Introduction to Solid State Physics

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Lecture – 67
Experimental observation of Bloch oscillations

In the previous lecture, I discussed the dynamics of a particle in a band and let me just recall what we did and then I will describe an experiment that shows this and why we do that experiment in a particular way.

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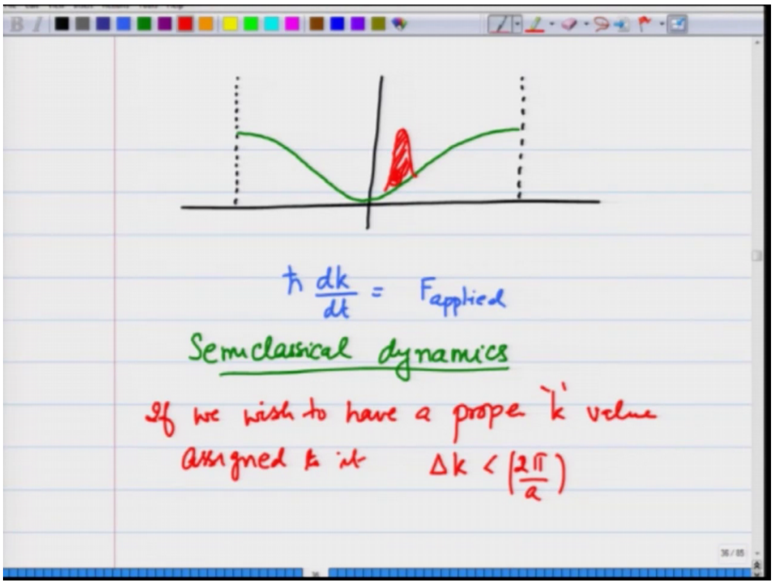

$\hbar \frac{dk}{dt} = F_{applied}$

Semiclassical dynamics

If we wish to have a prope 'k' value assigned to it $\Delta k < \left(\frac{2\pi}{a}\right)$

So, if I just look at one band and take a particle in it, what we showed by energy considerations that this equation of motion is going to be given by h dk by dt equals F applied. And if you recall long ago I had told you when we were describing bands and particularly when I was explaining metallic insulating behavior through bands I had said that this motion of an electron in a band is described through the applied force giving it energy and it is the electron moving higher and higher in the energy. So, same thing we use here to get this equation of motion.

So, as you apply the force this k increases; now what I had also told you we are doing semi classical dynamics. In that, when I look at a particle I am actually looking at, it as a wave packet and if I want to have a proper k assigned to it. So, if we wish to have a proper k value assigned to it then they spread in k should be much less than the spread of the Brillouin zone. Otherwise, it will have no meaning because after all, the way I described the motion in the previous lecture was that this particle moves in the Brillouin zone, it comes from minus pi by a 2 plus pi by a and goes back to minus pi by a because plus pi by a and minus pi by a are the same.

So, if you want to see that motion through the Brillouin zone, I should be able to see k at the level of this Brillouin zone and therefore, I should be identified k within this boundary of 2 pi by a and for that the spread in k of the wave packet should be much less than 2 pi by a or the extent of the Brillouin zone.

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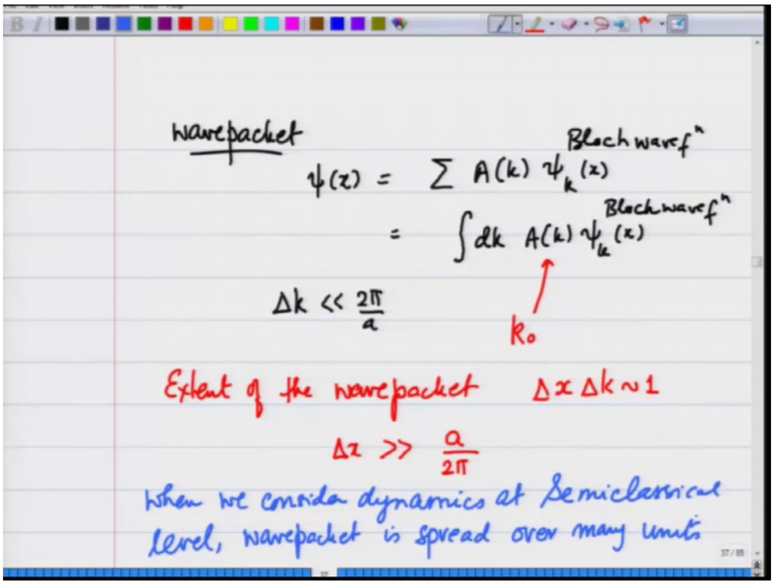

wavepacket

$$\psi(x) = \sum A(k)\,\underset{k}{\psi}(x) \quad \text{Bloch wavef}^{n}$$

$$= \int dk\, A(k)\,\underset{k}{\psi}(x) \quad \text{Bloch wavef}^{n}$$

k_0

$$\Delta k << \frac{2\pi}{a}$$

Extent of the wavepacket $\Delta x\,\Delta k \sim 1$

$$\Delta x >> \frac{a}{2\pi}$$

When we consider dynamics at Semiclassical level, wavepacket is spread over many units

Let me explain that further, after all what we are doing is when I make a wave packet right, this is some psi x this is done by mixing many psi k x Bloch wave functions which in the case of continuous k would be written as integration dk A k psi k x Bloch wave function.

So, if I want to have a well defined k which I want to check later, I should have this spread in k much less than 2 pi by a. So, amplitude A k here should be large for a particular say k 0 and then it should die off very fast. And therefore, the extent of the wave packet; that is the size of the wave packet is given by the uncertainty relation delta k delta x delta k is of the order of 1 and therefore, delta x will be much greater than a over 2 pi. And what that means is, when we consider dynamics at semi classical level, a wave packet is spread over many crystal constants or crystal sites. So, wave packet is spread over many units all the crystal.

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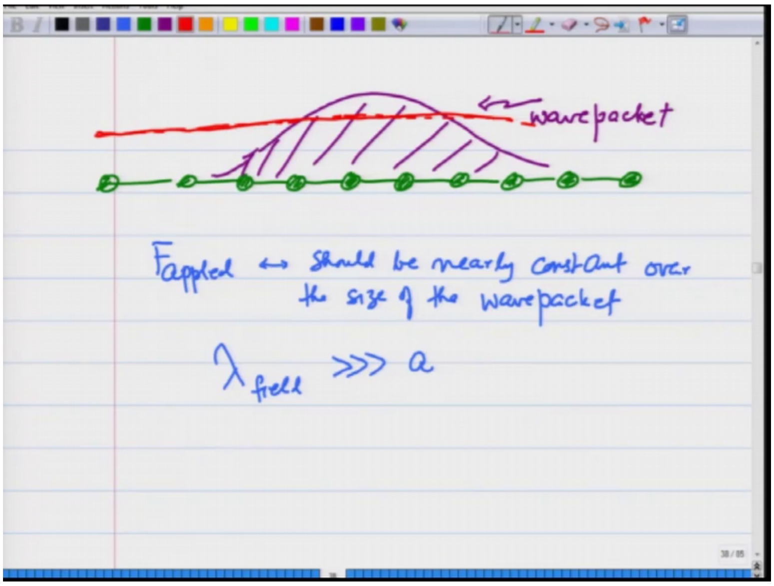

$F_{applied}$ ↔ Should be nearly constant over the size of the wavepacket

$\lambda_{field} >>> a$

So, wave packet is large. So, if I were to make a picture if I have this crystal like this. And to have a well defined k, wave packet has to be spread over a large number of crystal sites, this is the wave packet and when I apply a field to study the dynamics. Now we when we apply a field F applied, it should not be changing over the wave packet so, F should be nearly constant over the size of the wave packet and that means, lambda field or the force should be much much larger than a.

So, that if I see the field it is nearly a constant over the wave packet and then may it may just die down right. So, this lambda you can see is huge. Obviously, if I apply a static field, a constant electric field lambda is infinity and there is no problem in studying the dynamics of the wave packet. So, this is the kind of setup I wanted to give you, I derived all the equations everything and I wanted to now tell you, where it is applied and what is the region of validity.

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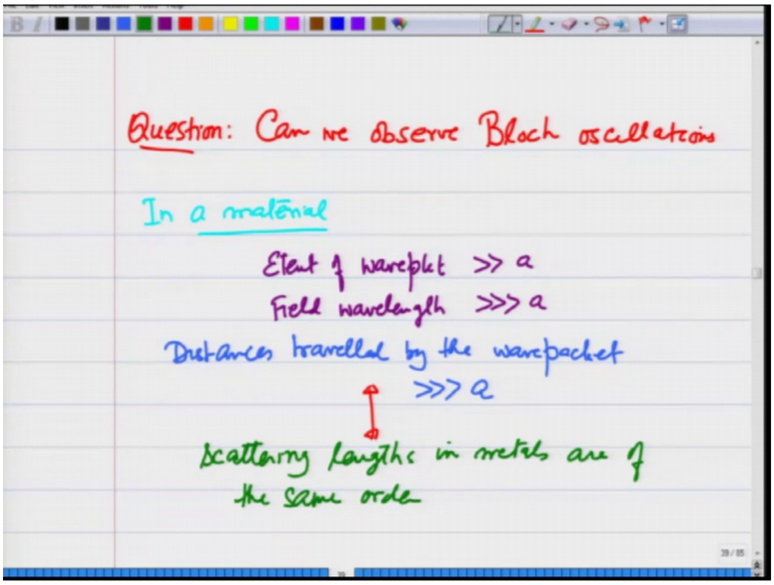

Question: Can we observe Bloch oscillations

In a material

Extent of wavepkt >> a

Field wavelength >>> a

Distances travelled by the wavepacket >>> a

Scattering lengths in metals are of the same order

So, now, the experiments question is, can we observe Bloch oscillations let us say in a material. So, in a material a metal or something right. Now, we already said that the extent of wave packet is much larger than a. Field wavelength is even larger or maybe a static field. Now extend a wave packet is much larger than a, it would also be moving through a distance much larger than a before I can see clearly that the wave packet has moved.

So, I can also argue that the distances travelled by the wave packet if I want to really see what distance traveled has to be also much larger than a, then only I can see that it has moved, but now there is a conflict. And the conflict is scattering lengths in metals or materials are of the same order.

So, before you observe that the wave packet has moved or the particle has moved, it let us be scattered its momentum, its position all those will be lost and then you cannot really verify whether your equations are correct or not. So, these experiments were done in a slightly different system and these are beautiful set of experiments one of which I will describe today.

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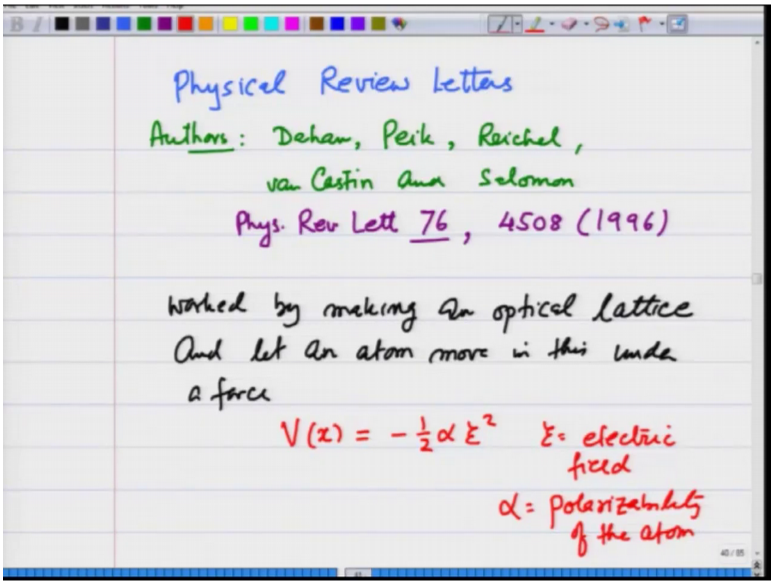

Physical Review Letters

Authors: Dehan, Peik, Reichel,
van Castin ana Solomon
Phys. Rev Lett 76, 4508 (1996)

worked by making an optical lattice
And let an atom move in this unde
a force

$V(x) = -\frac{1}{2}\alpha \mathcal{E}^2$ $\mathcal{E}$ = electric field

α = polarizability of the atom

And this experiment is taken from physical review letters I will upload this, paper also its very easy reading this was published in 1996, the authors are Dehan, Peik probably, Reichel, Van Caston and Solomon and the reference is Physical Review Letters volume 76, page number 4508, this was published in 1996. Many more papers have followed after this, but this was the first paper and a beautiful experiment. So, I will describe this experiment to you and then you can follow the rest of the papers just by typing frequency of Bloch oscillations things like those and Google and you will get many of those later papers.

So, in this paper what these people did is worked by making an optical lattice and let an atom move in this under a force so, let me explain each term to you. Roughly, if you want to understand the potential of an atom in electric field would be minus half alpha E square that is the potential energy where E is the electric field and alpha is the polarizability of the atom.

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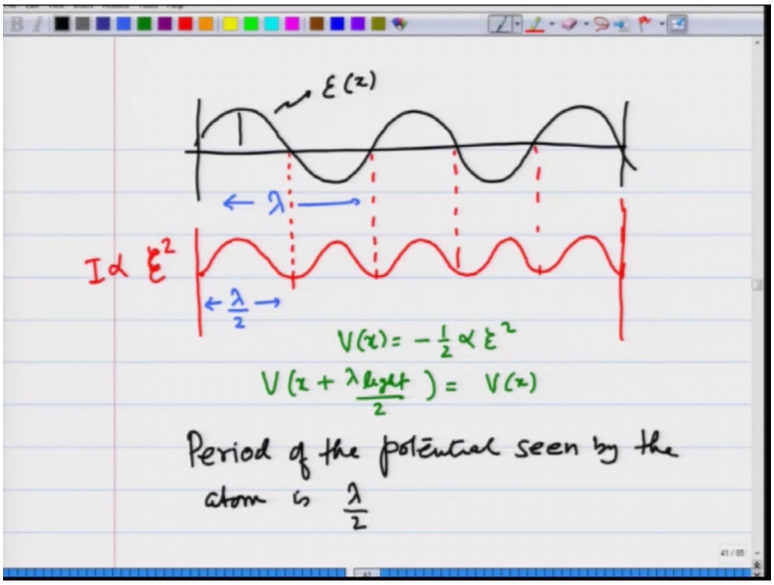

$V(x) = -\frac{1}{2}\alpha E^2$

$V\left(x + \frac{\lambda_{light}}{2}\right) = V(x)$

Period of the potential seen by the atom is $\frac{\lambda}{2}$

So now if I take a standing light wave; a standing light wave so that, this is this electric field E x; obviously, it oscillates back and forth. This electric field would give me an intensity let me just make it carefully which is proportional to E square and this look like this, this is E square intensity is proportional to this. So, if the wavelength of this light was lambda, the intensity has a period of lambda by 2, it repeats itself after lambda by 2 and an atom in this has the potential energy V x is equal to minus half alpha E square therefore, it sees a potential which is periodic of period lambda by 2.

So, V x plus lambda let me write light by 2 the same as V x. So, the period of the potential seen by the atom is lambda by 2, it is moving in a periodic potential and therefore, this is known as an optical lattice.

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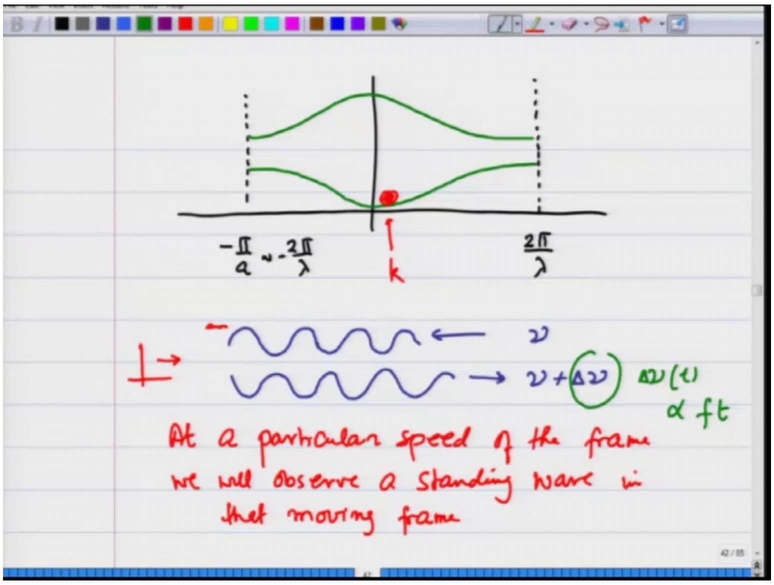

At a particular speed of the frame we will observe a standing wave in that moving frame

And if I were to plot the energy of this atom in the system it will look exactly like particle in a periodic potential, this band diagram in the Brillouin zone which is extending from minus pi over a which will be 2 pi over lambda minus 2 pi over lambda to 2 pi over lambda, in this the particle would be moving in a band like this.

So now the job is to put this particle here at some k and give it an acceleration. So, we have to prepare a wave packet initially and then give it an acceleration and this is another beautiful trick, these people do in this experiment. If I look at two waves counter propagating. This is going to the left and the other wave going to the right and have a frequency difference slight frequency difference delta nu, then the standing wave pattern will move, but if I sit in a frame that is moving to the right the frequency of the upper wave will look larger to it due to Doppler shift and frequency of the lower wave will look smaller to it due to Doppler shift at a particular velocity two frequencies would be the same.

So, at a particular speed of the frame, we will observe a standing wave in that moving frame. Now, suppose I make this delta nu let me take this delta nu time dependent linearly; suppose, this is sum f t being a constant. Then the frame would have to keep moving faster and faster with time, in other words it will have to keep increasing a speed or accelerate with time in order to see this standing wave.

So, I will be sitting in a non-inertial frame and therefore, all the particles in this frame would be experiencing a pseudo force minus m times that acceleration. So, this is how you produce this force on these particles in this wave. So, you take 2 lasers counter propagate them and make the frequency of one of the lasers changing with time and then in that moving frame it will be the particles in this will be accelerating backwards they will be experiencing a force and then you can transfer back to your original frame whatever you observe.

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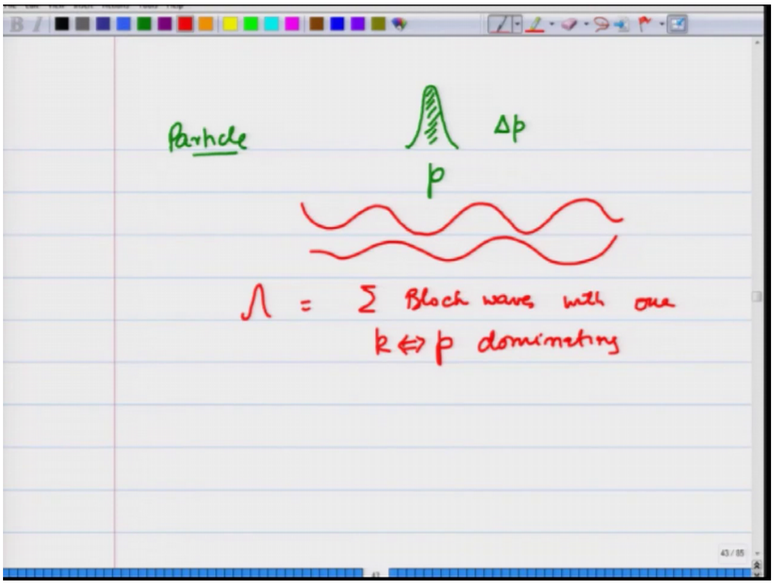

So, this will be the trick use so, what these people did is initially took a particle. Prepared a state with some momentum p in the x direction and some delta p which is small compared to you know whatever dimensions, whatever lasers they are using and then slowly turn this lasers on. So, that there was a standing wave and now the wave packet would be a collection of many Bloch waves; however, if you keep the field weak enough and change the laser intensity slowly.

So, that you do an adiabatic switching these are all words that I am taking from quantum mechanics, this will turn into a collection of the wave packet return into a combination of Bloch waves with one k corresponding to p dominating and then you chirp or increase the frequency of one of the lasers with time linearly.

Let the wave packet evolve and suddenly switch off the lasers; when you suddenly switch it off, then you will see in that wave packet or in the particle mainly the k it was at that point and this is how they measured things. So, after performing this experiment they started measuring the k values and the velocities and the amplitude of the oscillation and they confirmed with whatever is predicted through Bloch oscillation.

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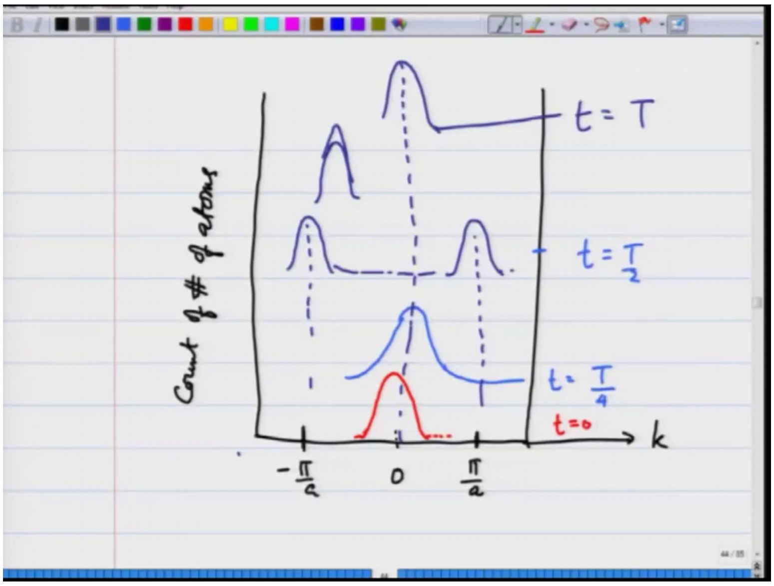

So, for example, when they measured k I am taking it from the paper suppose, I were to plot the count of number of atoms that were in a particular state and this is the k value are measuring k in the first Brillouin zone.

So, let me write this as minus pi by a to pi by a and they started and this is 0 with a wave packet spread in k space, but you see the spread is less in k space like this. So, number of atoms which is very large has k equals 0 and then; obviously, there is a spread some atoms have other k values also and this is at t equal to 0. Now, they turned their lasers on let the whole thing accelerate for some time and then they measured that this wave packet shifted to this side this is some at t equals the time period divided by 4.

They measured it more and more and at t equals T by 2 what they saw was 2 peaks, one at k equals pi by a and one at k equals minus pi by a what does that mean? That simply means that, once the wave packet reached the Brillouin zone we have already been saying that plus pi by a is equivalent to minus pi by a.

So, half the atoms went into plus pi by a state half the atoms went into minus pi by a state and as one evolved this peak started becoming larger towards minus pi by a. So, at higher temperature a higher at longer time this again started evolving and the peak became bigger and then finally, at t equals the Bloch period t equals the Bloch period to the peak came back to q equals 0.

So, this confirmed whatever we had predicted and also confirmed how at t equals T by 2, we got 2 peaks plus pi by a and minus pi by a because the 2 are equivalent. So, atoms had no choice, but to split. The probability of having particles with plus pi by a momentum is the same as probability of having minus pi by a momentum. Because remember when we are doing our band theory nearly free electron model at this stage, when we were at the Brillouin zone boundary the 2 waves mixed with equal amplitude.

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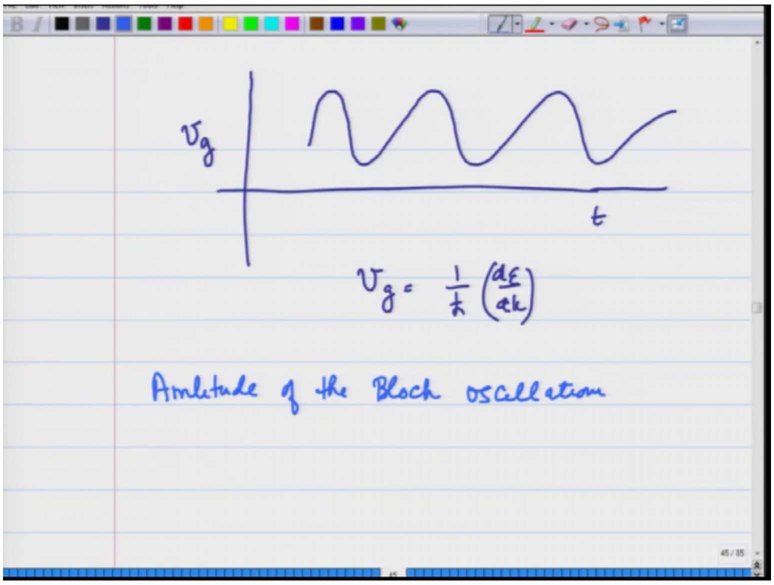

$$v_g = \frac{1}{\hbar}\left(\frac{dE}{dk}\right)$$

Amlitude of the Bloch oscillation

So, this is the first confirmation they did it. Then they also measured the velocity, how the majorities are described in the paper and they saw that velocity also change with time in a periodic manner. This is exactly what you expect from the, this is velocity let me call it v group, because v group is the particle velocity this is precisely what you expect from 1 over h cross d E by d k in a band and I had plotted this velocity yesterday. And finally, they also measured the amplitude of the Bloch oscillations and I will give this what the amplitude should be as an assignment problem to you.

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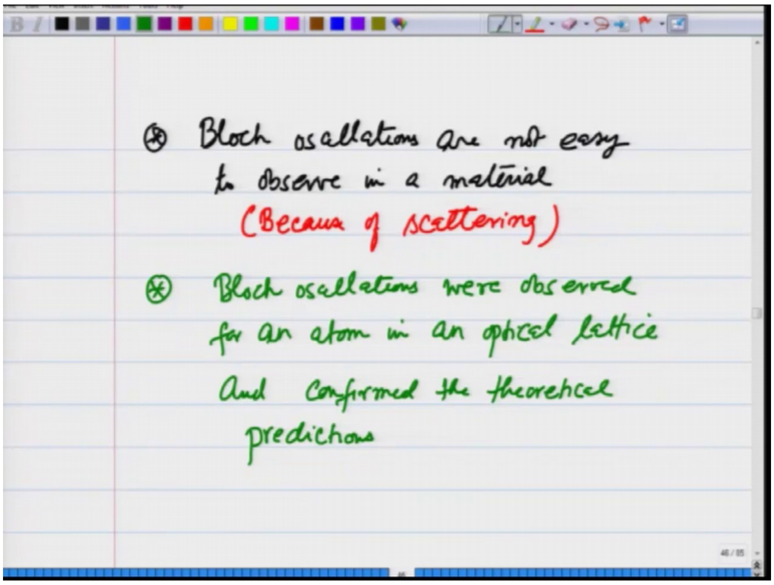

⊛ Bloch oscillations are not easy to observe in a material

(Because of scattering)

⊛ Bloch oscillations were observed for an atom in an optical lattice and confirmed the theoretical predictions

So, to conclude Bloch oscillations are not easy to observe in a material and this is because of scattering; it is very difficult to make a pure material. And therefore, Bloch oscillations were observed for an atom in an optical lattice and confirmed the theoretical predictions. With this I conclude the discussion of bands and the motion of particles in bands. Next week, we will start on semiconductors and discuss their properties, their thermal properties, their conductivity and things like those.

Thank you.