Introduction to Solid State Physics

Prof. Manoj K. Harbola
Prof. Satyajit Banerjee
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 06
Introduction to Hall effect in metals – Part I

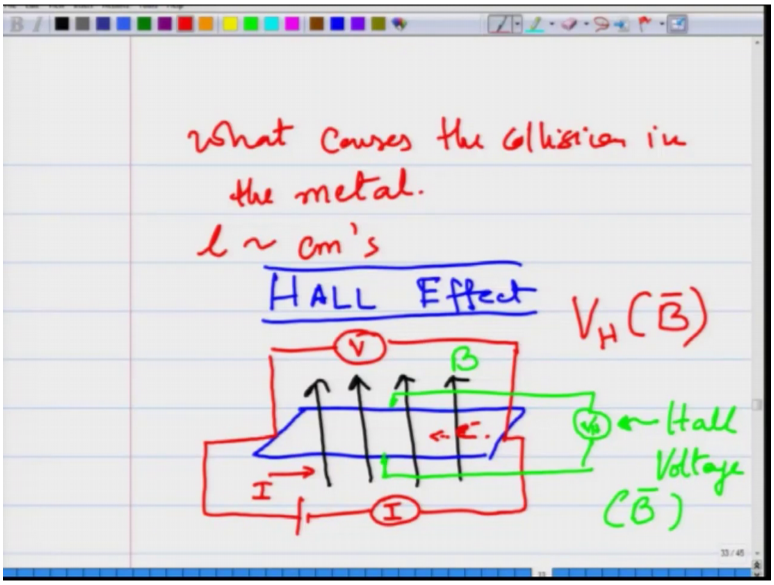
We saw that the Drude model gave us a way to understand how does resistivity appear inside the metal. It appears because of scattering, we wrote down the equation of motion which has a drag term appearing because of the scattering and hence found out the resistivity.

But as we went on further, we found that if you include realistically how the electron behaves; it is not really like a classical particle, but it is a quantum mechanical object which moves inside the solid. And if you include the distribution the energy distribution of the electrons inside the solid; if you use that quantum mechanically its velocity turns out larger. You will find that as you lower the temperature the scattering times are larger which will give rise to typical mean free paths which are much longer than the mean spacing between the ions.

And so therefore, although it is successful; there are some issues in which are not completely solved and which are not completely well known ok. So, what is the mechanism of scattering? It is not really coming out from the Drude's model and then hence that is one sort of a drawback of the Drude's model. We do not have a exact mechanism of the scattering we know that the electron scatter, but what generates a scattering; it could be disorder, it could be electron phonons, you will study about something called as phonons. So, the electrons where we scattering of phonons and therefore, there is a temperature dependence and so on which you will be exposed to in the part of this course.

You will study some phenomenas of these of this nature; you will be exposed to phonons. So, the phenomena of scattering of electrons inside the metal is not sort of well established, but be that as it may the Drude's model is still successful. And it is successful in describing another phenomena which is the phenomena of Hall effect.

(Refer Slide Time: 02:15)



This is a very important phenomena and it is also very useful phenomena; it has a lot of applications. In current day life, we are all surrounded by Hall sensors our mobile phones have it, our cars have it and they are various sorts of places they have Hall sensors installed inside it.

So, what is the Hall effect phenomena? So, what you do is in the Hall effect. So, now, we will describe you the Hall effect phenomena; the Hall effect phenomena basically has you take a very thin film of the material so that the electrons are basically moving in one single plane. Consider the plane of the screen and the electrons are moving only in the plane of this screen. It is a very thin film of the metal or the material.

So, you have a very thin film of the material and let us say that we apply a voltage across this. Because of the voltage which is applied there will be a current I which will start flowing through the metal; it will flow through the metal and you will get a current I. There will be a current I in the mental and as per convention the electrons will drift in the opposite direction.

So, there will be a drift of electrons this is as per convention that if the electron; if the current is in this direction the electrons are drifting in the opposite direction. Now because of this current I you will get a voltage drop across the metal which is what you have studied; Ohm’s law and we calculated this through the resistivity; that you will get a normal voltage drop along the direction of the flow of current and this is known ok.

Now apart from this configuration we will add something extra to this ok. So, you have applied an electric field and the system is responding to that electric field by generating a current. And you know that the current voltage relationship which we have calculated through the resistivity in the earlier lectures.

Now, apart from an electric field let us switch on a magnetic field and let us apply it perpendicular to this. We apply a magnetic field to this plane of this material. So, there is a magnetic field which is applied perpendicular to the plane of this screen ok. From below it we apply a magnet from below the screen we apply a magnet which applies a vertical magnetic field which is crossing perpendicularly to the surface of this film.

Now what you will observe is that apart from this longitudinal voltage which is in the direction of the flow of current; you will get an additional voltage which actually acts perpendicular to the flow of current that. And this voltage appears which I write it as V H; this voltage which appears across the sample and perpendicular to the direction of the flow; the direction of the flow of current is in this direction.

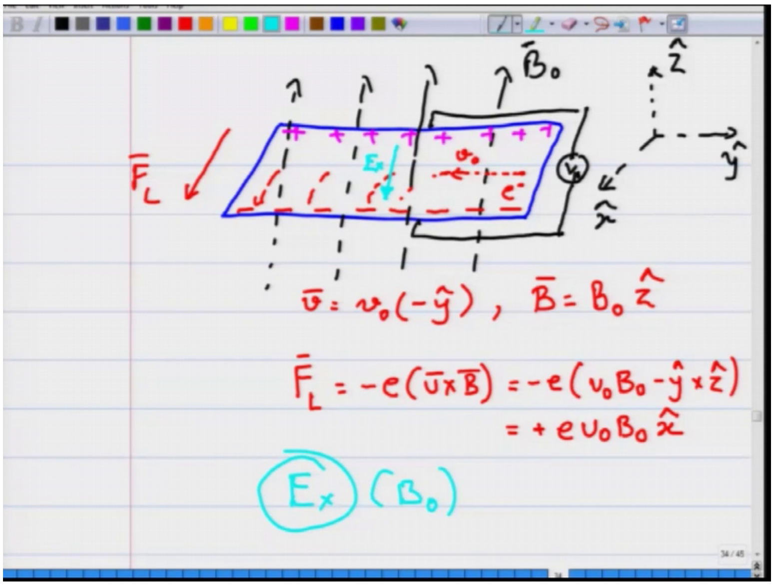
And you are finding a voltage which is being generated perpendicular the flow of the direction of the current; this is called the Hall voltage. And this is generated only when there is a magnetic field B applied to the sample. This Hall voltage appears only when there is a magnetic field which is applied to the sample and the Hall voltage is a function of the magnetic field.

If you measure this transverse voltage which is a occurring perpendicular to the direction of the flow of current; you will find that the Hall voltage is the function of the magnetic field. If you know the functional dependence namely how does the Hall voltage depend on the magnetic field; you can actually measure the Hall voltage and figure out what is the strength of the magnetic field.

And that is how the Hall sensors work; that is how Hall sensors which are present in our mobile phones, which is present in the inside cars and so on actually measure what is the magnetic fields in its environment; wherever the sensors are placed and they actually measure the Hall voltage. By measuring the Hall voltage you know how much magnetic field is present or is being experienced by that sensor. If there is no magnetic field, if B is 0 if the magnetic field you switch it off the Hall voltage will become 0.

So, the essential part of the Hall effect is that when you apply a magnetic field; there is a transverse voltage which is generated, which is transverse to the direction of the flow of current and that is the Hall effect or the Hall phenomena. Let us look at this in slightly bit more details that why does it occur. First let us look at it physically that what is happening; so let us draw the metal once again.

(Refer Slide Time: 08:03)



And assume that I have applied a voltage to this metal, first let us draw the coordinate axis. And I apply a magnetic field which is in the perpendicular direction which is perpendicular to the plane of this metal whose value is B naught ok.

This is a magnetic field I have applied and let us assume that the current is flowing in this direction; therefore, the electrons are flowing in the opposite direction. So, you have the electrons which are drifting in this direction. So, the velocity of the electron is let us say v naught they are moving with a velocity v naught which is in the minus y cap direction. And the magnetic field that you have applied is B naught z cap; it is along the z direction which is perpendicular to the plane of the screen. So, the magnetic field is along the z cap direction electrons are moving in this direction.

What will happen? What will the electrons experience? Because of the flow in a magnetic field you know that they will experience a Lorentz force; which is minus e into v bar cross B bar. This is minus e into v naught B naught minus y cap cross z cap. And in your cross product you know that this will be e v naught B naught; x cap.

So, what you get is a force which is acting along the x cap direction and as a result of this force; what will happen? The electrons will start getting deflected because there is a force which is acting; there is this Lorentz force which is acting along the x cap direction the electrons will get start getting deflected along this side. And there will be a buildup of electrons along this edge of the metal which I am showing as these negative signs because the electrons are building up on this side.

And because the electrons are drifting away from that part; you are leaving back positive charges on this end and so what you have is positive charges which are left on this end. So, one end becomes negatively charged the other end becomes positively charged; there is a buildup of negative potential and there is a buildup of positive potential at the two ends.

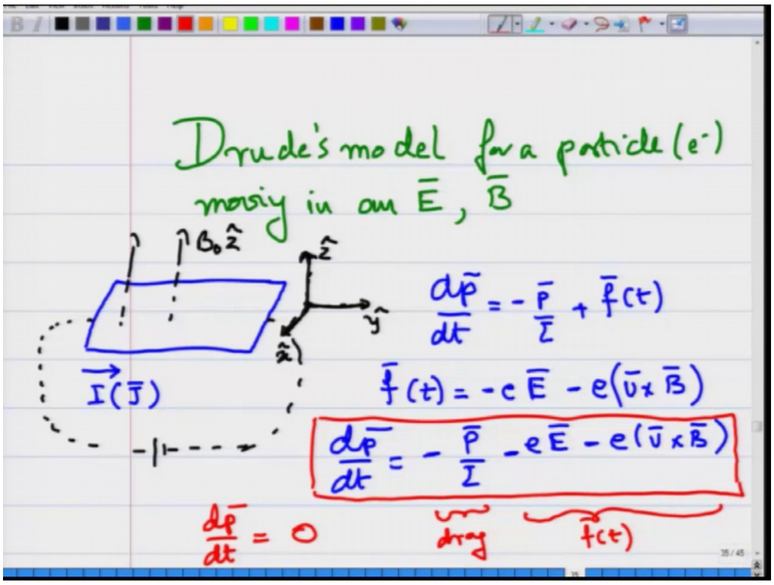
And because of this what happens is that what you will have? Naturally you are going to have a electric field which is acting in the x direction; you have an electric field which is generated and because of this you are going to get; if you measure a voltage across this, you will get a transverse voltage.

The voltage is not along the direction of the current; this voltage which is generated is purely because when you switch on a magnetic field the electrons get deflected and they collect along the edges of the sample. And then if you measure the voltage in the transverse direction which is in the direction perpendicular to the flow of current; you will get a voltage drop.

Because there is an electric field now generated in that direction as the charges have got deflected. There is a buildup of negative charge in one side and build up of positive charge on the other side and because of this, this is the Hall effect. So, we would like to calculate how much is this electric field which is generated inside the metal. And how does it depend on the strength of the magnetic field; how do we do about go about calculating this electric field.

So, for that we now go back again to the Drude's model of conductivity and let us write down the Drude's model for a particle which is moving in an electric field and a magnetic field.

(Refer Slide Time: 13:21)



So, we will write down the Drude's model for a particle which is an electron out here moving in an electric field and a magnetic field.

So, to do that first let us again draw and let us set up our equations clearly; this is your sample, this is your current or also your current density which you set up inside the metal. This is the direction of the flow of current; the electrons are propagating in the other direction. And what you have is you have a magnetic field which is applied to this sample which is B naught z cap and let me draw the coordinates x cap, y cap, z cap.

So, assume that I have set up a voltage, I have applied a voltage across it because of which electrons are flowing and there is a current which is generated ok. There is a current density J which is in this direction and you also place it in a magnetic field. So, because of the voltage there is an electric field which is present and apart from that there is also a magnetic field.

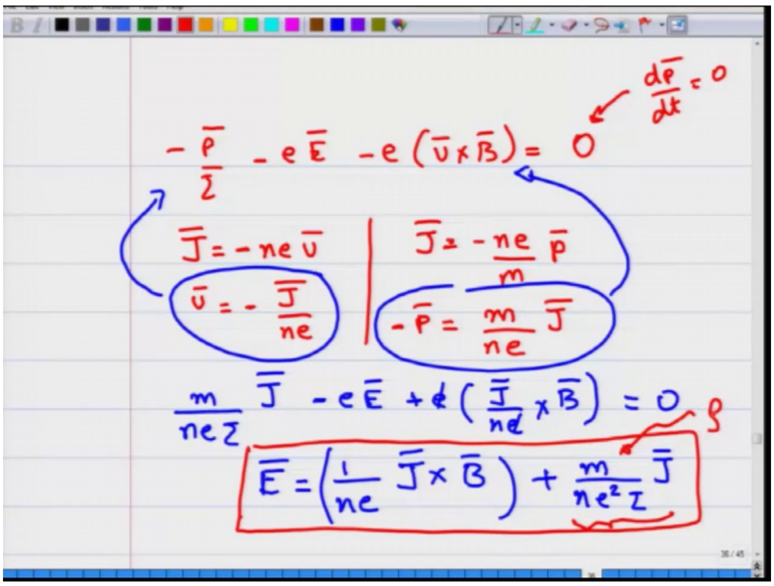
Now, let us calculate what is happening in this system. So, we start off again with the Drude's equation that the rate of change of momentum of the particle is minus P divided by tau plus f of t, where f is the force which is acting on the electron or the particle as a function of time. But here we are going to consider all static fields; electric field is static magnetic field is static, so they are not dependent on time. So, let us write down the force which is experienced by the electron; the force experienced by the electron will be minus e times the electric field.

And there is an additional term now; earlier when we had looked at only the electric field there was just the e which was present now we have a B. And so what is the other force which will come in? That will be the Lorentz force which is minus e v bar cross B bar; this is the total force which is experienced by the electron. And of course, we are looking at it between 2 successive collisions because as the second assumption of Drude states that, that is where the system will experience a force and it will respond to that force and then we will see a net effect on the system because of all these forces.

So, the force which is acting on the system is this and now we can put it back in the equation of Drude's model and you will get dp by dt which is minus P divided by tau minus e times the electric field; minus e v bar cross B bar ok. This is your full expression for how your momentum is going to evolve when you have applied a static DC electric field and a DC magnetic field.

And as before we are looking at static solutions not time varying solutions; we are looking at equilibrium phenomena that you have applied an electric and a magnetic field. And after a very long interval of time what is the current which is set up inside the system; it is a uniform current which is present it is not changing as a function of time. So, we are looking for solutions which are of the form dp by dt is equal to 0; where your external force f of t is being balanced by the drag force which results in uniform constant velocity or constant speed of the electron. So, let us put this now.

(Refer Slide Time: 18:17)



That minus P by tau minus e times the electric field minus e times v bar cross B bar is equal to 0 and this is coming because we are looking for solutions of the form dp by dt is equal to 0.

So, now let us work out this expression. If you remember that current density is minus n e into v. So, v I can write it again in terms of current density as minus J over n e and my velocity also can be written in terms of momentum. So, j is equal to minus n e over m times p. So, my momentum also can be minus of p can be written as m over n e into J.

So, we have 2 expressions here the velocity and the momentum can both be expressed in terms of J and you can substitute both of them into this. So, that we have now an expression in terms of J and the fields that are present. So, you will get first term will be m by n e tau into J minus e times the electric field minus e times.

Now, the velocity will be written as for the velocity; we replace this expression as J by n e cross B; this negative sign has taken care of this sign and this is equal to 0. And if you write down the expression; for now you can readjust the whole thing ok, you have a electric field and you have current density is on one side. So, let us reframe this expression and you just have to readjust it slightly; you will see that the electric field can be written as 1 over n e times J cross B; which is basically this term.

And there is another expression along with it which is m divided by n e square tau into J. I have just readjusted this equation so that I have taken this term on this side and then I have divided it by e; so that I get electric field. When you do that you get these 2 positive terms which is this and this and so you will end up with this expression; which is this.

Let us stop here and let us look at this expression in the next lecture and it will tell you a lot about the behavior of the system because now you have included the effect of the electric field as well as the magnetic field. Before we go further, I will just like to tell you that can you identify this term? This term is nothing else, but your resistivity rho which we discussed when we use that Drude's model to explain behavior of resistivity. So, with this thought we will go on to the next lecture.