Introduction to Solid State Physics
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Lecture - 68
Concept of hole as a current carrier in semiconductors - I

So, far we have completed our picture of how electrons move in a solid and we have understood metals, semi conductors and insulators through band theory. This week is going to be divided to semiconductors, because these are materials that every solid state faces should know something about and they also provide a very nice training ground for understanding the properties of materials, particularly those materials that are important from electronics point of view.

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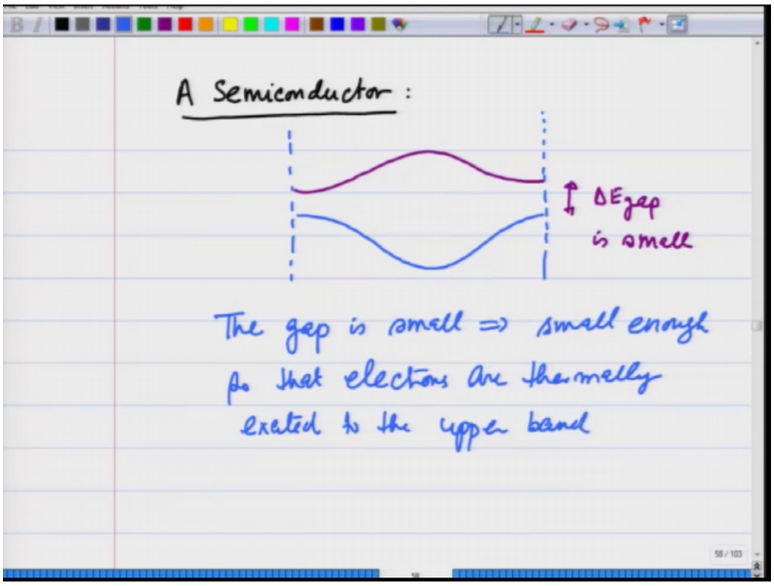

So, as we have discussed earlier, a semiconductor is a material where the band gaps are small. So, if I would just make two bands and I am not say probably, just make two bands within the first by zone and the upper band then this gap delta Eg is small. The gap is small means small enough so that electrons are thermally excited to the upper band. So, if I would to make a picture, it is like this.

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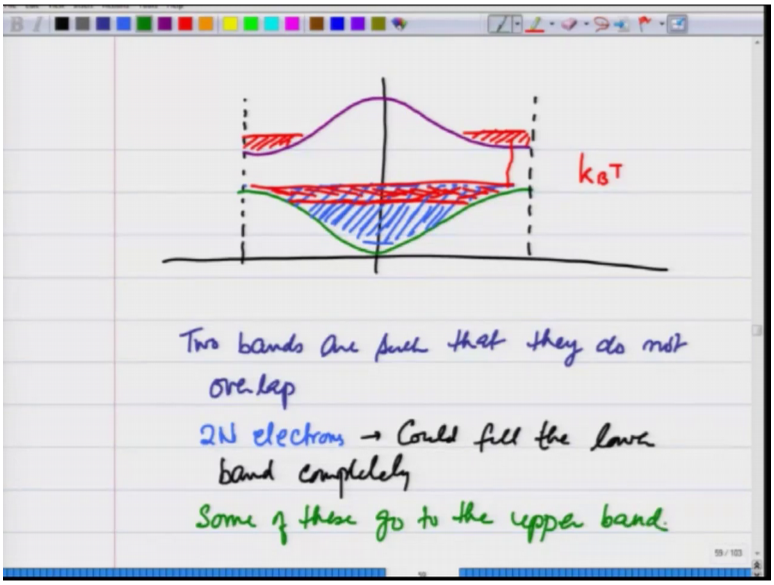

I have two bands and I am taking only one dimension. In higher dimensions these two bands are such, let me write these two bands are such that they do not overlap, otherwise we got a semi metal as we had discussed earlier. So, they are not overlapping. If I take 2 N electrons they will fill the lower band completely. So, let me show this by sharing it. This would have been a perfect insulator unless the thermal energy k B T causes some of these electrons to move up.

So, that part of the upper band is filled and in the lower band this part, the one shown by red is empty. The only part which is filled is the one shown by blue. So, this is filled and this red part has become empty. So, if that happens as a result what would happen is that could fill the lower band completely, but because of what we discussed just now, some of these go to the upper band.

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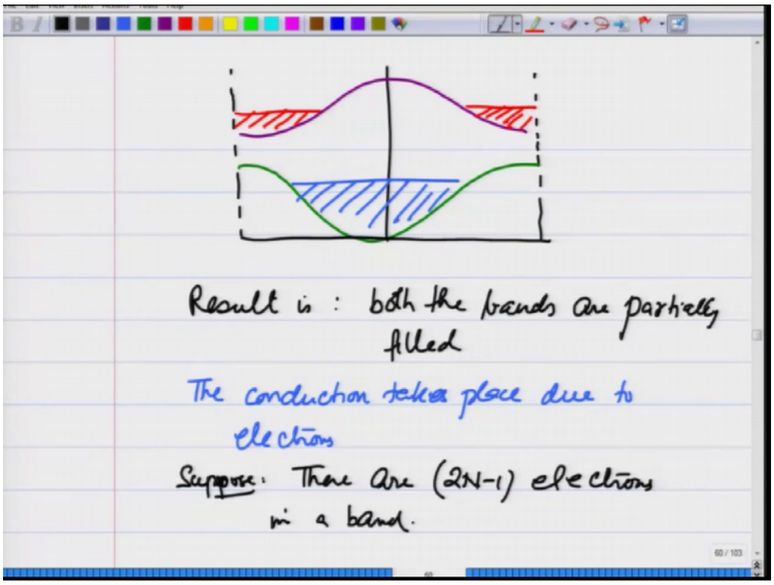

And therefore, let me keep making this picture again and again so that it is clear, what is going on and what I have shown is that, this band is very partially, some of these electrons have moved up. So, when they are thermally excited the result is both the bands are partially filled and we have discussed earlier that if I have partially filled bands, they conduct.

So, in a semiconductor where they are thermally excited electrons to the upper band, both bands become partially filled and they start conducting. The conductivity obviously, is going to be highly temperature dependent, because of the number of electrons or how partially the bands are filled depends on temperature, exponentially that we will see later in these weeks lectures, but right now, the idea is that the both bands become partially filled and when they become partially filled, they both conduct.

Now, we have seen earlier, the conduction takes place due to electrons, because these are the only charge carriers that carry current. However, there is a very important concept that arises, when we look at the band picture and particularly in semiconductors is that, I can either look at electrons or the absence of electrons to see how the conduction is taking place.

So, this just to give you a feel for it; suppose there are 2 N minus 1 electrons in a band; just suppose, there are 2 N minus electrons in a band.

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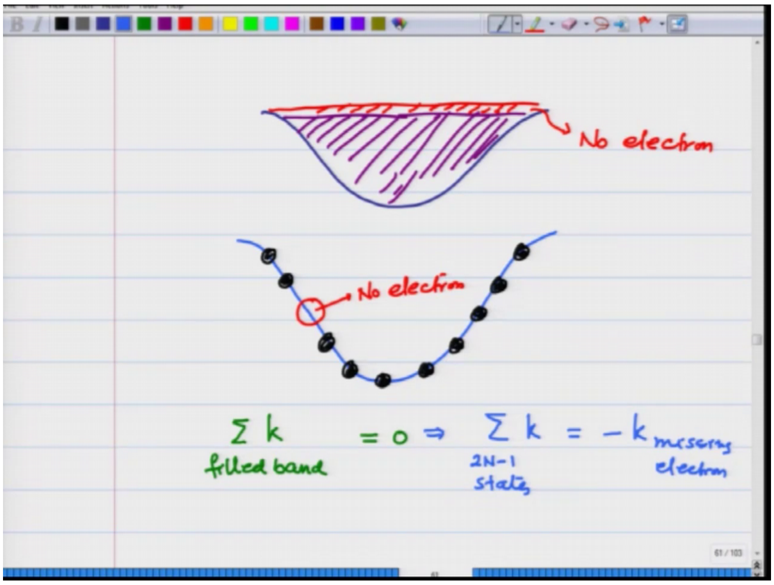

So, what would happen if I were to make this band, there are these electrons filling all the way up to 2 N minus 1 and there is 1 electron which is not there. So, 1 state is free, no electrons. I can also see it like this. Suppose there is this band and I have electrons sitting all over the place at each k value, but one place is empty. This has no electron.

When I look at the current or the motion of these electrons either I can look at all 2 N minus 1 electrons or I can see the missing electrons and see how it is behaving and from that I can calculate my things. This is because I can do that because summation of k in a filled band is 0 and therefore, if I fill 2 N minus 1 states, this implies that if I fill 2 N minus 1 states including this pin, this is going to be equal to minus k missing electron.

So, if I know the k of the missing electron I know precisely what the other electrons are doing that is, because the sum is always the same. Something like let me just give you an example.

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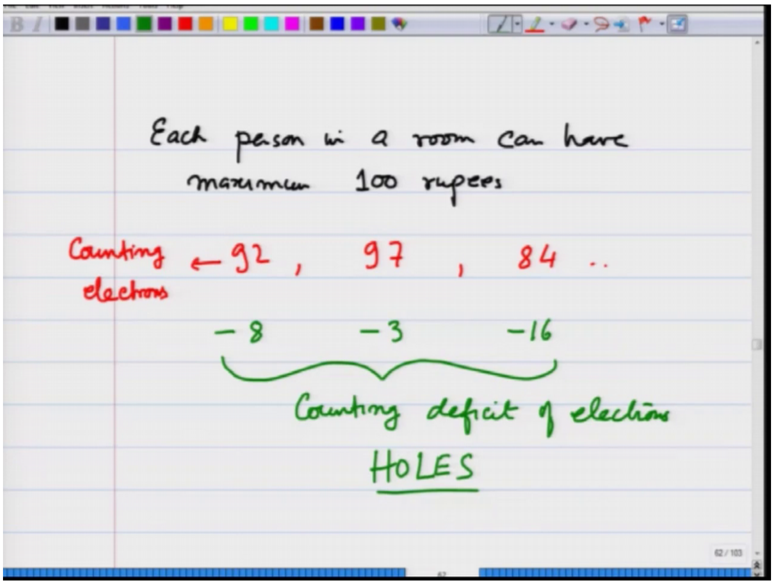

Suppose, we have that each person in a room can have maximum 100 rupees, maximum possible is 100 rupees and then I ask people how many rupees do you have? Somebody says let us say 92, somebody says 97, somebody says 84 and so on. Instead of that I could say minus 8, minus 3, minus 16, and so on. That would imply exactly the same thing, that I have 8 less than 100 92, 3 less than 100 97, 16 less than 100 84.

So, I could do all my calculations based on these negative numbers, the deficit of the rupees from 100. So, this would in a loose sense corresponds to counting electrons and calling this minus thing and the negative numbers who corresponds to counting deficit of electrons, which I am going to call holes.

So, concept of hole is nothing but a good way of bookkeeping and bookkeeping can be done well, if you have to count less number of, you know whatever you are dealing with. In an otherwise filled band, if there are very few electrons missing, it is better to work out things in terms of holes rather than electrons.

If the band is less than half filled, is better to count electrons both are equivalent ways of doing my calculations except that now, I have to precisely give the rules as to how my counting is going to be done and how these holes are going to be connected with deficit of electrons so, that I can do my mathematics correctly.

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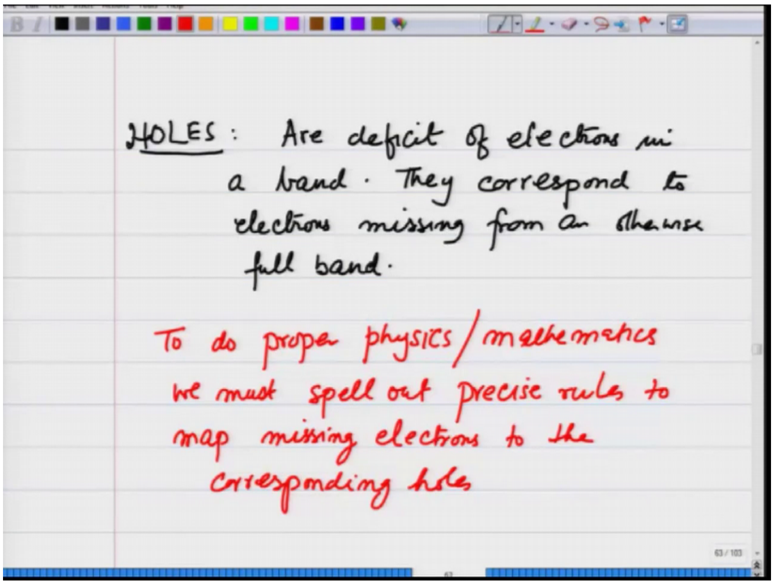

So, here it is holes are deficit of electrons in a band. They correspond to electrons missing from an otherwise full band. So, very few are missing and we call them holes.

Now, I have to therefore, to do proper physics, mathematics of transport or whatever we must spell out precise rules to map missing electrons to the corresponding holes. Once, we do that then we can do rest of the calculations in terms of holes alone.

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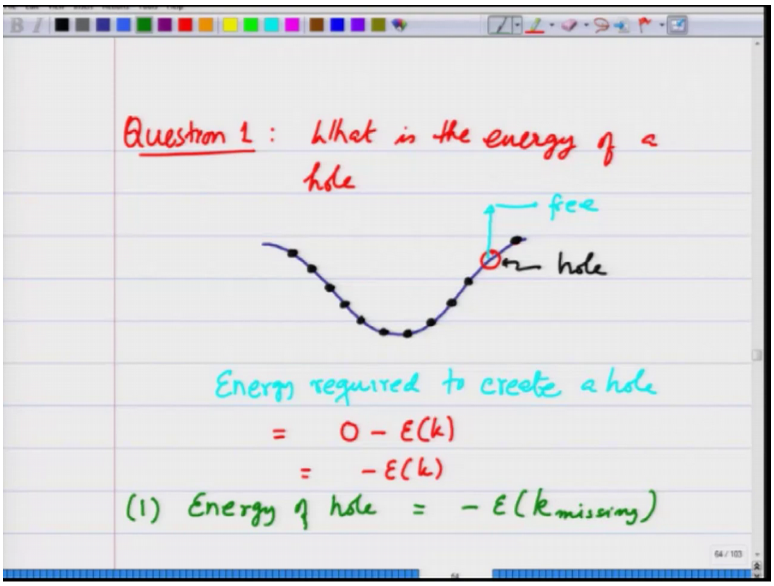

So, question number1; what is the energy of a hole? Remember, hole is corresponding to a missing electron. So, let us look at the band picture. I have this band in which I have electrons filling different k states, I am taking a few of them, but one of them is missing an electron. So, this is the hole, where the electron is missing from.

Now to create this hole what one had to do is, remove an electron from here and make it free. If the electrons are bound in the system, the energy required to create a hole will be equal to when electron is free its energy 0 minus the energy E k. So, this is equal to minus E k. So, one can immediately write rule number one, energy of hole. Now, we understand when I say a hole it corresponds to a missing electron is equal to minus E I will write k missing, by that I mean the case states from where the electron is missing. So, energy of a hole is minus. So, it is a positive energy if the electron is bound, then this has positive energy I will make a diagram little later and you will see.

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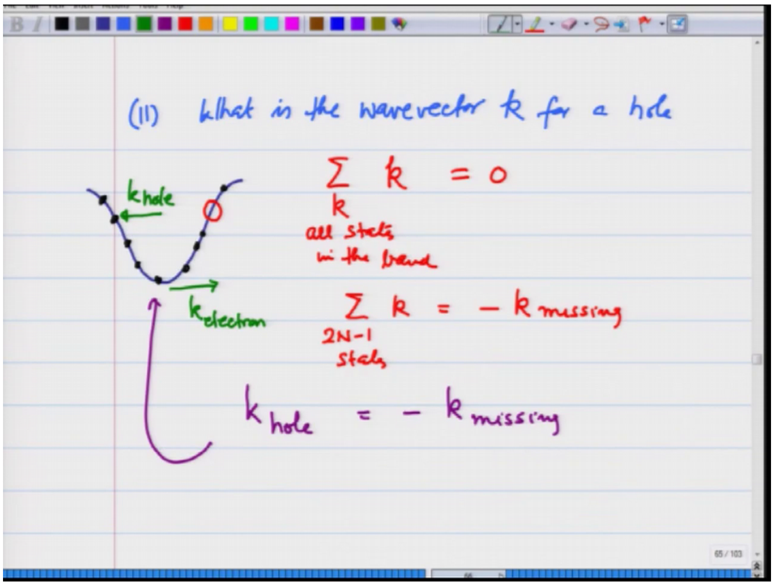

Number 2; what is the wave vector? So, after all I want to do the calculations just like I do for electron. So, I have to have all those quantities that I have for electrons for the holes also wave vector k for a hole. So, again let me make this picture of a band with all these electrons except one, which is missing and other electrons, then I know that summation k over the band by symmetry k, all states in the band is equal to 0 by symmetry.

And therefore, summation over 2 N minus 1 states k is going to be equal to minus k missing. Since, I want to represent these 2 N minus 1 filled electron states by the missing electron or by the hole.

So, I can say the k of the hole is equal to minus k missing. If I go back to this picture of the band, if this is k electron, where the state where the electron is missing from here is k hole, k hole is minus k missing electrons.

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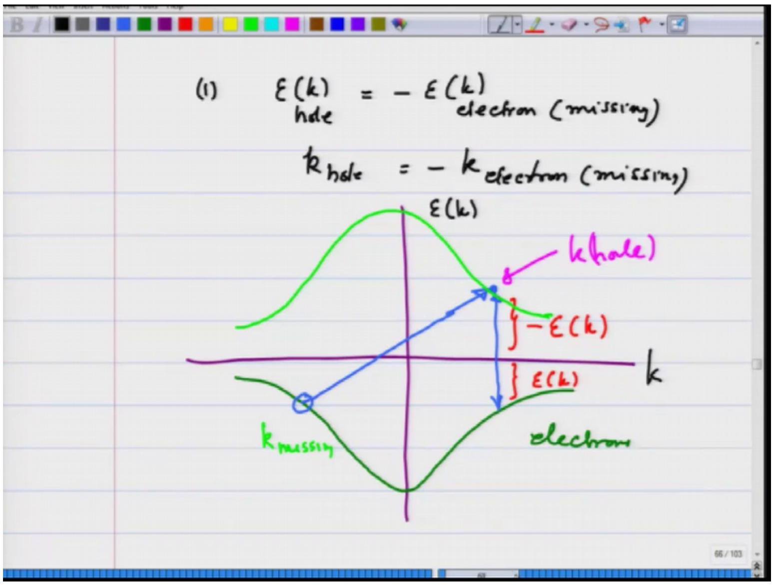

So, taking point one and point two together right, taking point one and point two together, that is point one was that E k of hole is equal to minus E k of electron missing and k of hole is equal to minus k of electron that is missing. Then if I would to make a band picture, if this is the band corresponding to electrons for each k out here, this is k electron, k hole would be opposite of that and the energy will be opposite of that.

So, let me make; this is E k, this will be minus E k and k also switches. So, I am going to have the whole band like this. So, this is k missing and this is out on the right is k hole. So, this is how the band picture looks. The band gets inverted, because it is minus k and it also gets inverted across the E k axis.

So, it gets inverted both across the k axis, let me write this here, and the E k axis is both it gets inverted and that gives you the whole band. So, we made two points, E k and k whole.

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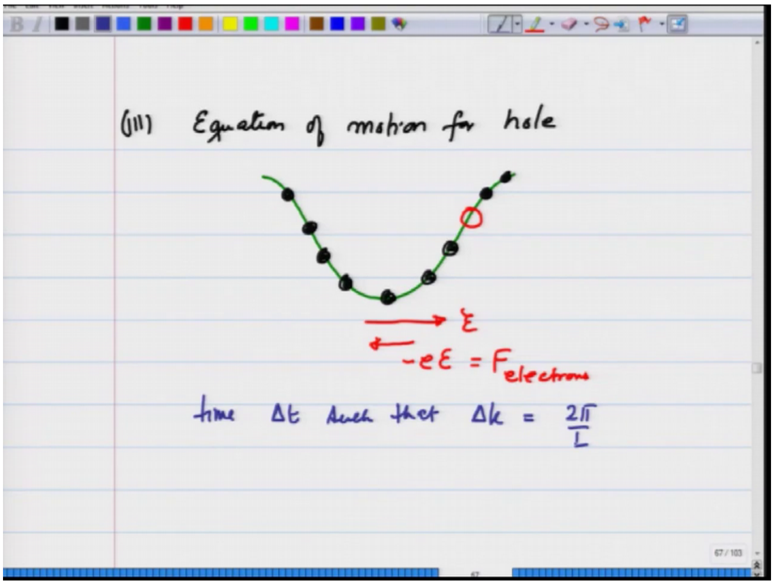

Next, third equation of motion for hole. Now, look at this, if I were to again make this band, in which I have these electrons and one hole and then electrons again and I apply an electric field in the positive x direction then the force on the electrons would be minus e E is equal to force on electrons. Let me take time, delta t such that delta k is precisely equal to the gap between two states. So, let us say 2 pi over L and let us see what will happen.

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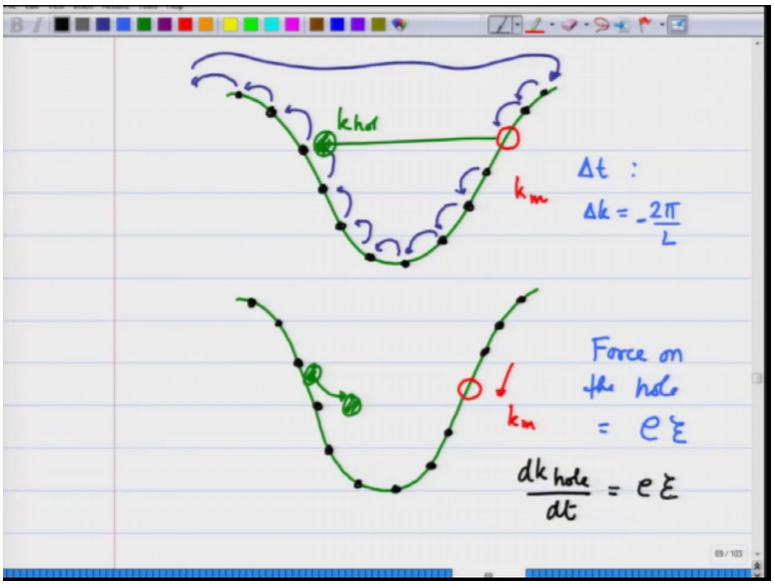

So, I will show this in the next slide. I have this band in which all these electrons are sitting except this hole and then the electrons and I am taking time delta t such that, delta k is equal to 2 pi over L and I should put a minus sign, because the force is in the other direction than the e. So, all these electrons will start hopping to the next side in that time delta t.

What about this electron the last electron? This will hop out and this goes back to its equivalent point on the other side. So, after all this hopping is done let us see what the picture is like. The picture is like all these levels, because there is been this hopping taking place are filled, except that the hole has now moved and I have three filled states to the right of it. So, hole has moved to the left.

So, if this was k missing, it has moved here. The corresponding k hole was here and this has moved this way to the right. So, electrons move to the left and hole is moving to the right. What does that tell you? That shows that the force on the hole was like e E that is in the positive direction, not negative, because it is moved to the right.

In other words, if I were to write dk hole by dt the equation of motion it will be e E. So, one can conclude that hole behaves as if it has positive charge.

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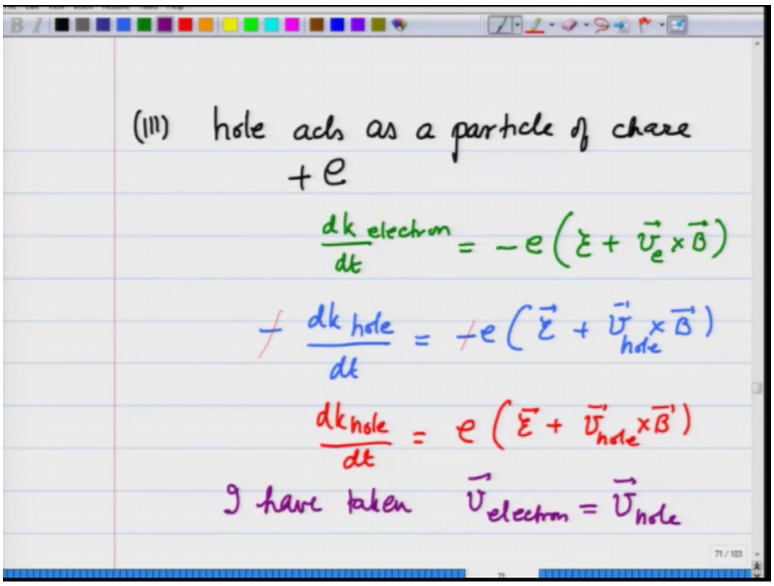

So, what we have understood from this point number 3 is hole acts as a particle of charge plus e. Now I am going to write it little explicitly mathematically, after showing you through all the band pictures and all that. I have dk dt for electron is equal to minus e the electric field plus V electron cross B, then I have minus dk hole corresponding to this missing electron divided by dt is equal to minus e E I should put a vector sign on top plus V hole cross B.

The minus sign goes away and I have dk hole dt is equal to plus e E plus V hole cross B, which shows that it is behaving like a particle of positive charge. If you are careful, you must have noticed that I have taken V, missing electron to be equal to V hole improving this. Let me show that next, I owe it to you.

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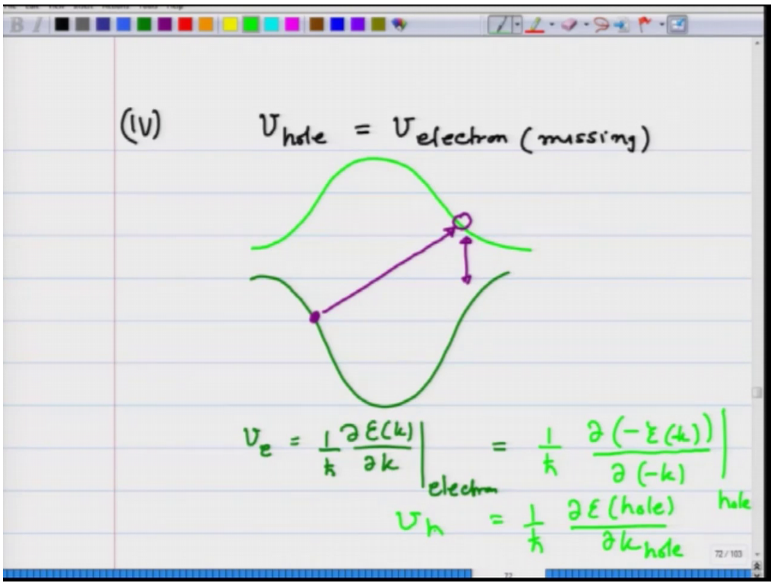

So, point number four is going to be that V hole is equal to V electron. When I say electron, I mean that missing electron and this is very easy to see, because I had made those bands right. So, if this is the electron band the corresponding hole band was exactly inverted of this, where the k of the missing electron is related to k of the hole out here and the energy is exactly opposite.

So, if I were to take V electron let me show it in the same color as the band V electron. This is d E k dk 1 over h cross and this is or for the electron, which I can write as 1 over h cross the whole energy is minus E k. I can put a minus sign also here, because of the symmetry d minus k and this is for the hole.

So, which I can write as 1 over h cross d E hole divided by dk hole. So, what you see mathematically is that the group velocity of the electron is exactly same as the group velocity of the hole. So, I can write at V e equals V h and this is what I use in the previous slide.

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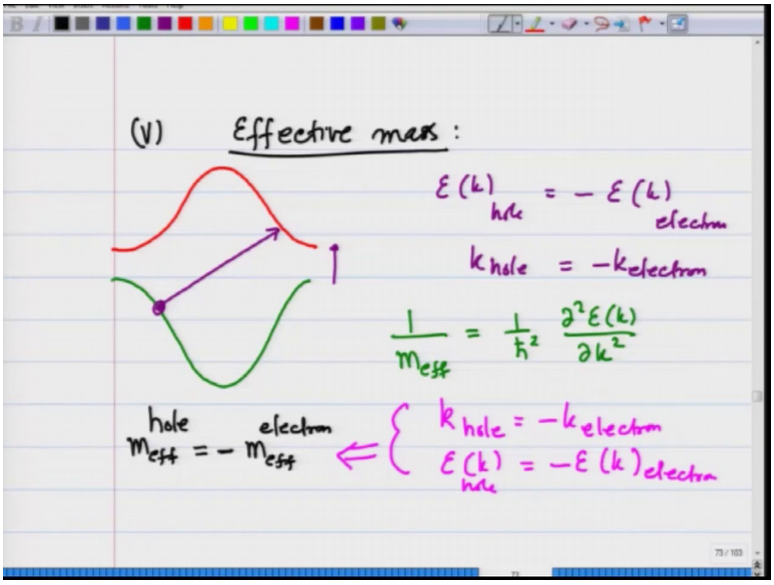

Number five ; we also talked about effective mass in that we talk about how an electron or those 2 N minus 1 electrons respond and that response will be equivalent to the response of this hole. So, for the effective mass, I can again use this band inversion. I have this band of electrons, which transforms to the whole band like this, k is related across and E is also related across; that means, E k hole is equal to minus E k electron and k hole is equal to minus k electron.

Effective mass 1 over m effective is defined as 1 over h cross square d 2 E k over d k square. Since, I am taking a double derivative with respect to k, if I take k hole which is minus k electron the double derivative with respect to k would not change sign whereas, E k hole is equal to minus E k electron. So, all this implies that m effective for the hole is equal to minus m effective for the electron.

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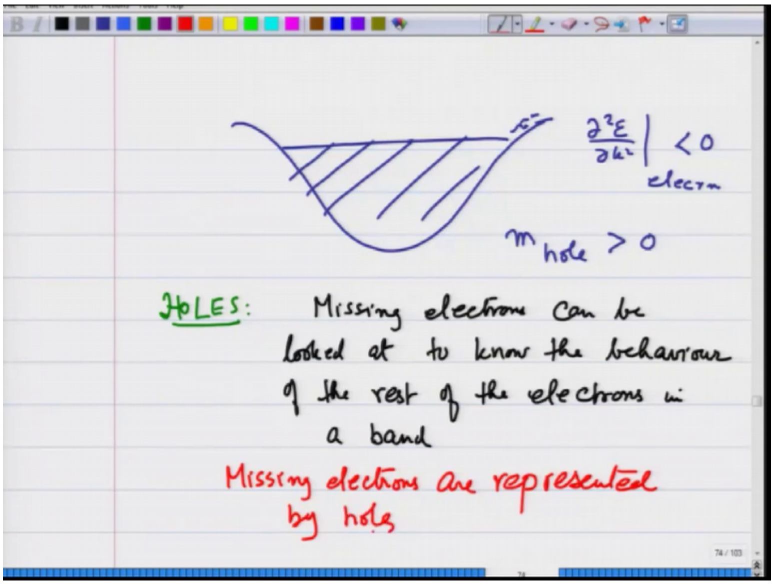

And that makes perfect sense, because if I look at the band, the top has d 2 E by dk square for the electron to be less than 0 negative and this is where the whole concept is useful, that rest of the band is filled and some of the states in the top of the band are free, then the mass of the hole is going to be positive.

So, we have a positive charge, a positive mass particle that is known as the hole, which actually represents the missing electron or the 2 N minus 1 electron, because of the one missing electron. So, to conclude the concept of holes we have shown that missing electrons can be looked at to know the behavior of the rest of the electrons in a band and this concept is useful only, if some of the electrons are missing from the top of the band, because that is when all the positivity of the mass and the charge comes through and these missing electrons are represented by holes.

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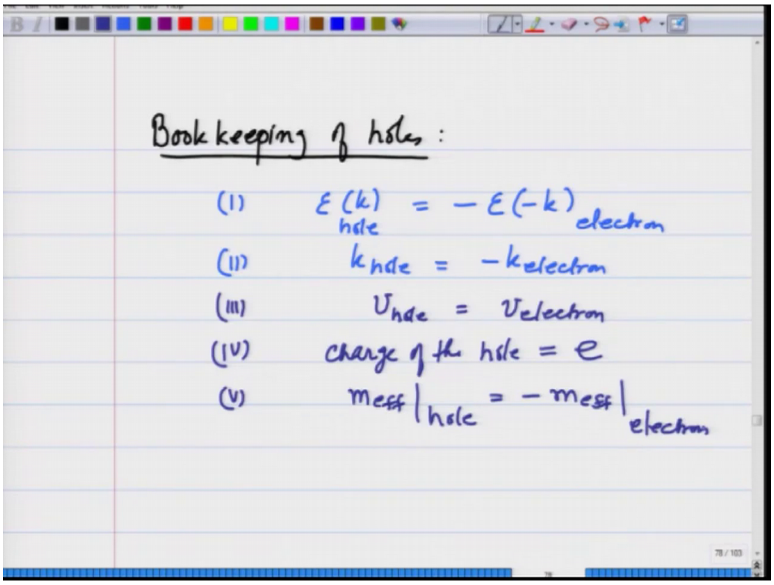

And these holes, so bookkeeping holes is done through number I, E k of hole is equal to minus E and, because the symmetry of the band I can write as minus k electron or the missing electron. Number II k hole is equal to minus k electron. Number III V hole is equal to V electron.

Number IV charge of the hole is equal to e plus e. Number V, m effective of hole is equal to minus m effective of that missing electron and with this we can completely describe on otherwise filled band, which is slightly empty with holes and this is our bookkeeping for the hole.

So, we will talk about holes in terms of they being genuine carriers of electricity and charge in the coming lectures and what we doing now, rest of the lectures in this is developed the theory for how many electrons, how many holes are there as a function of temperature and therefore, what are the properties of semiconductors.