Introduction to Solid State Physics
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Lecture – 05

Calculating the electrical conductivity of the metal using Drude's Model - Part II

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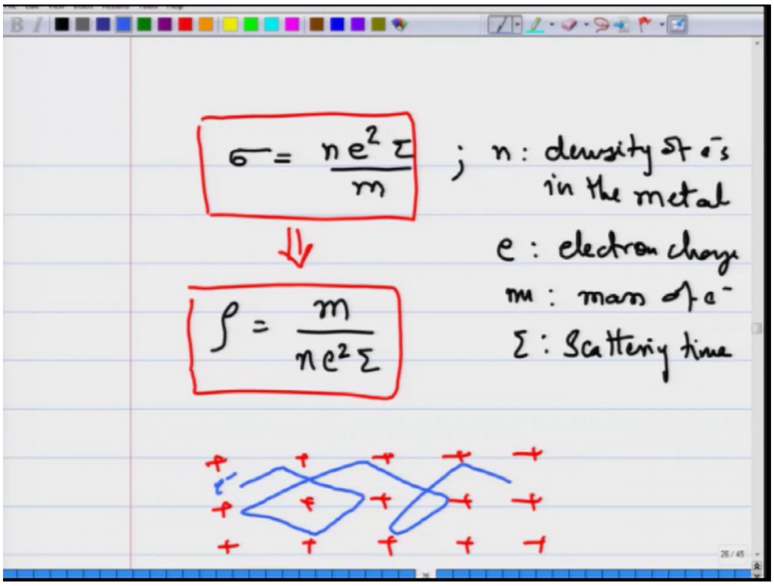

And this is the fundamental and one of the basic successors of Drude's model; that it gave us a way to think about how does resistance appear inside the metal. For the first time there was an understanding which started to develop as to how does resistance appear inside the metal? It occurs because of scattering inside the metal. Now let us go a little further and let us try and do some estimates; let us try and make some simple estimates with what we have calculated.

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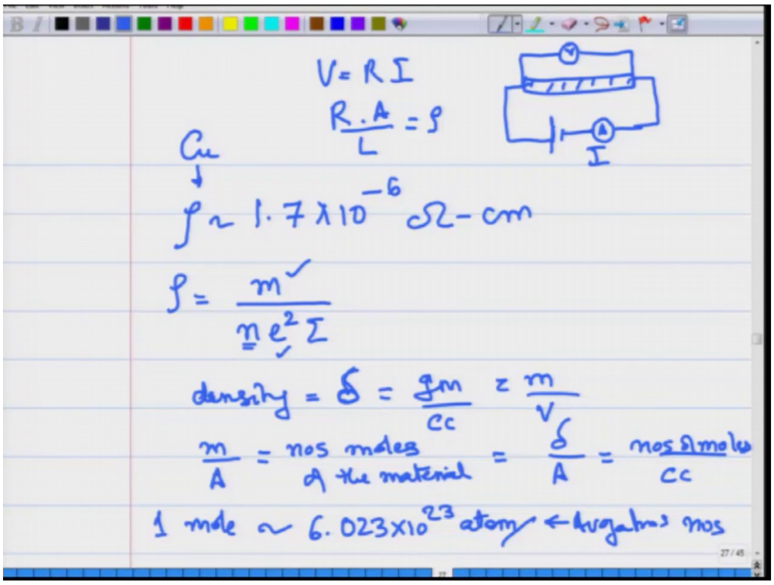

So, since we have this expression let us recall that for copper; you can again do the simple experiment which I had told you about that you take a piece of copper and what you can do is you have a piece of copper and then.

So, let us take this piece of copper and then you apply a voltage across it ; you measure the current that is flowing through the circuit and you measure the voltage which is appearing across this piece of copper; which is your basic Ohm's law. Then you can measure the resistance which is V is equal to R into I; you will get the resistance and from the resistance into A divided by L. A is the area of cross section of this wire and L is the length; you will get your resistivity rho. And in that way if you do the experiment very carefully you will get a fairly good estimate of your rho.

And so for copper the resistivity is at room temperature is known to be about 10 raised to minus 6 ohm centimetre. It is typically in this range 1.5 to 1.7 ohm centimetre for a very clean piece of copper. Now, if you recall that the resistivity depends on from the Drude's model n e square tau.

You know the mass of the electron, you know the charge of the electron what is this n? n is the density of the electron. How do you know that? People know and you can very well calculate the density, you know the density of the solid in gram per cc. So, let us say that the density of the solid is let us call it by some curly d; I do not want to use rho because rho I have already used it for resistivity.

So, instead of using the conventional notion of density I am just using some other slightly different notation for the density which is say gram per cc; which is mass per unit volume. And you know that if you take mass divided by the atomic mass of the substance; you will get the number of moles of the material. So, the density divided by A will give you number of moles per cc of the material. And 1 mole contains 6.023 into 10 raised to power of 23 atoms; this is your well known Avogadros number.

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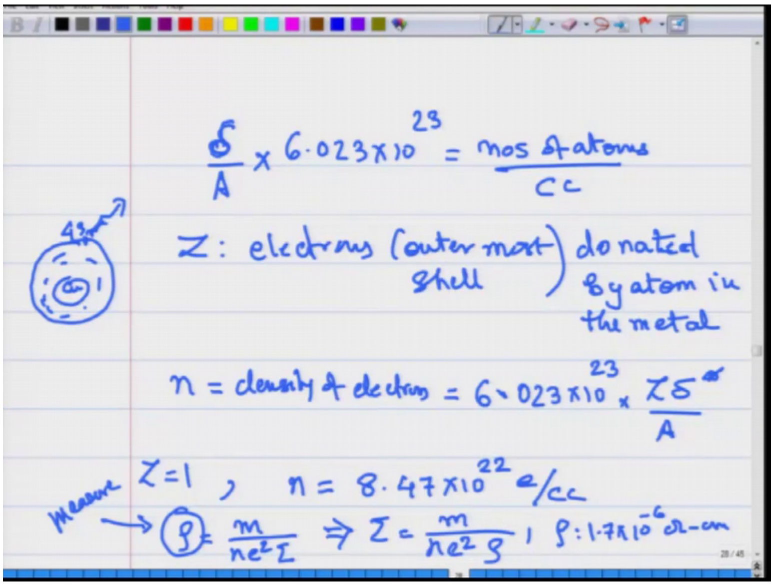

And hence if you the density divided by the atomic mass into 6.023 into 10 to the power of 23 is going to give you number of atoms per cc of the material. You easily get the number of atoms per cc of the material and now if you take copper for example, it has one outermost electron; which is in the Fourier shell this is the copper nucleus and then you have different shells. And if you go to the outermost electron it is in the 4 S orbit which has 1 electron which can be donated by copper.

And that is how copper becomes a metal by giving out 1 electron; you can get it to form a metal because that electron is delocalised, it moves across the entire lattice and each copper atom gives out 1 electron. So, if Z is the electrons of the outermost shell which is donated by the atom; which is donated by each atom in the metal, then n which is the density of electrons is the number of atoms per cc into Z because each atom in this is giving 1 electron a Z number of electrons. So, your expression for the density is 0.23 into 10 to the power of 23 into Z; density of the material divided by the atomic mass, where Z is the number of electrons donated by the number of electrons outermost electrons which are donated by each atom in the material.

Copper for example, donates 1 electrons; so Z is equal to 1. For copper and with that if you put it in this expression using the density of copper, you will get a density of copper n; a density of 8.47 into 10 to the power of 22 electrons per cc. So, this many electrons per cc is available in copper metal for conduction; this is the density. And now again let us go back to the Drude's model and look at the resistivity the resistivity is m divided by n e square tau.

So, rho is something you can measure; you can measure rho you know how to calculate the density m n e have already been determined experimentally. So, can we estimate tau? Tau gives us a value which is like m by n e square row. And now I had already told you the value of rho; for copper is 1.7 into 10 to the power of minus 6 ohm centimetre. You have per cc how many number, plug in the value of mass of the electron; you plug in the value of charge of the electron and you will get a value of tau.

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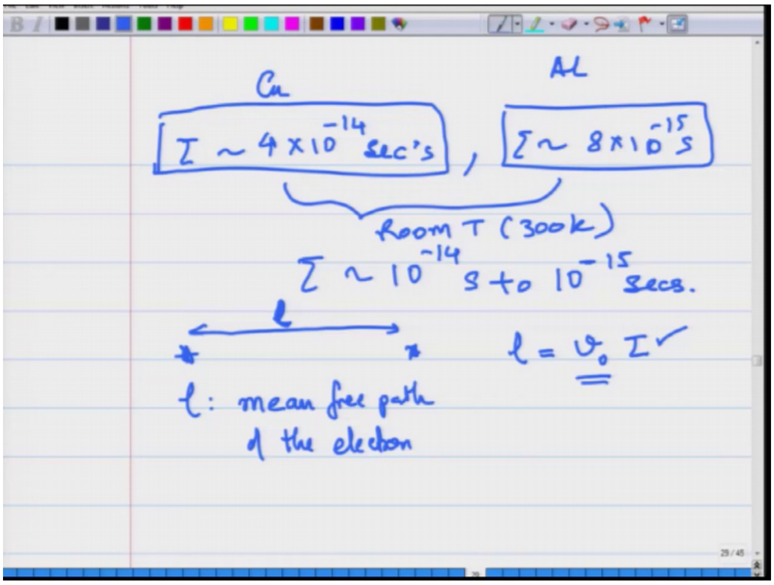

For copper which is like 4 into 10 to the power of minus 14 seconds. And this is what I was telling you is that the scattering time you can estimate now immediately that the scattering time of the electron that the time interval between 2 scattering events; which is experienced by the electron inside the solid is of the order of 10 raised to minus 14 seconds and this directly give you a way to calculate the scattering time ok.

Now you can actually check; you can go to aluminium and if you do it for aluminium this is for copper. If you do the aluminium calculations, you will get a tau for aluminium which is about 8 into 10 to the power of minus 15 seconds. So, that is why at room temperature of about 300 Kelvin; the scattering time typically is in the range of 10 raised to minus fourteen seconds to 10 to the power of minus 15 seconds; this is the typical scattering time that is found in metals at room temperature.

Now with this concept in place let us start exploring the outcomes of the Drude's model a little further. Since we already have some quantitative things let us try an estimate what is going to be the mean free path of the electron; which is what is the average distance which is travelled by the electron between 2 successive collisions? l is the mean free path; so, l is the mean free path of the electron.

The average distance which is travelled by the electron between 2 successive collisions is the mean free path. So, let us try an estimate the mean free path of the electron; very simply and you know from kinetic theory of gases that the mean free path were also you have calculated the mean free path. The mean free path is if v naught is the average velocity of the electron and tau is the time interval between 2 scattering events; the mean free path is nothing else, but the velocity of the electron in to the mean free path.

We know tau; it is already we have calculated can we estimate v naught. And here we again take use of the Maxwell's Boltzmann's kinetic theory of gases. So, what Drude did was again he you considered that the gas of electrons is exactly like; the kinetic theory of gases which has a Maxwell Boltzmann distribution. And from Maxwell Boltzmann distribution here also he considered that the electrons also have a Maxwell; Maxwellian Boltzmann like distribution which I had already told and given an introduction during the kinetic theory in the first part of my lecture.

So, here also the electrons are considered to have a similar sort of distribution. And you know that when you consider this Maxwellian distribution then what is the natural outcome; the natural outcome is the equipartition theorem which states that per degree of freedom per degree of freedom the particles have energy which is equal to half k B T.

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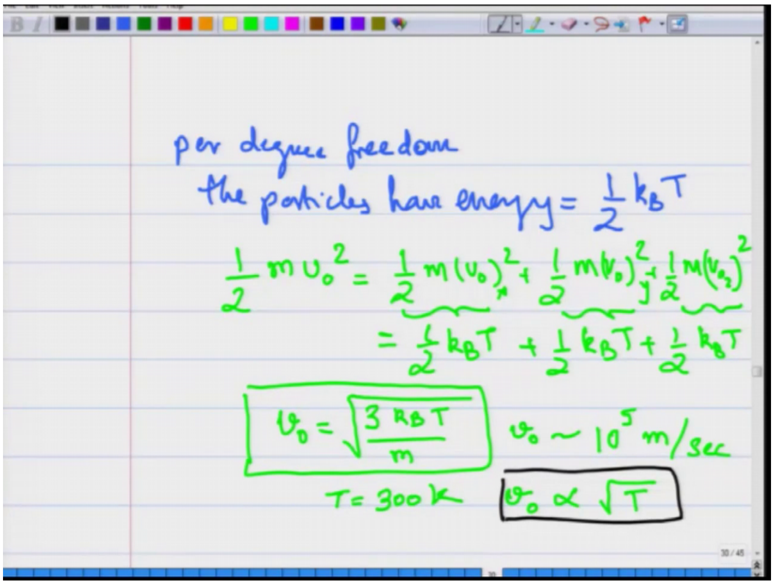

So, in this case the electrons which are also a gas of particles with Maxwellian like distribution. And we have said that they do not have any interaction Drude's model says that the particles do not have any interaction. So, then the average m v naught square; which is the kinetic energy the electrons only have kinetic energy and v is the average speed of the electrons inside the metal, m is the mass of the electron each electron will have a velocity v naught and m is the mass of the electron.

Then this can be written as half m v naught x the whole square plus half m v naught y; the whole square plus half m v naught Z the whole square. And each of this term is the energy associated with each individual degree of freedom because the electron can move along the x y or z direction. And each of this because of the Maxwell Boltzmann distribution can be replaced by half k B T. Just like the kinetic theory of gases for this electron gas also and then v naught is equal to square root of 3 k B T divided by m.

Where m is the mass of the electron k B is the Maxwell Boltzmann's constant T is the temperature. And this gives you an idea of; so T is the temperature at which the metal has been kept and this gives you a velocity which is if you put in the numbers at 300 Kelvin at a temperature of 300 Kelvin which is room temperature. You put in all the other constants you will get a velocity which is like 10 raised to 5 metres per second; this is the typical velocity with which the electrons are travelling in the metal.

So, now you have you can estimate your velocity of electrons and you also know the scattering time. Before we go further, the Drude's model which assumes the Maxwell Boltzmann's type distribution of energies inside the solid of the electrons the Maxwell Boltzmann type distribution of energies of electrons inside the solid shows that the velocity should be actually temperature dependent and it is proportional to square root of temperature.

So, this is one outcome of the Drude's theory which you should know that it tells that the velocity of the electron at the speed of the electron inside the solid is changes with temperature and the way it changes is by square root of the temperature. We will come back to that in a few minutes, but now let us to call the value of the velocity which is 10 raised to 5 metres per second.

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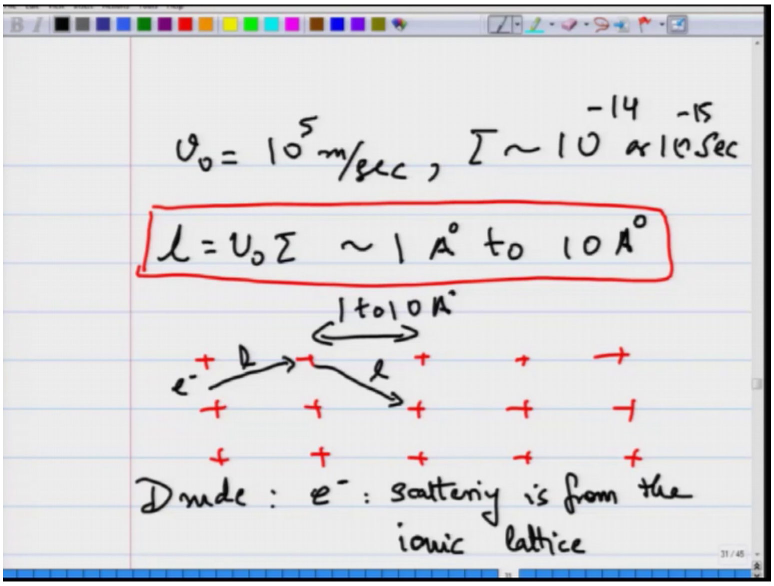

And so velocity is 10 raised to 5 metres per second and tau is like 10 raised to minus 14 or 10 raised to minus 15 seconds. Using this, the mean free path which is v naught into tau; if you just multiply the two, you will get it in the range of 1 angstrom to 10 angstroms.

The mean free path of the electron inside the solid which works out from Drude's model is in the range of 1 angstrom to 10 angstroms. Now what does this mean? This means that 1 angstrom is typically the spacing between the atoms; this is typically what you would expect that if you have the ionic cores inside the solid, they would be roughly spaced in this the spacing between these positive ion cores inside the solid would be in this range.

And you are getting the mean free path of the electron to be in this range; the mean free path of the electron is roughly in this range which is roughly the spacing between the ions inside the solid they are moving by a distance l and it is equal to the spacing inside the solid. So, Drude assumed that the scattering of electrons is from the ionic lattice. So, it looks as if as per Drude; the ionic lattice is going to cause this sort of a scattering, where the scattering length is equal to the period of the ions which are in the solid and this gives you a scattering length scale which is roughly the spacing. So, this is roughly the spacing of 1 to 10 angstroms. So, everything seemed quite right everything seemed very nice, but there are some limitations to this Drude's model and this is where the problem starts.

When you calculate the mean free path; the mean free path as per Drude model is given as 1 to 10 angstroms, but is it right? What do experiments say? And are the conclusions of Drude model really correct? So, we will just look at some of the limitations of the Drudes model. While it is very successful; it gave us the way to understand how does resistivity occur inside the material because of scattering of the electrons. There are some specific limitations which are found in this Drude model.

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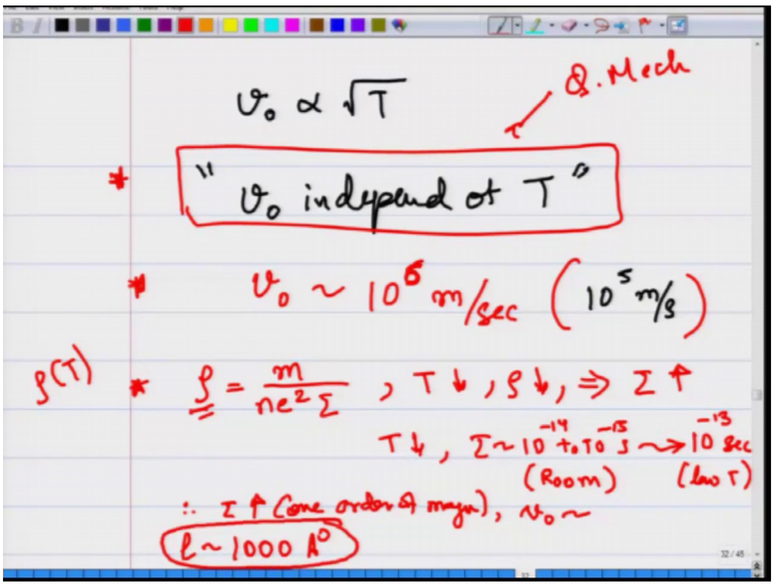

One of them is if you recall I said that the mean velocity of the electrons which was calculated in Drude model; turn out to be proportional to temperature square root of the temperature. So; that means, the velocity of the electron turns out to be dependent on temperature.

Now, this is as per if you consider the electrons classically; if you consider the electrons as classical billiard balls which are hard spheres, which are sort of randomly moving around and colliding with the lattice and then continuously moving around; then it is a valid assumption that these velocities these electrons are particles will have a Maxwellian like distribution. But we know that the electrons are actually quantum particles they are fermions and they follow a different statistics which is the fermi dirac statistics; they do not have a Maxwell type Maxiwellian type statistics, but they have a fermi dirac statistics.

And in subsequent lectures we will use that to improve our model of the behaviour of the electron inside the solid by introducing quantum mechanical concepts. And if you do that then the velocity actually turns out to be independent of temperature ; it is completely independent of temperature. Apart from that if you do the calculations using quantum mechanics; then we will show that the velocity turns out to be 10 raised to 6 metres per second.

And if you recall that using the Drude model of Maxiwellian statistics; we had 10 raised to 5 metres per second. So, it is at least 10 times larger there is a 10 times increase in v naught furthermore. So, these are some of the problems that you encounter which is not explain by Drude model; that the velocity of the electron, the mean average velocity of the electron inside the solid is much larger than 10 raised to 5; it is 10 times larger ; v naught is independent of temperature.

And if you actually measure the resistivity of the material as a function of temperature; you will find that the resistivity decreases. So, the resistivity which is equal to m over n e square tau as T is lowered, the resistivity decreases which implies the tau increases. So, with lowering of temperature; tau which is roughly 10 raised to minus 14 to 10 raised to minus 15 seconds at room temperature increases to around 10 raised to minus 13 seconds at low temperature. Because the resistance is changing as a function of temperature the resistance goes down which means tau increases.

So, it goes from 10 raised to minus 14 to 10 raised to minus 15 seconds to 10 raised to minus 13 seconds. And therefore, tau has increased by one order of magnitude compared to Drude's model calculation; as well as the velocity of the electron has increased by one order of magnitude. So, if you calculate the mean free path now; the mean free path will instead of 1 to 10 angstrom which was obtained in Drude's model, now can increase to almost 1000 angstrom; 100 to 1000 angstroms; there is a large increase in mean free path; you can get by.

Because the velocity of the electron once you include the quantum mechanical effects you will find that the mean velocity of the electron is independent of temperature; it increases to around 10 raised to 6 and scattering time also increases.

So, the mean free path is no longer about 1 angstroms, but it is few orders of magnitude larger. This is no longer the spacing between ions inside the metal; the spacing between ions inside the metal is about 1 to 10 angstroms, but the electron is moving over distance is much larger than that and it can scatter over distance is much larger than the spacing between the ions.

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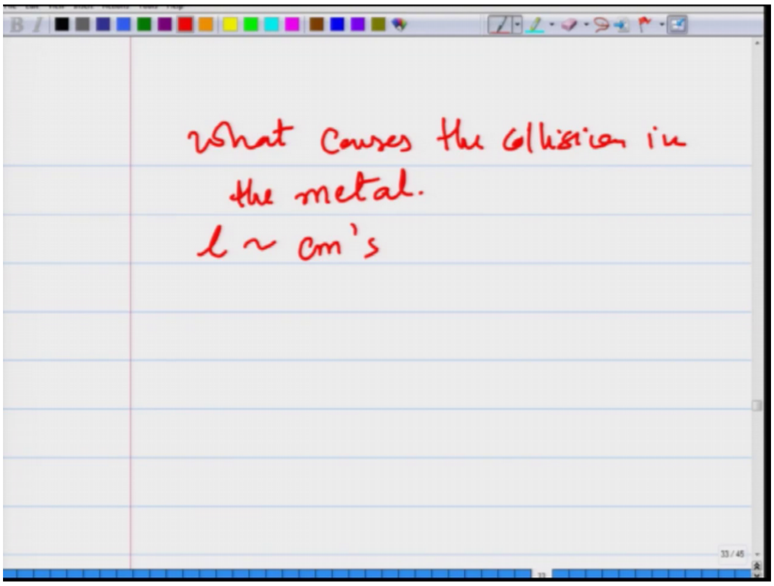

So, it is actually not clear what causes the collisions in the metal. In fact, if you make the metal pure; if you make it very clean you can reach sometimes mean free paths of the order of centimetres; people have reported mean free paths which are reaching up to few centimetres.

So, clearly there are some inadequacies in the Drude's model first of all; the velocity is independent of temperature which is a known fact and of course, we will also show that. It is not dependent on temperature as square root of T; velocities are mean velocities of electrons are much larger and mean free paths are much larger than spacing. So, one of the things which are there in Drude model is not clear what actually leads to the scattering of electrons.

And this is one of the features which actually holds back the Drude's model, but to a large extent to gain an understanding of the behaviour of conductivity inside the metal and how does it vary, what are the parameters which control it; you have a very good model which is even use now is an approximation to get an idea of what is going on inside the metal as a starting point the Drude model by itself works quite well.