Introduction to Solid State Physics

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Lecture – 41
Wave equation in a continuous medium and generalization to a discrete medium

So far, what you have learnt in the lectures by Professor Satyajit Banerjee is that a solid is crystal line.

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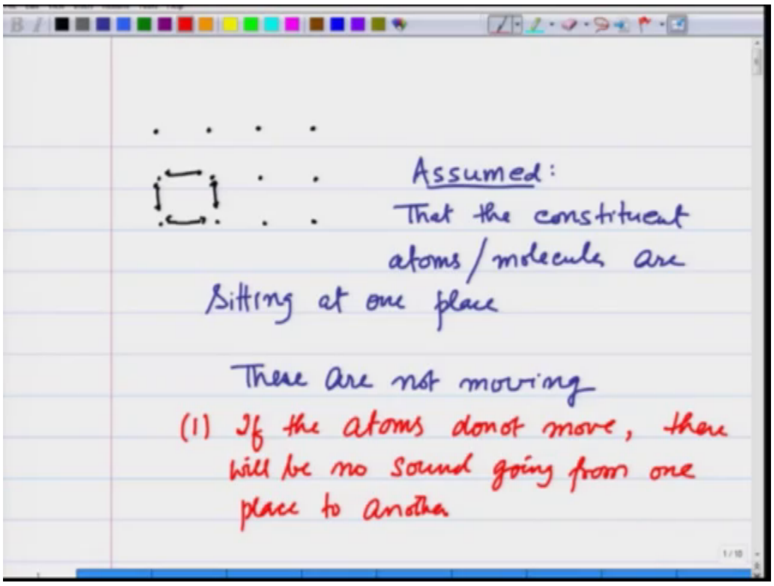

In that it has atoms arranged in a very regular fashion I am making a simple one out here and they are bound together by some force. Now, you have done x-ray diffraction in crystals and other things in all this it was assumed that the constituent atoms those are atoms or molecules, let me write molecules are sitting at one place. What is that mean? that means, these are not moving, right. So, these are fixed at one place.

What we are going to start from today's lecture onwards is what happens if they move? How do I know that these atoms and molecules move? So, the evidence is that 1, if the atoms I am writing atoms, but let us say this is basis or you know molecules or whatever that unit is sitting at each lattice points; if the atoms do not move, there will be no sound going from one place to another ok.

Let me explain that thought; you see when I am talking to you the sound that is going to the camera or the sound that is going to the speaker or the to the mic at the camera is transmitted through the movement of molecules of air between the speaker and me and this movement is important for transportation of energy. Similarly I know if I take an iron bar hit it on one side you hear the sound on the other side, if the atoms were static this would not be transported. So, any transport of any energy or anything requires movement of these atoms.

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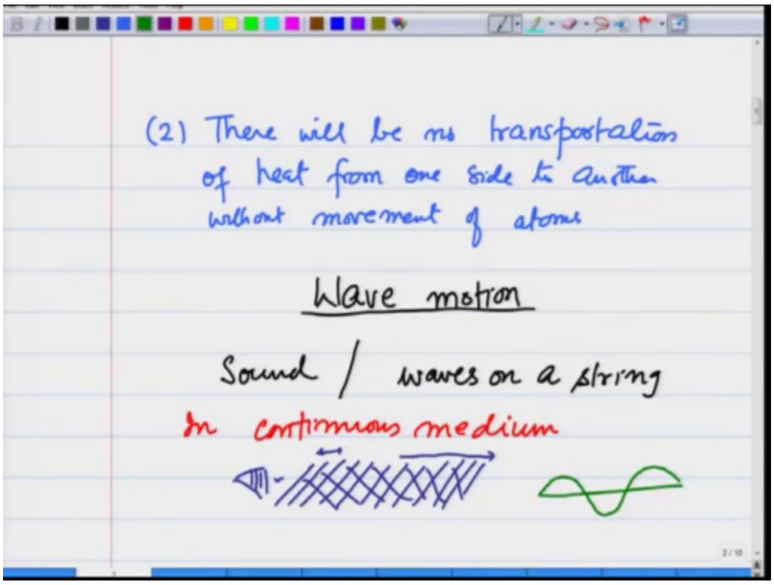

Let me give little more. So, on the same line there will be no transportation of heat from one side to another without movement of atoms because heat essentially is movement of atoms molecules. This room is hot or the room you are sitting in has certain temperature because the atoms molecules which are moving have certain energy k t 3 by 2 k t or whatever, right.

So, you need this movement for transportation of heat and later we you see that electrons in a solid also carry heat from one side to the other. You also you know that they carry current from one side to the other, so, all this requirement requires movement. So, we need to understand the movement of these atoms constituent atoms, if we have to understand the phenomena of transportation of any kind of energy from one side to the other and that is what we are going to start studying from today onwards in this week and next week's lectures.

To motivate how these atoms move let me recall for you what you may have already studied in your previous courses and that is the wave motion. The wave motion that you have studied in the past is that of sound you may have done experiments in your twelfth grade on a string and so on.

All these motions of waves are in continuous medium; let me explain what I mean by that. When you consider sound in that case there is some sort of air or water or wherever you are speaking, right and it is when you speak from here. So, let me make a speaker it disturbs this entire mass of whatever fluid or gas is there and that transports the energy.

In this case we neglect any distance between molecules constituent molecules or atoms and we consider this to be a continuous medium. Similarly, when I talk about wave on a string right you have see in this waves being like made like this you have also done you know problems with string harmonics and things like those, again when we talk about waves on this we consider this string to be a continuous medium; there is no discreteness.

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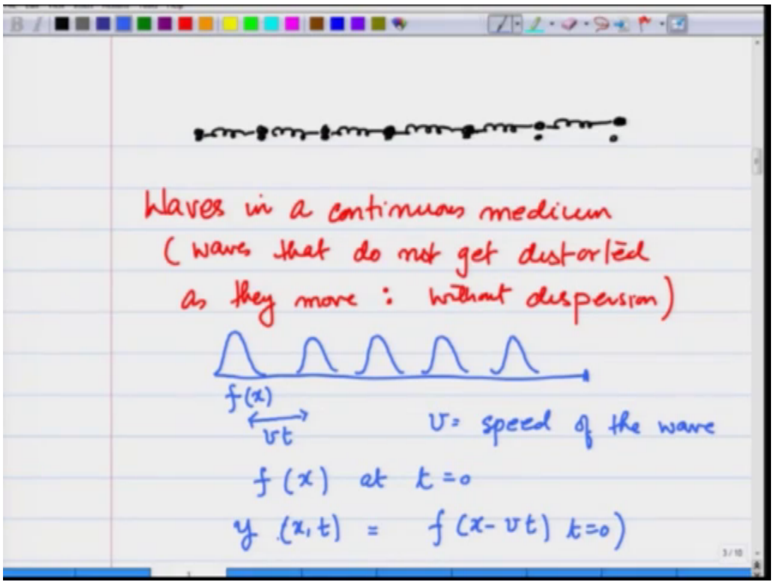

And this is different from movement of atoms in a crystal which are let us say connected through a spring I am just making it symbolically later we will see how exactly we deal with this.

Here the constituent atoms are separated by certain distance and I cannot take it to be continuous unless and we will see later the wavelength happens to be very large. So, it requires slightly different treatment; however, I am going to motivate that through discussion of waves right now that you know of that is, waves on a string or waves in continuous medium.

So, when we talk about waves in a continuous medium and I am going to talk about waves that do not get distorted as they move what is known as technically without dispersion. So, these waves move without distortion. So, how do we represent these waves. If let me start with a string if I have a string and I give a pulse here, right you can do this experiment at home no take a string and just give it a pulse and this pulse travels as time progresses without any distortion, that is what I meant that it is moving without dispersion.

Then the shape f suppose this is given as f x this remains unchanged that it moves, alright. As time progresses, however, it covers a distance. How much distance does it cover? In time t, it covers a distance v t, where v is the speed of the wave. So, the function f x which was at t equal to 0, it has shifted it has shifted by v t. So, the displacement let me call it y as a function of x and t will be given as f x minus v t because the displacement at x and t would be what the displacement was at x minus v t at time t equal to 0.

So, let me also write here to be explicit time t equal to 0. So, a travelling wave without dispersion is necessarily a function of x minus v t. It will be written in a slightly different form again.

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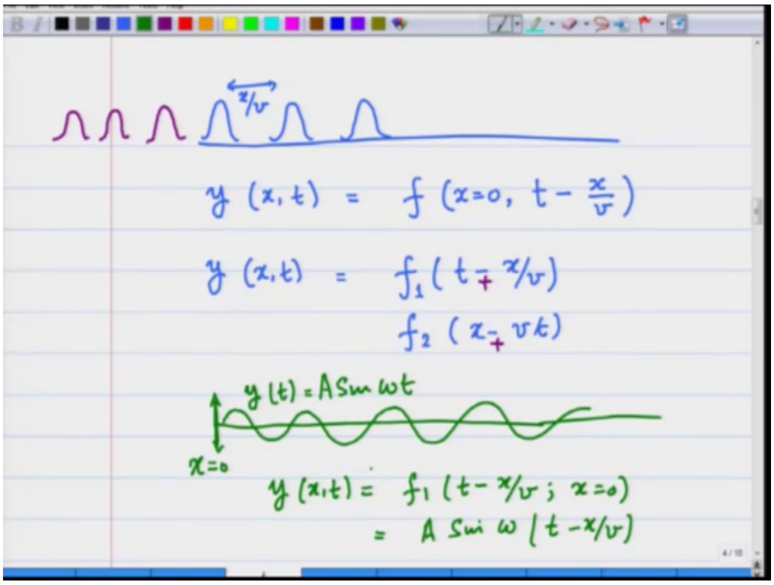

So, this is travelling undistorted on a string I can also say that y at x and t is the displacement which was at x equal to 0 at time t minus x over v because it took this much time to travel from one point to the other. So, in general for a wave displacement can be written as f x t equals a function of t minus x over v or some other function, right. So, let me write it f 1 and f 2 x minus v t. I took the wave to be travelling to the right; if it is travelling to the left no problem I will just change the sign.

So, with this sign I will be describing a wave which is travelling to the left as I am showing on the upper part of the screen pulse is now travelling to the left, this is a form. Let me see now what happens if I take a string and the start shaking this point let us say this is at x equal to 0 and this string is infinitely long and I start shaking it up and down with a displacement y as a function of time t equal to sin of omega t with an amplitude A alright.

So, I am taking the string and shaking it up and down at x equal to 0. Then I know this disturbance will be travelling further down. So, I will start seeing a wave like this right you will start seeing that shape and the displacement y x, t will be given as f 1 t minus x over v at x equal to 0. At time t the displacement at point x would be what it was at time t minus x over v at x equal to 0. So, this will become A sin of omega t minus x over v; this is a displacement. Let me write it more clearly in the next slide.

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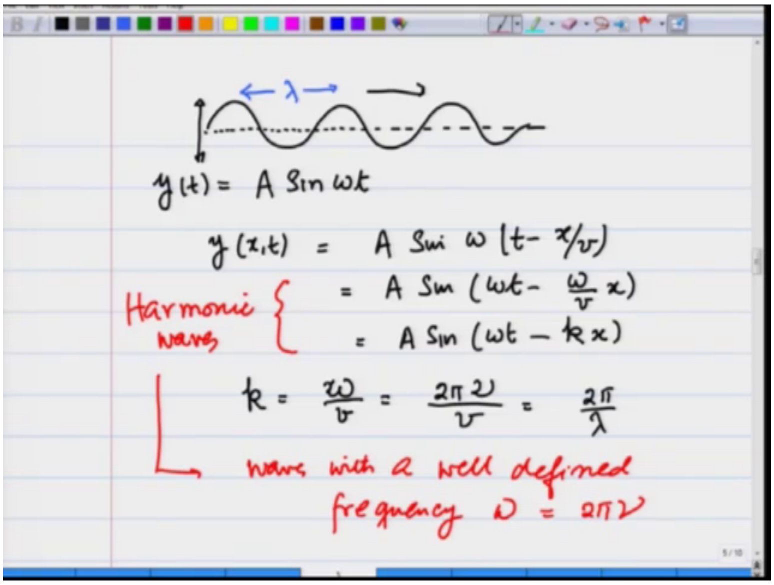

What I am doing is I have taken this string and I will start shaking it up and down with a displacement y t equals A sin of omega t. And then y x, t as this disturbance travels down the string this way will be given as A sin of omega t minus x over v which I can also write as A sin omega t minus omega over v x alright which I can write as A sin omega t minus I will call this quantity k x; where k is omega over v omega is 2 pi the frequency let me call it nu divided by v and you recall from your twelfth grade v over nu is lambda. So, this is 2 pi by lambda, where lambda is the wavelength again I will show it on the top of the screen lambda is the distance between two similar points this is wave, right.

So, there is a misconception among lot of students that a wave necessarily is sin omega t minus k x that is not true I have shown that is a very specific wave that is when one end is shaken in simple harmonic motion. So, these are known as harmonic waves, but these are one particular kind of waves. I started this lecture by showing you a pulse which is also a wave you can take that string and just give it a jerk and that pulse will travel that is also a wave, right except that is not a sin wave that is not a harmonic wave, right. So, this is a very specific kind of wave that has a very specific frequency.

So, harmonic waves; let me write this our waves with a well defined frequency omega which is equal to 2 pi nu. So, omega you can call angular frequency nu is the frequency in hertz and so on; so, that is a harmonic wave. So, this is the kind of thing that happens in a continuous medium.

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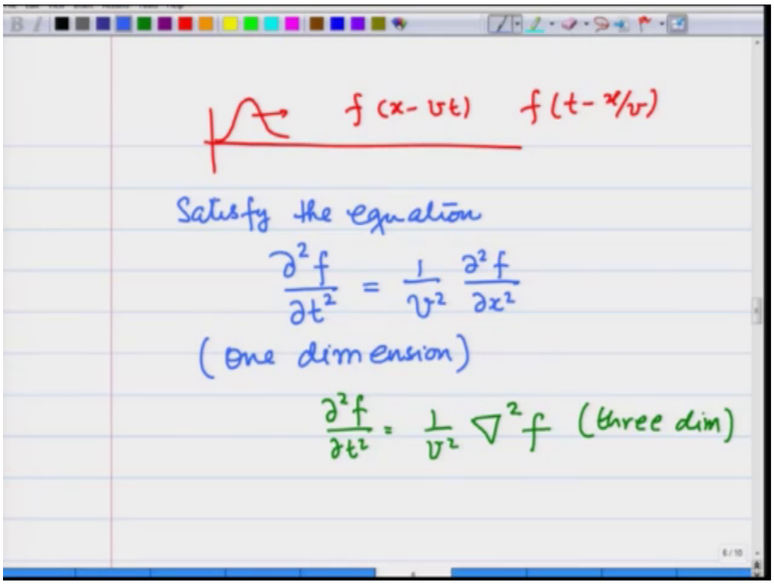

Now, let me also show you be a little mathematical and show you when these pulses travel down right with this function f x minus v t or function t minus x over v. What you can show that these functions satisfy the equation partial derivative of f with respect to partial derivative with respect to time the double derivative is equal to 1 over v square d 2 f by dx square and I am writing this in one dimension.

I will take the risk of writing it in three-dimensions and I would expect you to kind of mesh up your mathematics that is in three-dimensions the wave equation is going to be d 2 f by dt square is equal to 1 over v square Laplacian of f, this is the three-dimension analog I am not going to use this Laplacian thing.

So, do not worry I am going to use the one dimensional equation because idea is to convey to you what is going on. So, this is the equation that is satisfied by the wave, alright. Let me go to the next page.

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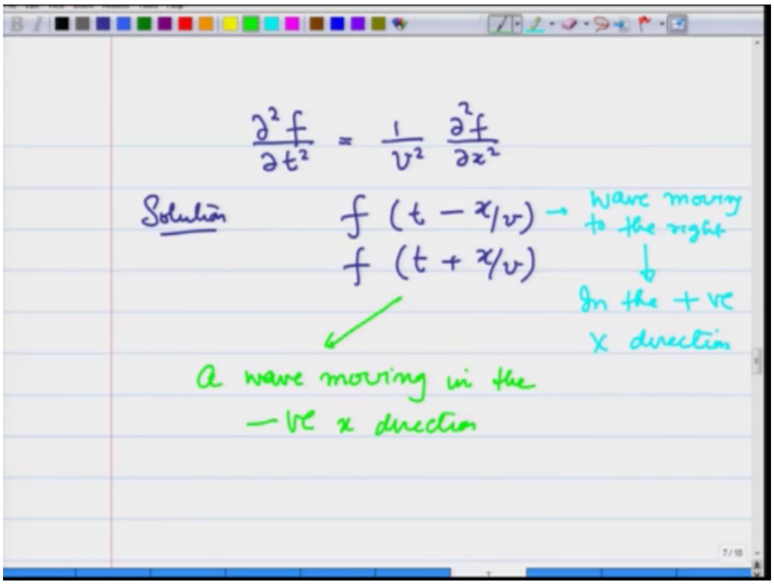

So, we have come to the point where we say that a wave satisfying this equation 1 over v square d 2 f by dx square and the solution we already know is going to be either f t minus x over v or f t plus x over v this represents a wave moving to the right; right is not the right word. So, let me just write it in the positive x direction and this one represents a wave moving in the negative x direction.

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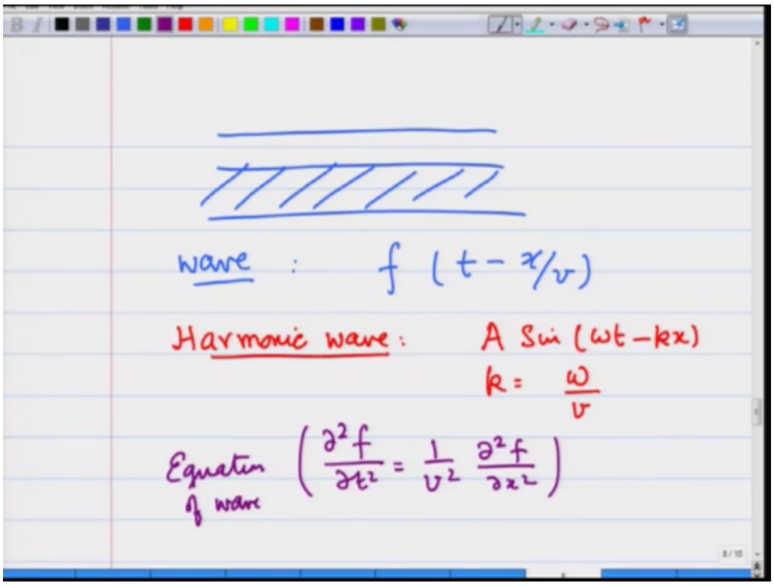

So, let me just now summarize what I have done so far, what I have shown you if I have a continuous medium would be a string or could be air in a pipe which you have seen in a twelfth grade or even the sound that is propagating when we are talking the wave or the disturbance that travels that is what a wave is given by a function f I am going to use this form x over v.

A particular form is the harmonic wave which is some amplitude A sin of omega t minus k x where k is omega over v and the equation satisfied by this wave is d 2 f by dt square is equal to 1 over v square d 2 f by dx square; this is the equation of wave. So far so good; however, remember what we are getting at we want to find the solution for a system which is not continuous.

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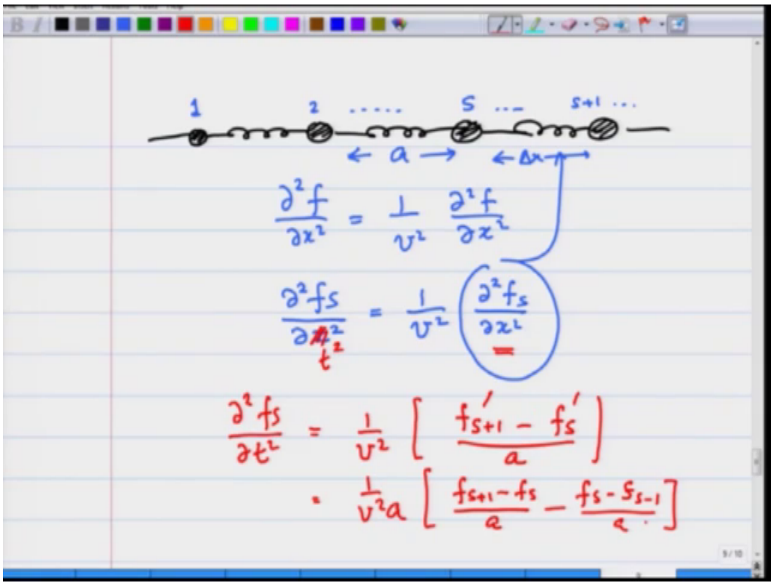

The system we are after is discrete in that these atoms or whatever basis are they are sitting like this and so on. So, this is not continuous and I would like to study the system for that just to motivate how we are going to study this I am going to write this wave equation d 2 f by dx square is equal to 1 over v square d 2 f dx square. Just motivate you intuitively how I can write the equation of motion for the displacements of these atoms.

So, let me label them let me call this 1, 2 and so on maybe this is s-th one, this is s plus 1 and so on. So, I am looking for the displacement of the s-th atom and experimentally we know that there is a speed associated with which this disturbance will travel because the particles do not travel the disturbance travels and I am going to have d 2 f by dx square.

Now, you notice since the medium is not continuous this quantity is not well defined. I cannot have d 2 f s by dx square because this delta x is finite this is a lattice spacing. So, I have to write this in a slightly different form. Sorry, I made a mistake here this should be t square, so, I am going to now change this to a finite difference.

So, what I am going to do is write this d 2 f s by dt square that is perfectly fine because t varies continuously as equal to 1 over v square and if I use the finite difference formula so, I can write the second derivative as the derivative at s plus 1 site f prime minus the derivative at site s divided by the distance a. The derivative let me write these constants outside f s prime I can write as f s plus 1 minus f s divided by a minus f s minus f s minus 1 divided by a, alright.

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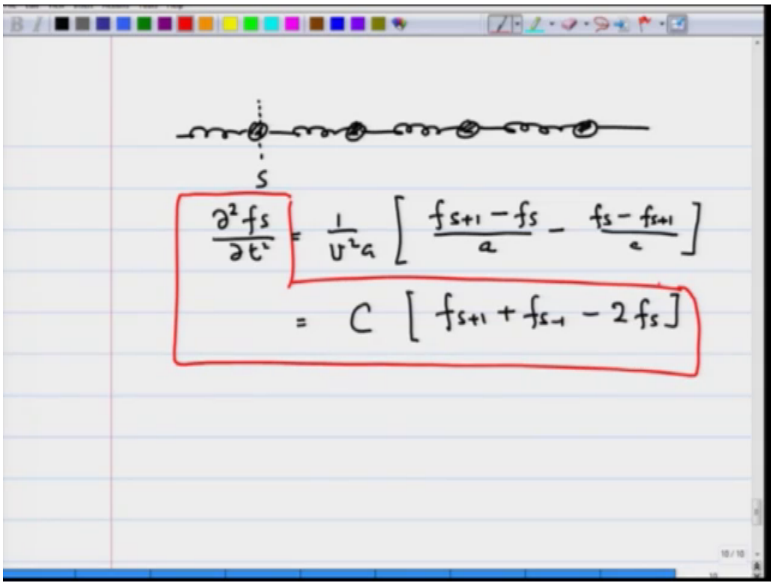

So, what I have done is for this finite system I have written that if I want to know the displacement at s-th site I have written that d 2 f at this point is going to be 1 over v square a f s plus 1 minus f s divided by a minus f s minus f s plus 1 divided by a which comes out to be some constant right which I can say is 1 over v square a square times f s plus 1 plus f s minus 1 minus 2 f s.

So, this would be the discrete system counterpart of the wave equation and you will see this is the precisely the equation I am going to get in a crystal and then we will develop the solution.