $e v_{\text{electrons}}$  over filled band is equal to 0. Because it is the current, it is the charge carrying capacity that we are talking about and therefore I have to talk in terms of current if I want to talk about a hole carrying current and that is what gives me  $v_{\text{hole}} = v_{\text{electron}}$  and from this I will also derive the formula for  $m_{\text{effective}}$ .

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The image shows a whiteboard with the following handwritten equations:

$$\frac{dV_{\text{electron}}}{dt} = \left( \frac{dV_{\text{el}}}{dk} \right) \cdot \frac{dk}{dt}$$

$$= \left( \frac{\partial^2 E}{\partial k^2} \right) \cdot -e (\vec{E} + \vec{v} \times \vec{B})$$

$$= - \left( \frac{\partial^2 E}{\partial k^2} \right) \times e (\vec{E} + \vec{v} \times \vec{B})$$

A pink arrow points from the second equation to the third. Below this, the derivation continues:

$$\frac{dV_{\text{hole}}}{dt} = - \left( \frac{\partial^2 E}{\partial k^2} \right) e (\vec{E} + \vec{v}_{\text{hole}} \times \vec{B})$$

The word "missing" is written in pink under the  $e$  in the above equation. The final result is:

$$\Rightarrow m_{\text{eff}}^{\text{hole}} = - m_{\text{eff}}^{\text{electron}}$$

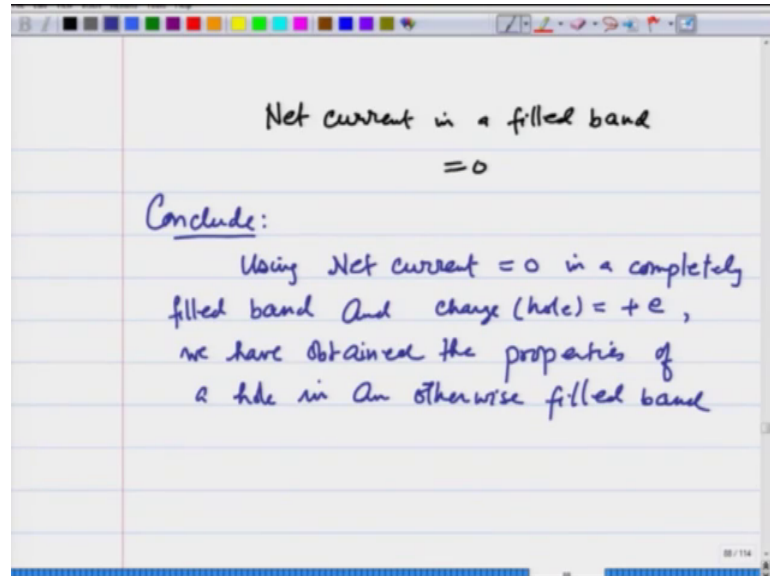
Because if you recall how we derived  $m$  effective; we said that  $d v_{\text{electron}} / dt$  and this is I am talking about the missing electron now is equal to  $d v$  by  $d k$ .  $d k$  by  $d t$  and let me write this electron here which is equal to nothing but  $d^2 E / dk^2$  and  $d k$  by  $d t$  is nothing but minus  $e$  the electronic charge times the electric field plus  $v$  cross  $B$  that is the force on it.

Now, I can write this as minus  $d^2 E / dk^2$  times  $e$  times  $e$  plus  $v$  cross  $B$ . I am writing this with a purpose by eliminating this minus sign from here and bringing it right in front of  $d^2 E / dk^2$ . I can now transform this equation for the hole. So, I can now transform this into an equation for the hole. So, I can write  $d v_{\text{hole}} / dt$  and here I am using the fact that  $v_{\text{electron}}$  is same as  $v_{\text{hole}}$  is equal to minus  $d^2 E / dk^2$  and this is at the missing electron site times  $e$  which is charge of the hole  $E$  plus  $v_{\text{hole}}$  because  $v_{\text{hole}}$  and  $v_{\text{electron}}$  are the same cross  $B$ . And this immediately implies that  $m_{\text{effective hole}}$  is equal to minus  $m_{\text{effective electron}}$ ; the state from where the electron is missing.

So, what I have tried to do in this lecture is, give you a feel for the hole from a difference perspective. In the previous lecture I had taken the change in the  $e$  with respect to  $k$ ; I had actually made the band for the hole, and then took all the mathematical derivatives and showed you the properties of hole with respect to the missing electrons. In this

lecture what I have done is actually looked at the current and use that to derive the properties of the hole and the; what I use is the net current in a filled band is equal to 0.

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So to conclude, using net current is equal to 0 in a completely filled band and charge of the hole is equal to plus e let me just explicitly write that plus; we have obtained the properties of a hole in an otherwise filled band. So, this is another way of bookkeeping for the  $2N - 1$  electrons in terms of a hole.

Thank you.