Introduction to Solid State Physics

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Lecture - 04
Calculating the electrical conductivity of the metal using Drude's Model – Part I

We had begun looking at the behaviour of an electron as it passes through the solid. And we wanted to understand how the motion of the electron occurs inside the solid, which will give us an understanding of the conductivity of the solid.

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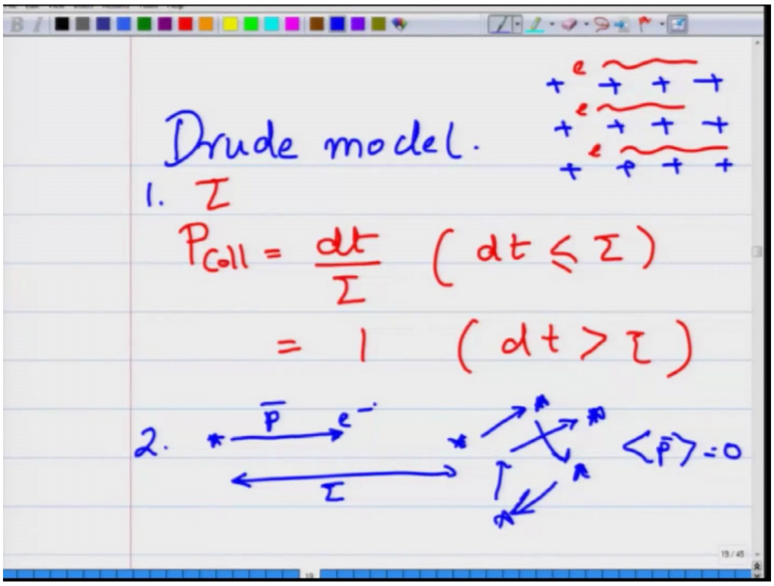

And for that purpose, Drude had come out with his Drude model. And we had discussed these points. And the points were that first you have these positive ions, which comprise the metal and the electron actually sort of moves through this lattice of positive ions. This collection and the ordered arrangement of these positive ions, the electrons is actually moving through this. And as it moves the electron undergoes collision. And there is a mean collision time tau, which the electron, which is the time between two successive collisions experienced by the electron. And using that we had written down the probability of collision within a time interval dt, the probability that the electron undergoes a collision is dt by tau if dt is less than or equal to tau; and it is equal to 1, if dt is greater than tau.

The second assumption was that the electrons between two successive collisions. So, if there is a collision and the mean in time interval between these two collisions is tau, the electron between these two collisions moves independently of any forces it has a momentum p. And after a collision the momentum gets completely randomised. So, with successive collisions the momentum of the electron gets completely randomised.

And so if you look at the net average momentum of the electron after a time interval, if you calculate the average momentum of the electron over a reasonably long interval of time, it will be 0, because the scattering has completely randomise the momentum. But in between the scatterings the electron will move with a certain momentum p. And it is only between these two successive collisions if there is an electric or magnetic field, the electron will experience a force, because of that electric or magnetic field. This was the second sort of assumption.

And the third assumption was the independent electron assumption or the free electron assumption that the electrons, this gas of electrons which is there inside the solid is actually a free electrons. These electrons are free; they are not interacting with each other. So, it is a completely free electron. And almost this is like the kinetic theory of gases, which has been applied to the electron gas inside the metal.

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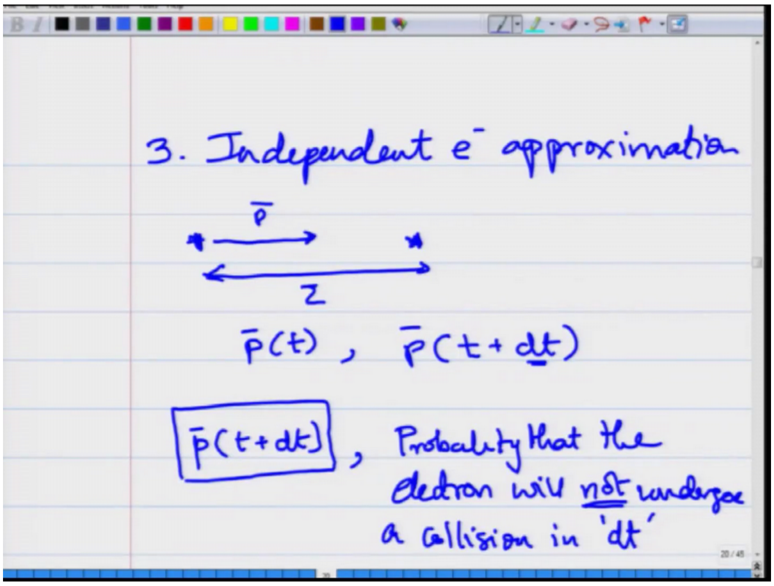

So, we have the independent electron approximation or the free electron approximation. So, these were the three basic postulates or three basic assumptions of Drude. And using this he wanted to calculate the conductivity. So, let us see how he goes about calculating the conductivity of the electron. As we know that the electron is going to move with a certain momentum p between two successive collisions. If the collision time interval tau is tau, then between these two successive collisions the electron will move with a certain momentum p. So, if I know the momentum at any time t of the electron, can I calculate the momentum at any time t plus dt?

So, after an infinitesimal time interval dt from time t from that instant of time t within time dt, what is the momentum of the electron. So, you clearly know that within the elapsed time dt, there is a possibility of collision. And if there is a collision, which is going to be encountered, because of a probability of collision in the interval time dt if a collision is going to be encounter, it will change the momentum. So, the average momentum of the particle has to be weighted by the probability that the electron does not undergo a collision within time dt. So, if you have to find the momentum of the electron within this time t plus dt, you have to write it in terms of what is the probability that the electron will not undergo a collision in this dt interval. So, what is the probability that the electron will not undergo a collision within this dt interval.

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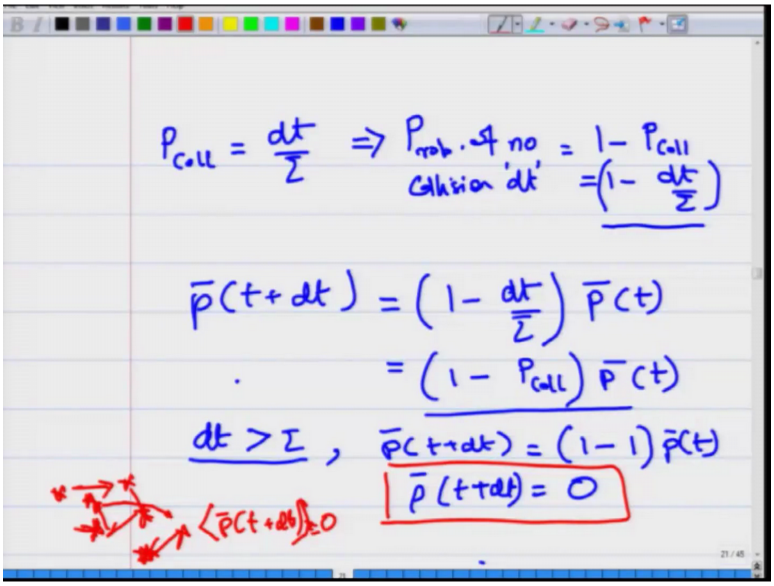

And if you recall the Drude's model had given already the probability of collision is dt by tau, which implies that the probability of no collision in dt interval is 1 minus p collision which is 1 minus dt by tau. So, given a time interval dt, the probability that the electron will not undergo a collision is 1 minus dt over tau. And only then the particle will have a certain finite momentum, because you know that if the particle undergoes collisions then its momentum will get randomised.

So, only within if it does not undergo a collision within time interval dt, if it does not undergo a collision, then only there is going to be an evolution of the momentum. It will go from point p, it will go from a momentum of p at t plus dt. So, the momentum at time t plus dt is going to be written as 1 minus dt by tau times the momentum at time t. So, this momentum at time t is going to be weighted by the probability that the electron does not undergo a collision. Let us look at this a little bit more closely that you might also want to write it as 1 minus probability of collision in time dt times p into tau.

Now if dt is much greater than tau, what does this expression say what is going to be the momentum at t plus dt. The momentum at t plus dt is going to be 1 minus 1 into p of t which is 0. So, the momentum at time t plus dt if dt is much greater than tau, will be equal to 0. The net total momentum the average momentum of the particle will turn out to be 0, because the particle in this if it is long enough, there are scattering which the electron will undergo its momentum keeps on changing.

And as it undergoes multiple scattering, the average total momentum at time t plus dt will become equal to 0. This is expected as per Drude's theory this is expected, as per Drude's model this is expected, and that is what you get. So, this expression actually tells you that that if you are going to look at the momentum at time t plus dt, you have to consider what is the probability that the particle does not undergo a collision and that is why we have this term. So, with this expression of how is the momentum at time t evolving to a time t plus dt, we have now a starting of expression in the Drude's model.

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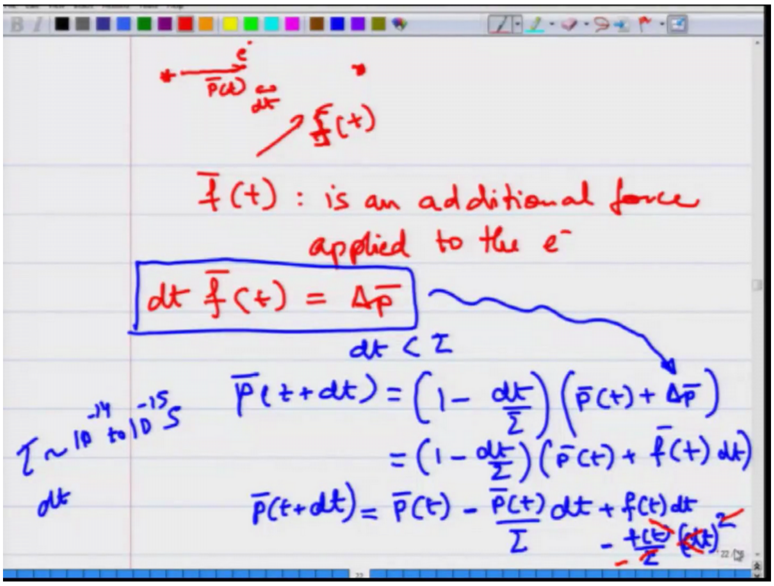

Now, we have a scattering centre and the particle is at time t the particle has some momentum this is the electron, it has some momentum at time t and in a time interval dt. We would like to find out what happens to the momentum of this particle. And what is the Drude's second postulate says? The second postulate says that if you are looking at this process within the time interval tau namely before a scattering occurs of the particle. Then this electron will be susceptible to the effects of electric and magnetic field. So, within this time interval tau, if you apply some additional force f of t which acts on the particle, then the particle momentum may change because of the action of this additional force, f of t is an additional force applied to the electrons.

So, only between the scattering before scattering or within the scattering time interval if there is an additional force, which is applied to the system then that will change the momentum of an electron. So, how will the force f of t change the momentum of an electron? The force f of t will change the momentum of an electron; how much will be the change in momentum in time dt? The change in momentum will be the force times the time interval t this is from basic Newton's law, that the change in momentum will be related to the force that is acting into the time interval dt.

So, therefore, we can write down for dt less than tau, if you are looking at a time is less than tau before successive collisions occur, if you are looking at it within this time interval, you can write the momentum of the particle in the presence of an external force f of t as 1 minus dt by tau into the momentum of the electron at time t plus the change in momentum which happens because of the external force which is applied. This is nothing else, but 1 minus dt by tau p of t plus f of t dt which I can write it as p of t plus dt is equal to p of t let us open out this bracket minus p of t divided by tau dt plus f of t dt minus f of t by tau dt square ok.

And now what we will assume is that, because the time interval dt is very small, if you recall the typical scattering times, which I had told you about tau was of the order of 10 raise to minus 14 to 10 raise to power of minus 15 seconds. So, dt has to be much smaller than this value only then there is going to be a possibility of no scattering and it will respond to these forces. So, if dt is much smaller than this, you can safely assume that you can avoid or you can neglect terms, which are of the order of dt square. So, these dt square will be a term which will be much smaller than dt, and therefore, you can safely approximately assume that you can drop these terms.

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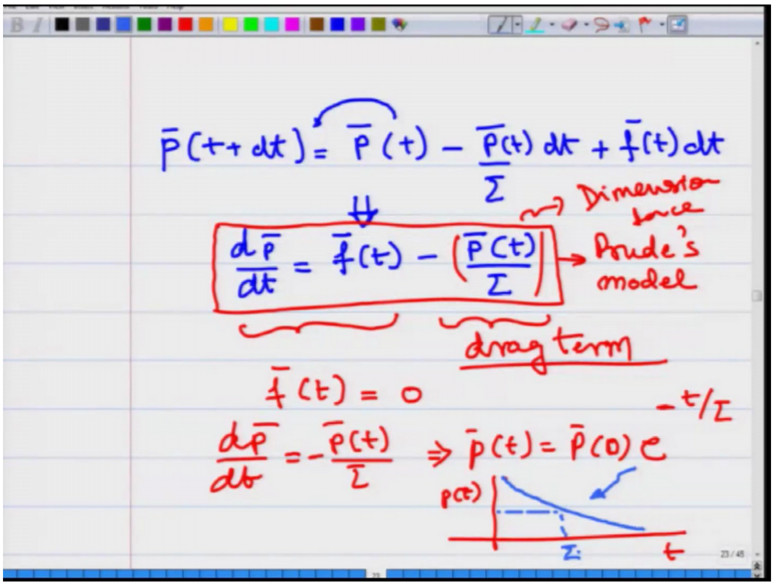

So, under this approximation we can write our equation p of t plus dt is equal to p of t minus p of t by tau dt plus f of t dt. You can take this term on this side, you will get a delta p. And you divide it by dt and then you can show from this expression that dp by dt is equal to f of t minus p of t by tau. And this is the expression for changing the momentum of the particle, when it is acted upon by a force f. If you see that the first this part of the expression is nothing else but your Newton's law that under the influence of a force there is a change in momentum.

And what is this additional term? This additional term is coming because of the scattering. And you can see that it is coming with a negative sign. And you are also aware of these types of expressions. So, this is again dimensions of a force. This term again have dimensions of force with the negative sign. And so what does it correspond to? This corresponds to a drag term. So, this is the drag term which is acting upon the particle or the electron which is moving.

So, if you apply a force to the electron, the electron responds to that force, but over and above that there is a drag term which is the force electron is feeling because of scattering. So, if you apply a force the electron moves, but it is not a free motion. The movement is such that it experiences a drag and the effect of that drag, which is coming from collisions is p by tau.

So, if we do not apply any external force if f of t is 0, then you can see that this expression is dp by dt is equal to p of t by tau with a minus sign. And the solution for this is very well known. You are aware of this solution is whatever is the original momentum of the particle at time t equal to 0 e raise to minus t by tau. So, if you give an initial momentum to the particle, what does this solution say what happens to the momentum as a function of time if you start with an initial momentum then the momentum exponentially decays down with a typical time scale tau because of scattering.

So, you give an initial momentum, but slowly the momentum is going to get randomised, because of the scattering, and then the momentum comes down to 0 because of scattering. So, this is completely consistent with what has been said regarding the Drude's model.

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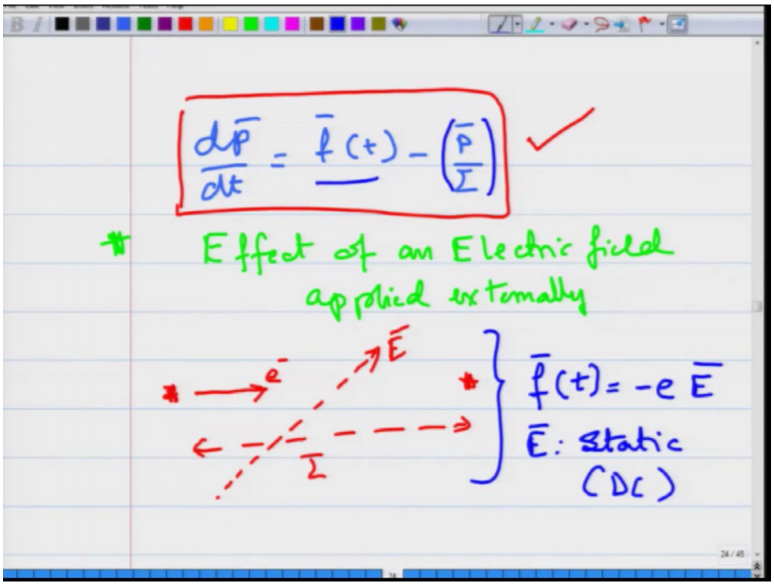

Now, let us go back to the original expression that dp by dt is f of t minus p divided by tau. And let us look at the consequences of this expression. And the first thing that we will look at is the effect of an electric field applied externally to the medium which is experienced by the electron. The electron experiences an electric field. So, let us look at the first effect of what is the effect of an electric field, which is applied to the electron.

Again the idea is that you have scattering centres, you have these scattering centres. Let me exaggerate it and show it, this is the time interval between two scattering centres, the electron is moving ok, this is the motion of the electron. And you are going to apply an electric field E whichever way. So, the moment the electron experiences the electric field its motion will change. So, we will investigate this problem in more details using this equation.

So, using the Drude's model and the assumptions of the Drude's model, we have derived an equation of motion of the electron, which is moving through the metal. And the effect of collisions now as we saw turns of like a drag term, it comes up like a drag term of p over tau, which actually drags the motion. And there is an effect of force which is coming from the external force, which is acting on the electron and that is going to give rise to the net change in momentum of the electron.

Using this we are going to analyse the problem of when you apply an electric field or you apply a voltage across the sample, how does the medium respond to it, and we will look at the outcome of that. So, basically between two successive collisions, you have an electron which is moving which experiences an electric field. And let us analyse this problem in the context of this equation of motion.

Now, in this problem the first thing that we can write down is what is the force experienced by the electron in this electric field. It is minus e times the electric field. The electric field is the static electric field, it is static, namely, it is a DC electric field. And we will look at DC conductivity when you apply a constant voltage what happens to the metal. So, we are applying a constant electric field the force is constant, it is minus e which is the charge of the electron into the electric field.

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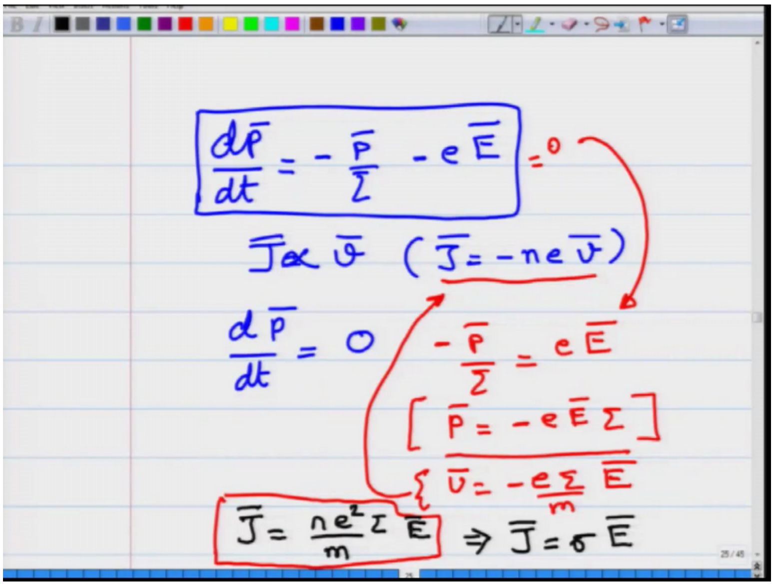

So, let us substitute it into the equation of motion. The equation of motion of the electron is the rate of change of momentum of the electron is minus p by tau, which is the drag term minus e times the electric field. This is a direct outcome of the equation of motion. Now, you have an electric field which is moving the electron. So, when an electron moves it generates the current. So, as the motion of the electron occurs in the direction of the electric field, it will generate a current. And the current as you know your current density is proportional to the velocity of the electron, because J is equal to minus n e times v. So, it is proportional to the velocity of the electron.

So, if you want a constant current the velocity should be constant, then only you will get a constant current, because rest of it is all constant. So, the velocity should be independent of time. But how do you get an independent velocity? If you want to get a constant current. You know that if you apply 1 volt across a metal which has say 1 ohm resistance then you will get a current of 1 amperes through that metal. And the current is 1 amperes, it does not keep on changing with time it is constant at 1 ampere, so which means that the velocity of the electron becomes constant.

How does it become constant? The velocity of the electron can become constant, because there is a drag term, the drag term actually makes the balances out the effect of the external force which is applied, and therefore there is no rate of change of momentum. So, if you want to get what is the static properties of in the presence of a constant DC field or a in the presence of a constant electric field, the system attains a constant momentum or dp by dt is equal to 0, where the drag term balances the externally applied force.

And from the above equation, we can see that minus p by tau should be equal to e times the electric field or p should be equal to minus e times, the electric field into tau, this is the direct outcome of putting this equal to 0 and you will end up with this. So, the momentum of the electron acquires a constant value, this is a constant value if your DC electric field is constant then rest of the terms are constant. So, the electron is now moving with a constant velocity. And this tells you that the velocity of the electron is minus e tau by m times the electric field.

And as I said that the current density is J is equal to minus n e times v. So, if you substitute this expression for the velocity into the expression for the current density, you will get your expression for your current density which has been set up in your sample, because of the applied voltage. The amount of constant current that you start getting or the constant current density is nothing else, but n e square by m into tau times the electric field, which you already know that J we had while working out and giving, you an idea about ohms law we had found that the ohms law can be rewritten as J is equal to the conductivity times the electric field. This is the well known expression where sigma is the conductivity. And from Drude’s model you have J is equal to n e square tau by m.

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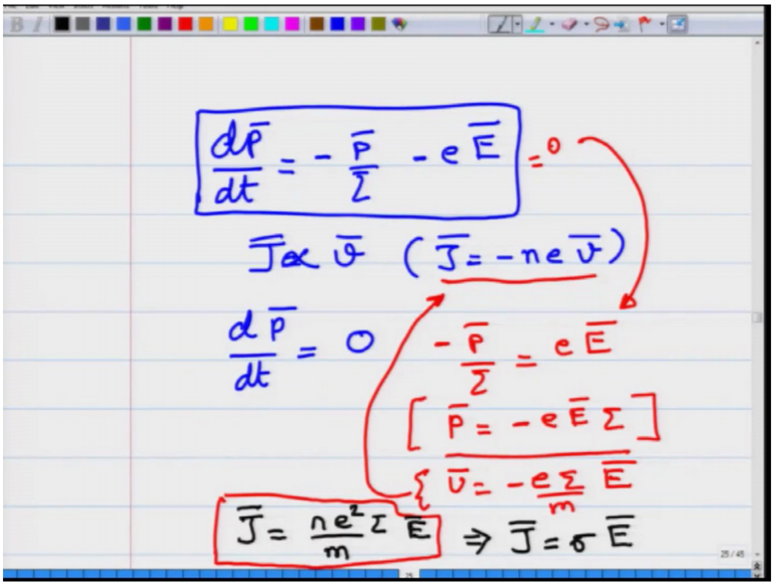

So, you know that you get your expression for the conductivity of the electron is n e square tau by m, where n is the density of the electron in the metal; e is the electron charge; m is the mass of electron and tau is the scattering time. A natural outcome of this is your resistivity of the material is nothing else, but one over conductivity is n e square tau. So, using the Drude's model, you directly have a way to calculate the resistance or the resistivity in other words the conductivity of the material.

And you can see that the resistivity is a function of the scattering time. It is crucially related to the scatterings, which are occurring in the solid. As the electron moves the idea is that, as the electron moves through this lattice of positive ions the electron is undergoing scattering. And as the electron undergoes a scattering as it moves through the solid it gains a resistance, because it is not allowed to move freely, it is dragged by the collisions and the collisions lead to resistance.