Introduction to Solid State Physics
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Lecture – 45
Waves in a crystal considering interaction among atoms beyond their nearest neighbours

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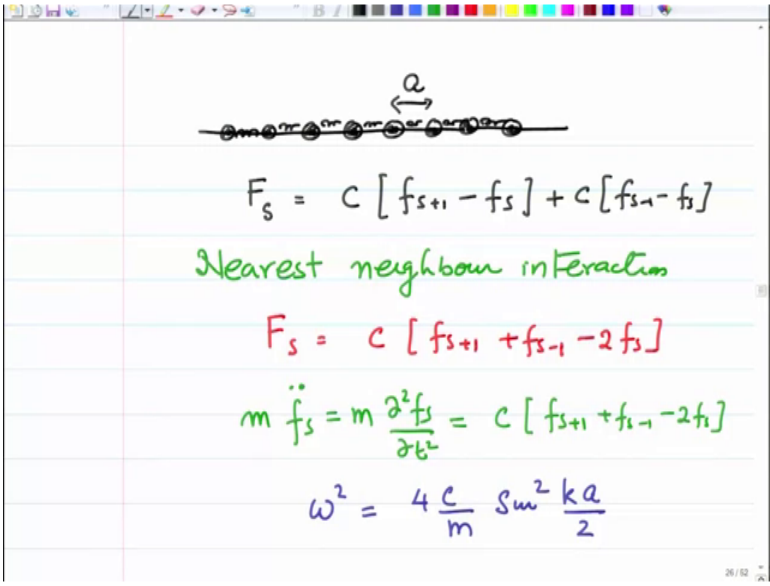

So, far what we have considered is chain of atoms or a one dimensional crystal with the atoms interacting through a force, which is proportional to the stretch of the assumed spring between them. So, we assume that the force on the atom at F site is proportional to c f s plus 1 minus fs plus c fs minus 1 minus fs; that is the only thing that we assumed was the nearest neighbor interaction and we derived from this, that the force Fs is c fs plus 1 plus fs minus 1 minus 2 fs.

And therefore, the equation of motion that we obtained was m fs double dot, where double dot is the second derivative of the displacement with respect to time and this was equal to c fs plus 1 plus fs minus 1 minus 2 fs. And assuming a wave like solution we obtained omega square is equal to 4 c over m sin square k, a by 2 where a is the distance between the atoms I am showing in the top this is. What we have done so, far and then we of course, defined the Brillouin zone and showed you how k is defined only within the Brillouin zone we want to address certain general aspects now.

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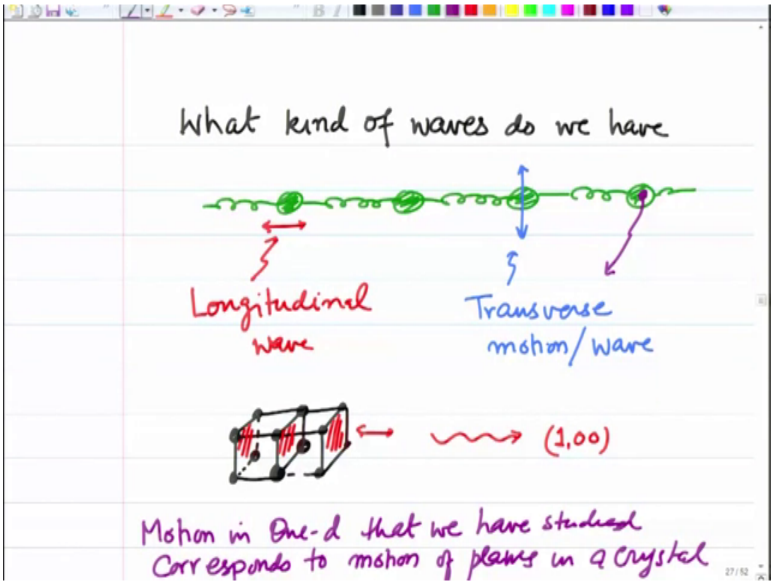

So, what kind of waves do we have? That is the first question I am going to answer. So, again looking back at this chain of atoms, I can have these atoms displaced in the direction of wave propagation and this will be a longitudinal wave. I can also have the atoms oscillating perpendicular to the direction of the chain, and this would be transverse motion or transverse wave. Transverse direction it can be in 2 directions; one is what I have shown in the plane of the screen the other displacement could be perpendicular to the screen. So, this is also transverse.

So, I could have transverse or longitudinal waves. You may also be wondering so, far as to how this one dimensional chain is related to what happens in a crystal. So, in a crystal what I am going to have? Suppose for simplicity I have a cubic crystal that is the atoms are sitting at these corners. What this one dimensional motion represents is the displacement of these planes. So, I could have for example, this plane oscillating like this and the wave propagating like this. This would be a longitudinal wave in the direction of 1 0 0. Similarly I could have waves propagating along the diagonal I could have wave propagating along the body diagonal or phase diagonal.

So, different planes which you have studied earlier can oscillate and propagate a wave and this one dimensional motion that we have studied so far or the motion in 1 dimensional crystal that we have studied so far would be actually the motion of these planes and that is precisely what happens in a crystal. So, let me write this the motion in one d that we have studied corresponds to motion of planes in a crystal. So, this is how we connect. So, what we have done is not a futile exercise is actually related to the motion of planes in a crystal and later, I will show you towards the end of this week's lectures that this is precisely what we study experimentally. So, that is one point I wanted to make.

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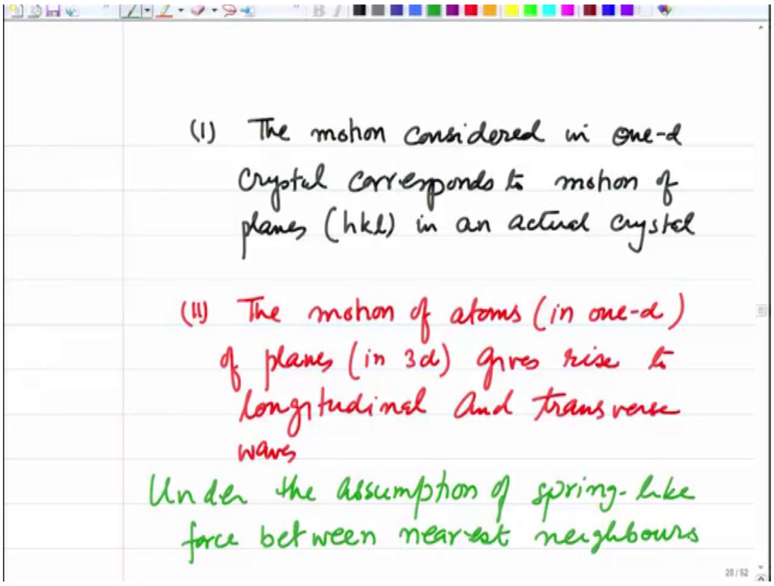

So, 2 points I have made is one that the motion considered in one d crystal corresponds to motion of planes, which are given by hkl you have studied that in an actual crystal. And point number 2 is that the motion of atoms in one d and of planes in 3 d gives rise to longitudinal and transverse waves. In a longitudinal wave the motion of planes is along the direction of the wave and in a transverse wave is perpendicular to the direction of the wave. And all this that we have studied is under the assumption of spring like force between nearest neighbors that is the simplification we take.

So, I want to take 1 step beyond it and see what happens, if all the planes or all the atoms were interacting how this dispersion relation be modified that is now the exercise I am going to take up. So, now, what we are going to see is let me make these planes right.

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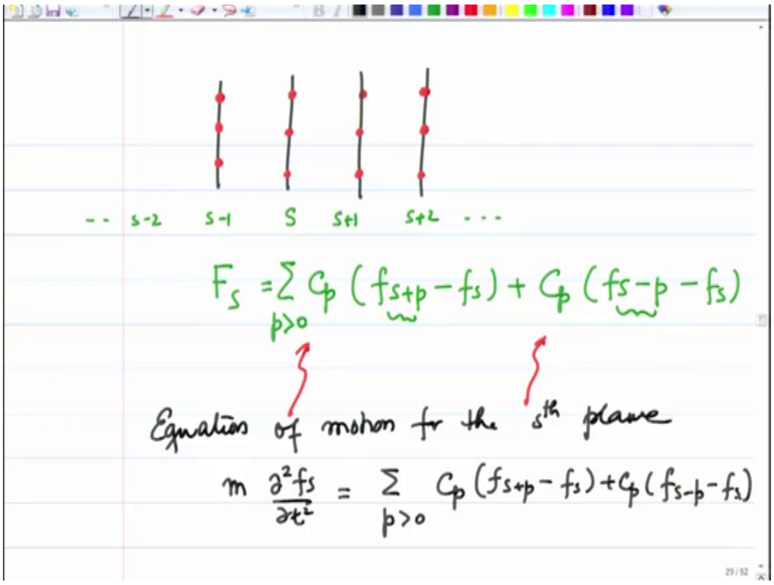

This is a plane of atoms let me make these atoms in a crystal is to make the point that these are real calculations that we are doing and so on. And what I am going to assume now is that, suppose I have the sf plane is going to interact with s plus 1 s plane s plus 2 plane and so on.

And similarly s minus 1 plane s minus 2 plane and so on and the force on the ss plane is going to Fs equals C p due to the pf plane minus fs plus C p fs minus p minus fs. This is force due to the plane on the negative side plane on the positive side I have assumed that for the force by the ps plane, the coefficient of force is C p it changes with each plane and then I have to sum over p.

Since I have taken p on the negative side also so, I will sum over p greater than 0 only. So, you can check all the direction of the force and everything is correct. Now the equation of motion for the sth plane is going to be m d 2 f s by dt square is equal to summation p greater than 0 C p fs plus p minus fs plus C p f s minus p minus fs.

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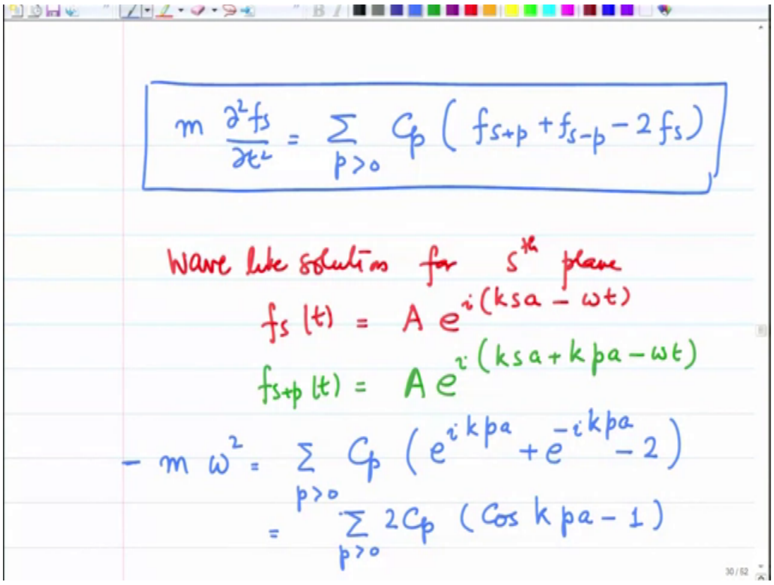

So, the equation of motion that I have is m d 2 fs over dt square is equal to summation over p greater than 0 C p and now I can take all the terms together I can write this f s plus p plus f s minus p minus 2 fs this is the equation of motion. Again I am going to assume a solution which is wave like. So, wave like solution for sf plane is going to be fs as a function of t is equal to some amplitude A e raised to i k s a minus omega t.

And therefore, for s plus p s plane is going to be A e raised to i k s a plus k p a minus omega t. And when I substitute this in the equation above what I am going to get is m omega square is equal to summation p greater than 0 C p e raised to i k pa plus e raised to minus i k pa minus 2 and this leads to summation over p greater than 0 2 C p cosine of k pa minus 1 there is a minus sign in front.

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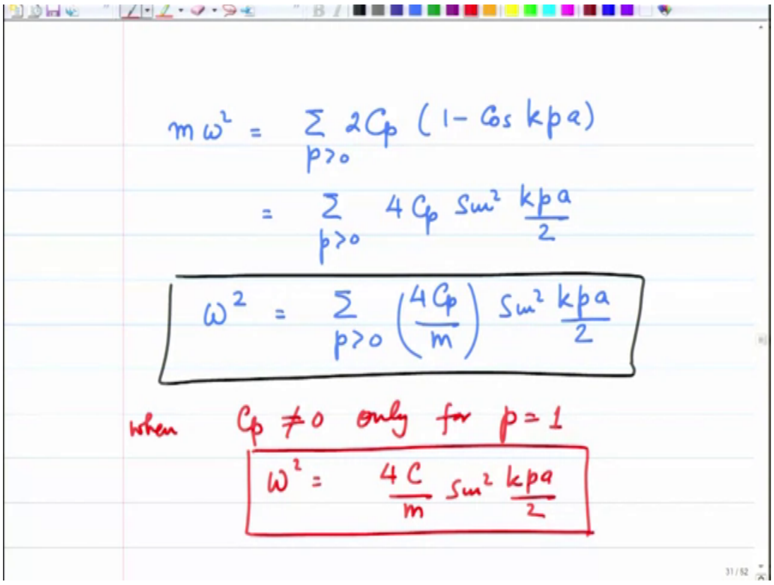

And therefore, the equation of motion gives me m omega square I take the minus sign to the other side summation p greater than 0 to C p 1 minus cosine k p a, which is nothing, but summation p greater than 0 4 C p sin square kp a by 2. And therefore, in this case omega square becomes summation p greater than 0 4 C p over m sin square kp a by 2. This is the relationship that we get if we assume still spring like force between different planes but all the planes included.

In a special case when C p is not equal to 0 only for p equals 1, you get omega square equals four c over m sin square kp a by 2, we get the previous relationship that we had obtained assuming only nearest interaction. So, we can do things in general and write the frequency of the travelling waves or the disturbances when other planes are also interacting, notice that I am still assuming a spring like interaction between the planes. So, this is a generalization of what we did earlier.

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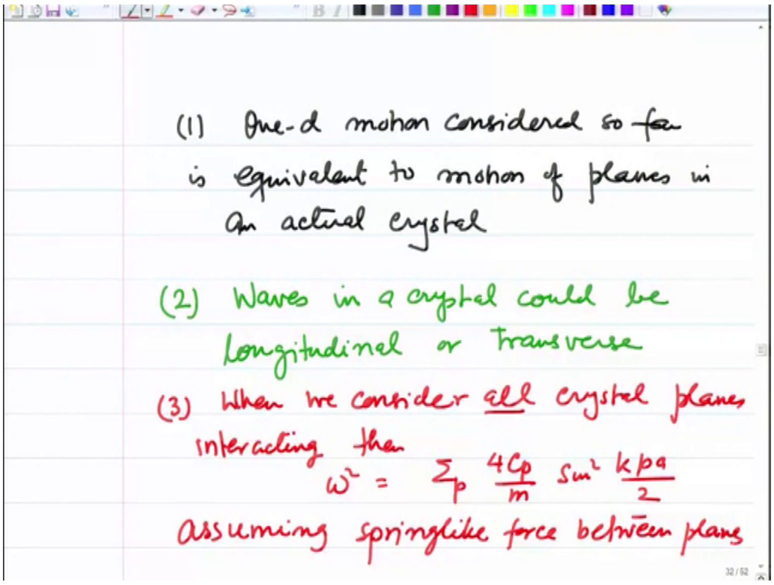

So, I end this lecture by noting again three things one that one d motion considered so, far is equivalent to motion of planes in an actual crystal. Point number 2 that waves in a crystal could be longitudinal or transverse and point number 3 when we consider all crystal planes interacting then omega square equals summation p 4 C p by m sin square k p a by 2 assuming spring like force between planes.