Introduction to Solid State Physics
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Lecture – 44
Group velocity of waves and speed of sound in a crystal

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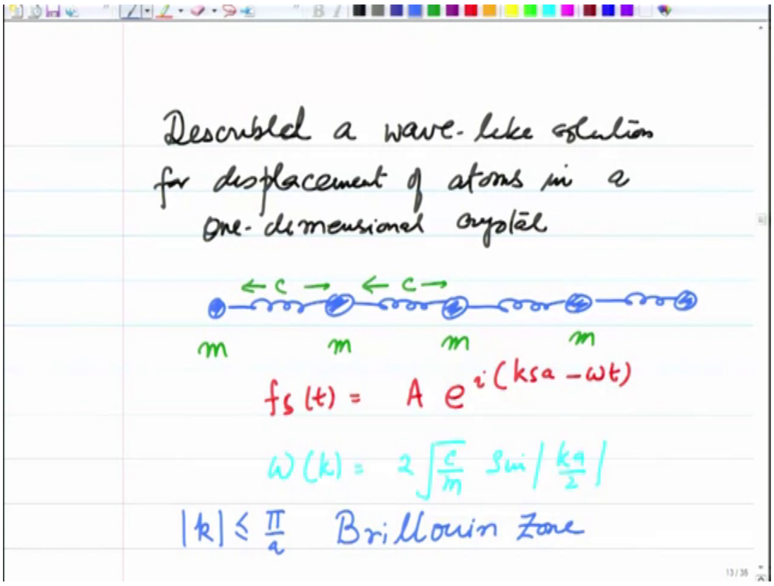

In the previous lecture, I described a wave like solution for displacement of atoms in a one dimensional crystal, where I had these atoms identical atoms each of mass m interacting only with nearest neighbors through a spring like force, with a constant of force being given by c and we assumed the wave like solution which was f at st site as a function of t being A e raised to I ks a minus omega t.

Remember in the previous lecture at the end I had told you that this is a notation one follows the exponential form because it makes life easy in terms of mathematics finally, I understand that I am going to take only the real part or the imaginary part of this. Let us stick with the real part and the frequency omega is given as a function of k as 2 square root of c over m sin of mod k a by 2. And then we learned that the only relevant k is actually mod of it is less than pi over a which is known as the Brillouin zone. And all quantities which are k dependent in a crystal actually can be described by k between minus pi by a and a and which is known as the Brillouin zone. So, everything and after that it becomes periodic example we saw is that omega is periodic with respect to k.

If I plot the solution; so in this lecture what we are going to do is explore the property of this solution.

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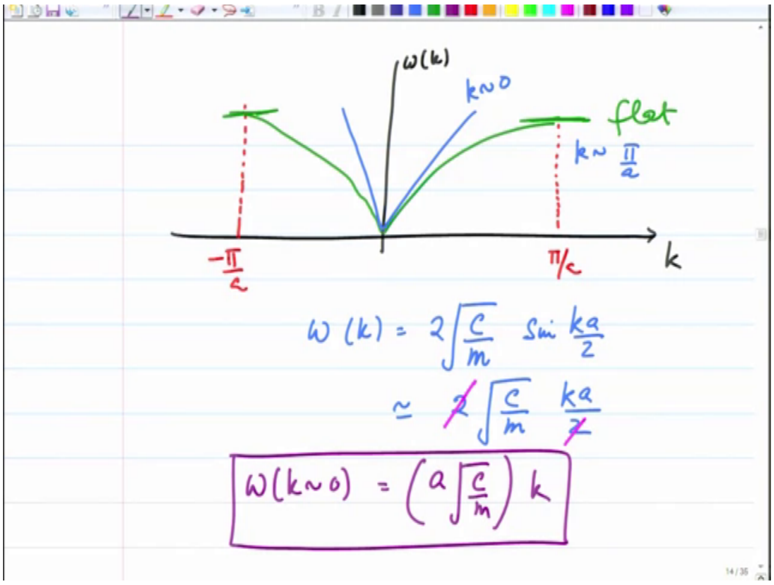

So, if I plot the solution omega k versus k, and I am going to now restrict myself to this Brillouin zone; so, I am going to plot the solution between minus and a on the 2 sides let us notice some features it becomes flat at the Brillouin zone and it is linear near k equals 0, this is a solution for near k equals 0 this is k around pi by a; it becomes flat the omega curve because omega is maximum at that point.

So, let us see how does it look near k equals 0. Near k equals 0 omega k, is I will consider only positive omegas now 2 root c over m sin of k a by 2 which can be written as 2 square root of c over m k a by 2, I will cancel these 2 and you get omega k near 0 as a root c over m times k as we said earlier it is linear and; obviously, near the Brillouin zone is flat what about the velocity? Is better to call it speed here of the wave.

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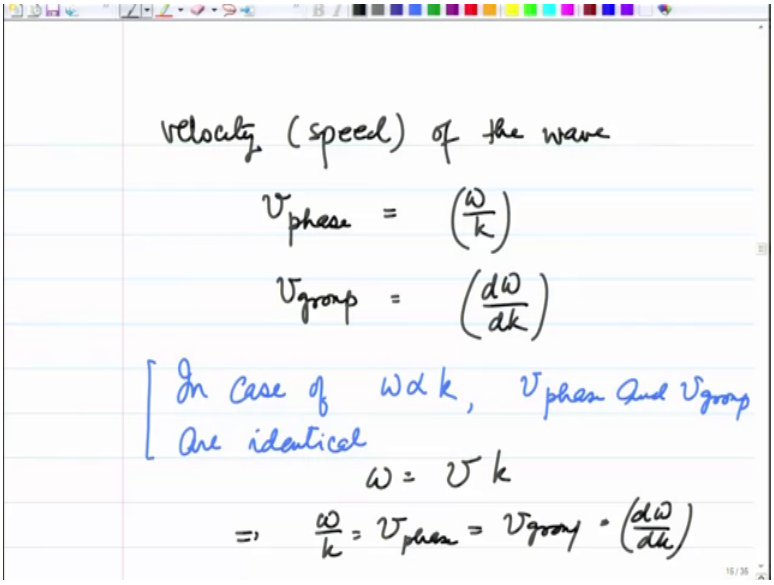

Well the speed is the 2 kinds of speeds v phase which is equal to omega over k, but it is a meaningless quantity in this context, because the information actually travels with something called the group velocity, which is d omega over dk. In case of omega being proportional to k, v phase and v group are identical.

So, this is known as linear dispersion or dispersionless medium, in which case there is no dispersion when I speak to you the sound is the same the speed of sound is the same irrespective to the frequency and therefore, you do not really hear any distorted sound. So, the feature when omega is proportional to k is that. So, suppose let us say omega is equal to some constant right let us call it v k, then omega over k is v phase and this is equal to v group which is d omega dk. So, phase velocity group velocity everything is the same for each frequency.

Whereas a frequency as dispersion other than linear then they change right. Now this velocity being the same for each frequency means you can hear no matter what distance you are standing at the same sound right it does not come with a delay for different sounds that is what it means.

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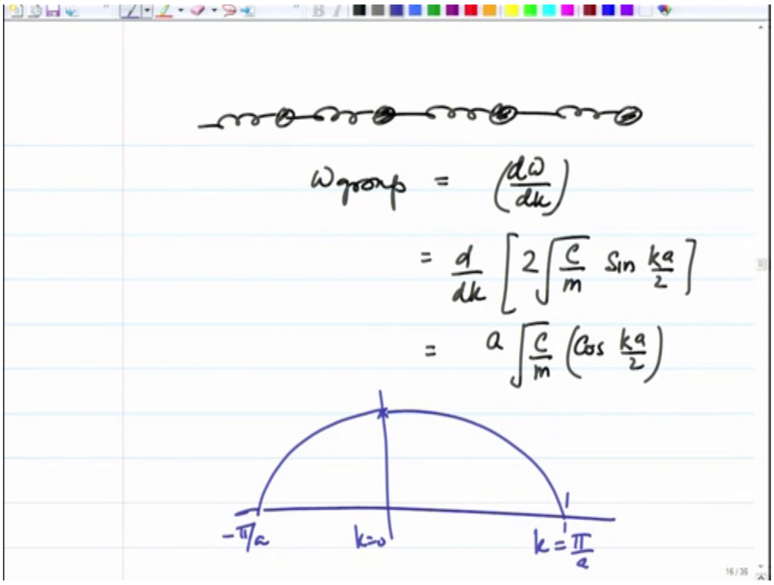

Now what does it mean for our case you know is when I am taking this lattice, then omega group is equal to which is d by dk of 2 root c over m sin of k a by 2.

So, this becomes equal to a root c over m cosine of ka by 2. So, this is a group velocity and if I plot it is maximum at k equal to 0 and then goes down like a cos curve and becomes 0 at pi by a and same thing on the other side it becomes 0 at minus pi by a. So, information and any lattice would travel with group velocity.

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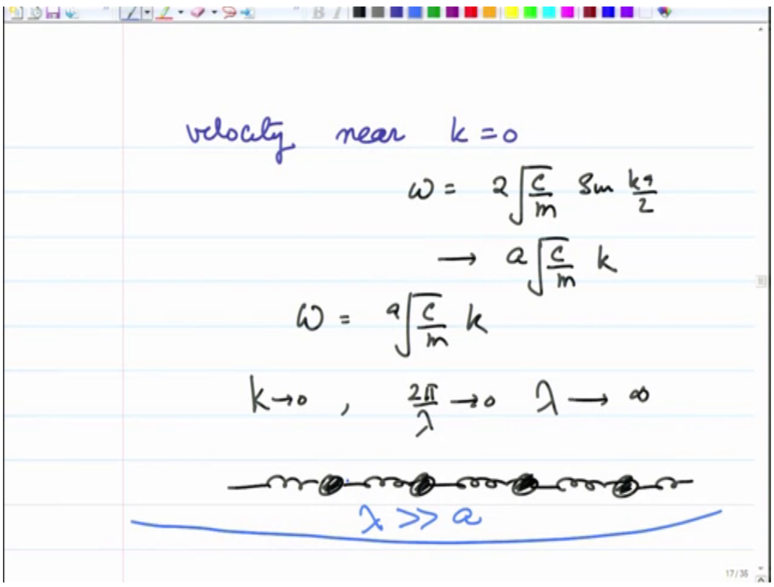

Let us now understand the velocity near k equals 0; near k equals 0 we saw that omega is equal to 2 root c over m sin k a by 2 and it goes to a root c over m k. So, omega is equal to a root c over m k, k equals 0 means if k tends to 0 this means 2 pi over lambda tends to 0 or lambda tends to infinity. What we are talking about is the waves with very large wavelength. pictorially what it means is when I have this lattice the wavelength we are talking about is really huge it is much larger than a that is what k equals 0 limit means and when lambda is much larger than a; it sees this whole medium almost as a continuous medium.

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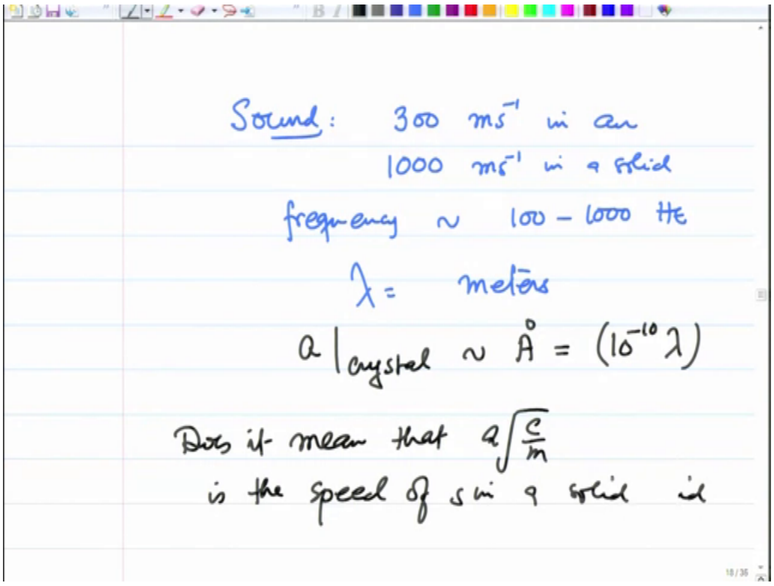

Now, let us take some numbers let us worry about sound. Sound roughly the speed is 300 meters per second in air, let us say of the order of 1000 meters per second in a solid it is much larger, but let us me take 1000 meters per second and frequency of our sound is almost of the order of 100 to 1000 Hertz. So, lambda of sound is of the order of meters right and a in a crystal is of the order of angstrom which is then equal to 10 raised to minus 10 lambda; so for sound the medium is almost continuous.

So, does it mean that a root c by m is the speed of sound in a solid from what we have done so far it does appear like that. Because what we have seen is for k nearly equal to 0 right pi by a is way ahead the dispersion is linear and the slope is v. So, it indeed describes this v or d omega by d k or omega by k for k tending to 0 is indeed the speed of sound in a solid medium.

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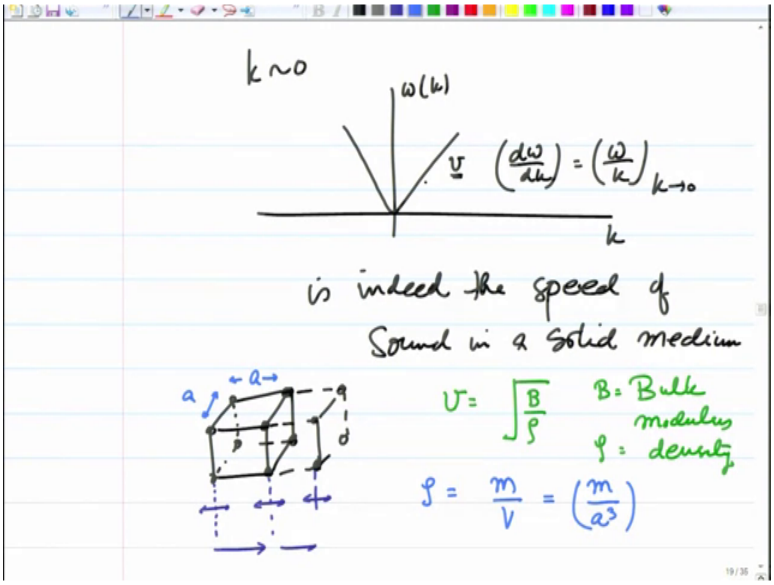

Let me convince you of that by a different argument; let me take a solid which has cubic structure.

So, it is made up of these cubes with the atoms sitting right here and then this gets repeated. There the one dimensional wave that I am talking about can be thought of as these planes in a cubic lattice moving. So, this could be for example, this plane out here, this plane out here, this plane out here, the wave is moving in this direction and these planes are getting displaced like this.

Conventionally what you have learnt in your previous classes, the speed of sound is given by a square root of B over rho where B is the bulk modulus and rho is the density. Let us calculate this in this cubic case. If I do that rho would be equal to mass divided by the volume; if I take the side of this cube to be a, then the density of the solid would be equal to each atom is divided into 8 different cubes.

So, mass for each cube is going to be m divided by a cubed, this is the density let us also calculate the bulk modulus.

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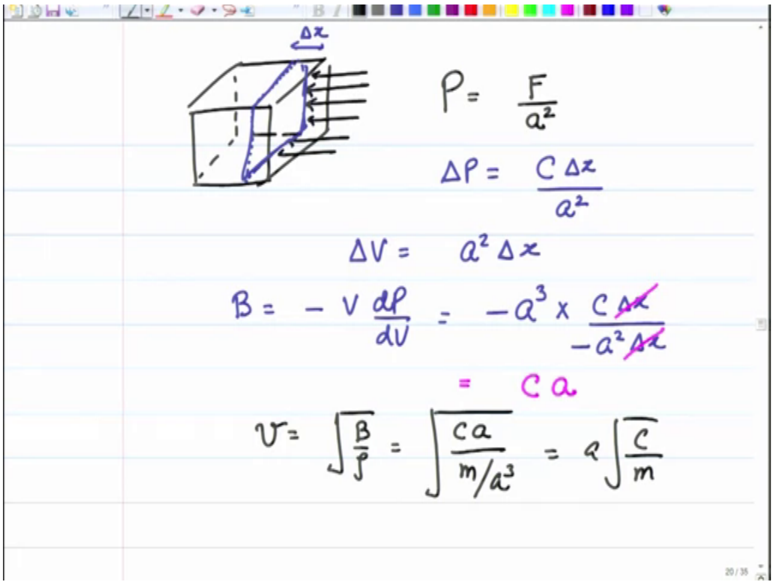

If I take this cube and compress it on one side by force F, then the pressure would be equal to the applied force divided by a square. Now the force when it is applied displaces this plane by distance delta x, and according to what we have done this force should therefore, be equal to C times delta x and the pressure becomes this in a way is that extra pressure applied so, I am going to call it delta P.

How about delta v? The volume change is going to be a square times delta x and therefore, the bulk modulus which is minus V dP over dV is going to be equal to minus a cubed minus takes care of whether the volume is decreasing or increasing, dP is delta p c delta x divided by volume is decreasing minus a square delta x and this comes out to be this delta x cancels comes out to be c times a.

And therefore, the speed of sound v which is a square root of B over rho is going to be equal to square root of ca over m over a cubed which comes out to be a square root of c over n. Which is the same that we got from the dispersion relation between omega and k calculate it from that wave equation. So, what I have shown you indeed that corresponds to the speed of sound.

So, low k or large lambda modes in a solid describe the propagation of sound in that medium.

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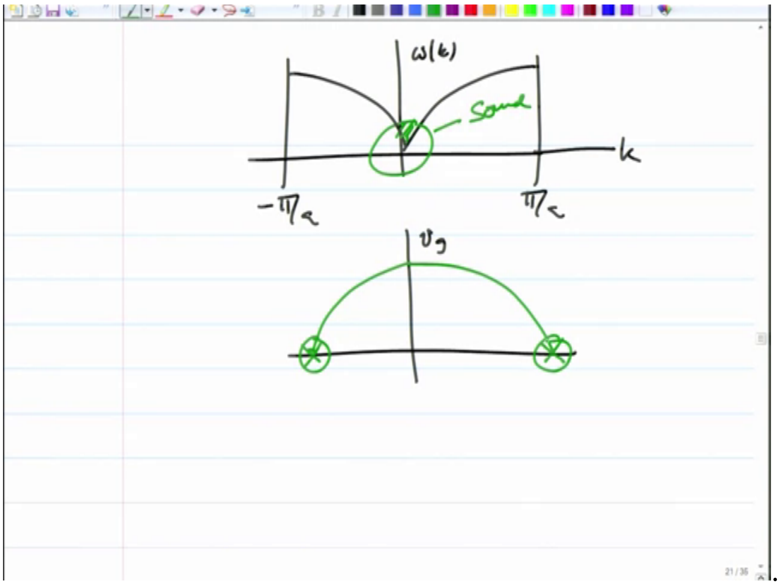

And given this omega versus k behavior minus pi by a pi by a this is k this is omega k, and the group velocity vg which goes down at these points right what we have learned is at these point these modes describe the sound. And this speed of sound is given by the slope of omega versus k curve at k nearly equal to 0. And at case at near the Brillouin zone the information does not really propagate because the group velocity becomes 0.

So, these are the takeaways from this omega versus k behavior and there will be many more things that we will be exploring, but for the time being what I have shown you is that for large lambda this describes the speed of sound this describes the propagation of sound. From now what we have derived from is a microscopic point of view and for k near Brillouin zone does this the lattice does not really carry in from any information from one place to the other.

Thank you.