Introduction to Solid State Physics
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Lecture – 30
Bravais Lattice Types Part-II

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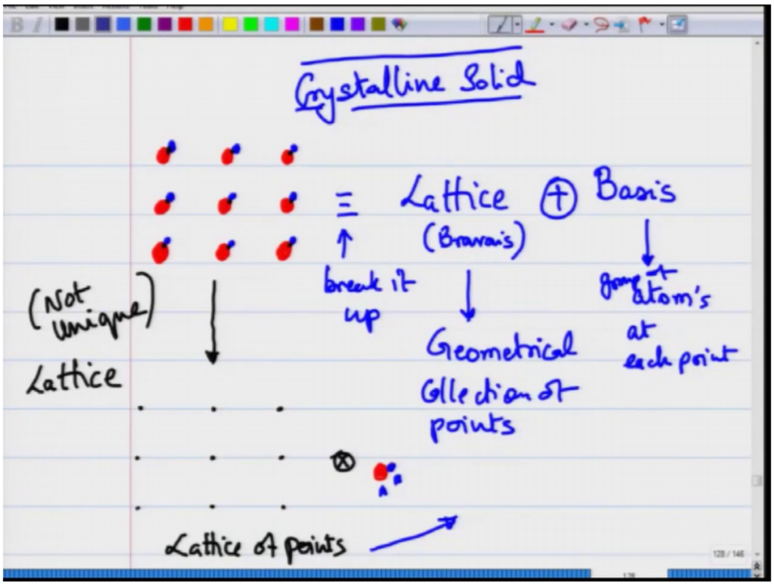

Welcome back, we had begun trying to study the crystalline solid which is most of solid state physics a large part of solid state physics deals with crystalline solids, where you have a collection of atoms which are placed periodically in the lattice. And we had seen for example, here you have a periodic structure of two atoms which are placed periodically inside the lattice we can breakup this structure into one which is a geometrical structure which is a lattice of points.

So, for example, one way to choose it is to replace between these two atoms you take a point and for all the atoms you do this and then you will get this lattice of points. And so, you define a geometric structure which is called a Bravais lattice and on each of these lattice points to make the crystalline solid you can put your atoms on each of them and that is called the basis. So, basis is the collection of atoms which you can put on each of the lattice points, and from there we had seen what is a Bravais lattice.

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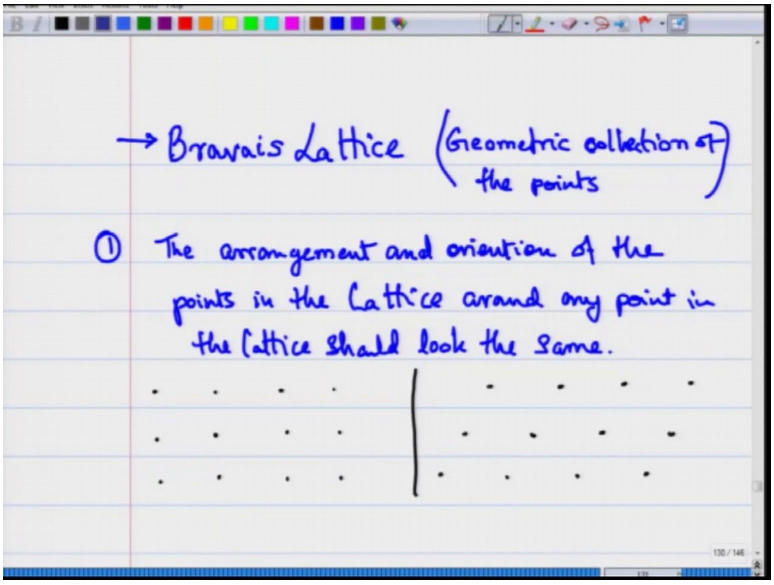

The Bravais lattice was something which is a collection of points in which at every point if you look at the collection of atoms or the arrangement of atoms around it, it looks exactly identical. In terms of arrangement, in terms of configuration orientation it has to look identical that has to be a Bravais lattice.

So, from a crystalline solid you have to first get a Bravais lattice and from the Bravais lattice if you add on each at[om]- on each point of the Bravais lattice if you add your atoms you will get the crystalline solid.

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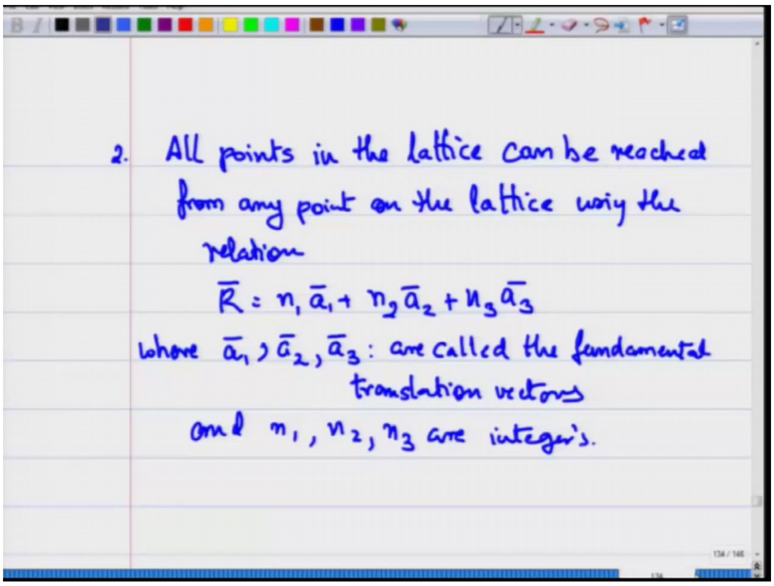

So, we had taken a look at some of these things and another important aspect of the Bravais lattice is that from any point in the Bravais lattice you can move to any other point using a translation vector, where this translation vector is written in terms of three either fundamental vectors a 1 a 2 a 3. And your translation vector then can be written as n 1 a 1 plus n 2 a 2 plus n 3 a 3 where n 1 n 2 n 3 are integers.

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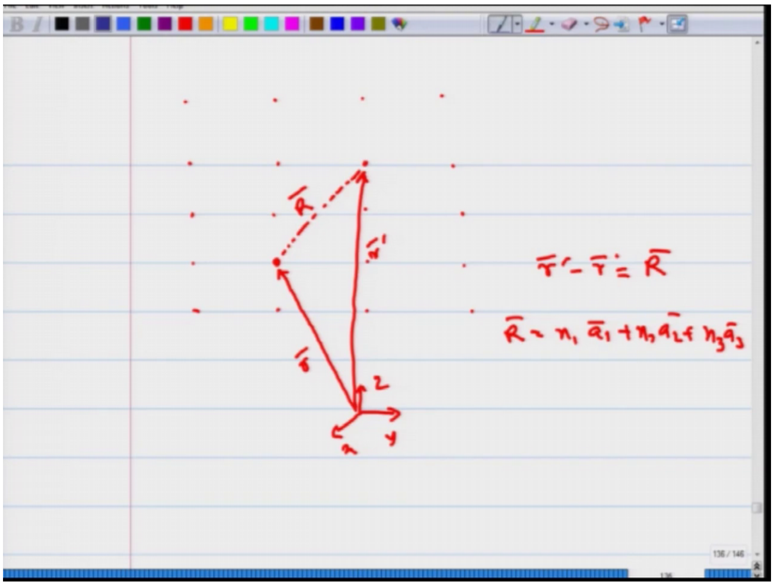

And what this meant was that you can go from any point to any other point in your Bravais lattice and these two points are connected by a translation vector, where this translation vector can be written in terms of.

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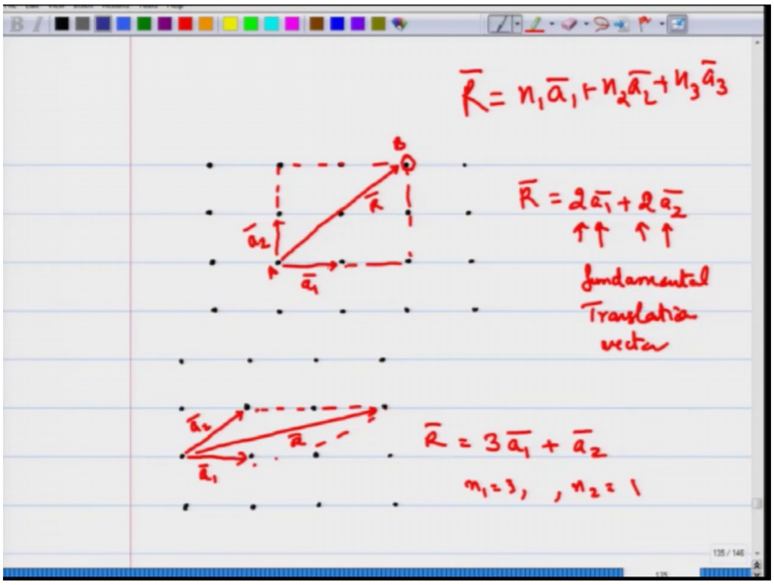

These fundamental translation vectors a 1 a 2 and a 3.

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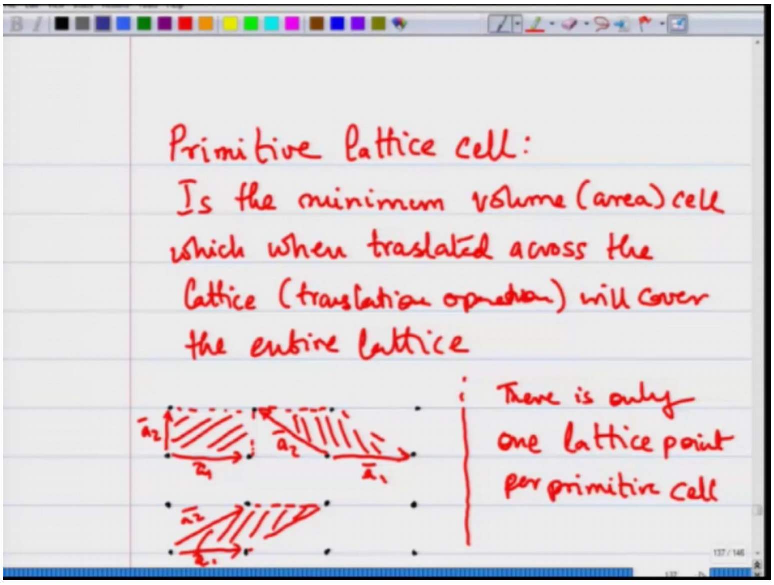

And then we had seen the concept of a primitive lattice, which is the minimum volume or cell which when translated across the lattice with the translation vectors covers the entire lattice without leaving any gaps in the lattice. And in the primitive cell or the primitive lattice the vectors which form this are called the primitive lattice vectors, and one very important point of the primitive lattice cell is that you can have only one lattice point per cell.

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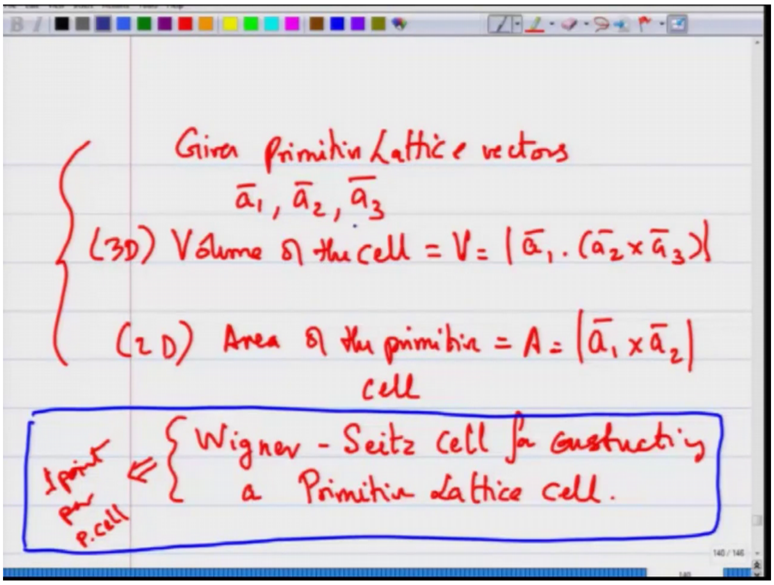

So, with this we had actually defined a way also that if you have a 1 a 2 a 3 as a primitive lattice vectors you can define the volume you can define the area in two dimensions ok. And you have one lattice point per volume of the primitive cell, if v is the volume of the primitive cell then you have one lattice point per primitive cell. And further more we had found that there is one way of actually constructing a primitive cell, given any lattice you can actually construct the Wigner Seitz cell where we had seen how to construct it.

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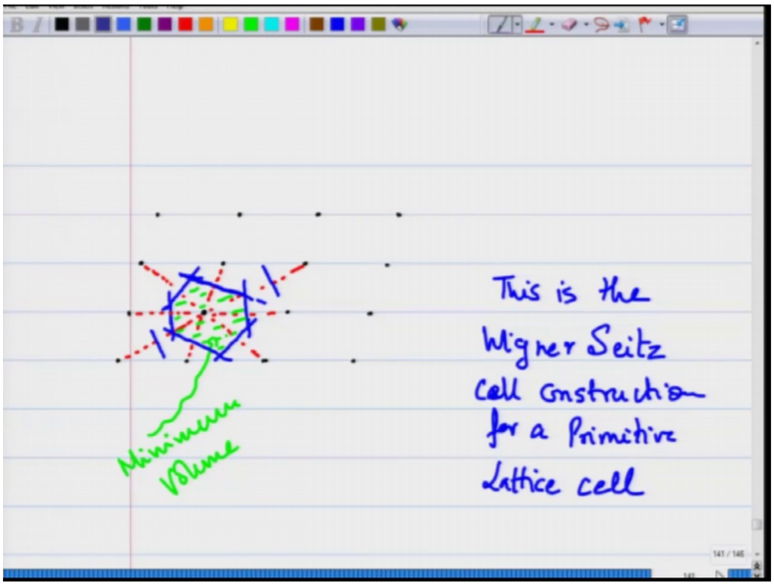

That you join to a line to the nearest neighbors, find the midpoint draw perpendiculars and then the intersection of the perpendiculars encloses an area which is closest to this point. So, all the points in this are the closest to this point and this area which is enclosed in green is the primitive lattice associated with this lattice point and there is one lattice point per cell and with this we had sort of built up all the required tools to study a crystalline solid.

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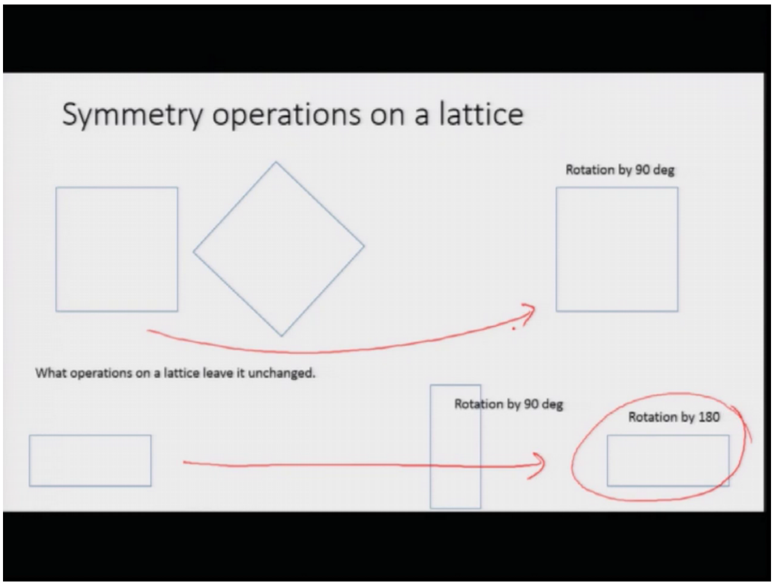

Now look at some important aspects of this crystalline solid. So, if you have these lattice of points, these lattice points obey some symmetries. Now what do I mean by symmetries? These are certain operations which you do on the lattice, it will take the points to back to themselves and let me give you some examples. One symmetry operation that you can do on a lattice is rotation; for example, suppose you have this cubic or let us say this is a square lattice of points as I imagine that there at the corners of each of these points there are individual points which are sitting at the corners of this. And if I now do a rotation so, let me try and do a rotation of this.

If I rotate it by some arbitrary angle then you get this new collection of points, this one here, one point here, one point here, one point here and you can also consider it as a very big lattice you can rotate the entire lattice of points. But after you do the rotation do you think that this lattice is the same as the original lattice, it s not its certainly isn’t ok. However, for this square lattice of points where you have points sitting at each individual corners of this if I rotate this cubic structure or the square structure by 90 degrees as shown here. If I rotate it by 90 degrees I will get a structure where I will not be able to distinguish these two lattices. So, certainly under rotation by 90 degree this cubic or this square lattice structure is invariant. So, this is one such symmetry operation which I can do on the square lattice which will keep it invariant.

. So, I can take this sort of Bravais lattice as a very special lattice; for example, this sort of a square lattice that it is invariant under rotations by 90 degrees; if I have a rectangular lattice then if I rotate it by 90 degrees I will go from this sort of a structure to this sort of a structure it is invariant, these two are very different structures. So, it does not rotation by 90 degrees for this particular rectangular like shape, rectangular like lattice isn't invariant, but if I rotate it by 180 I can again make it invariant for example, if I rotate this by complete 180 degree rotation I will get this structure.

So, if I go from the rectangular structure to my 180 degree rotated structure, then it remains invariant; similarly a 90 degree rotation leaves these structures invariant.

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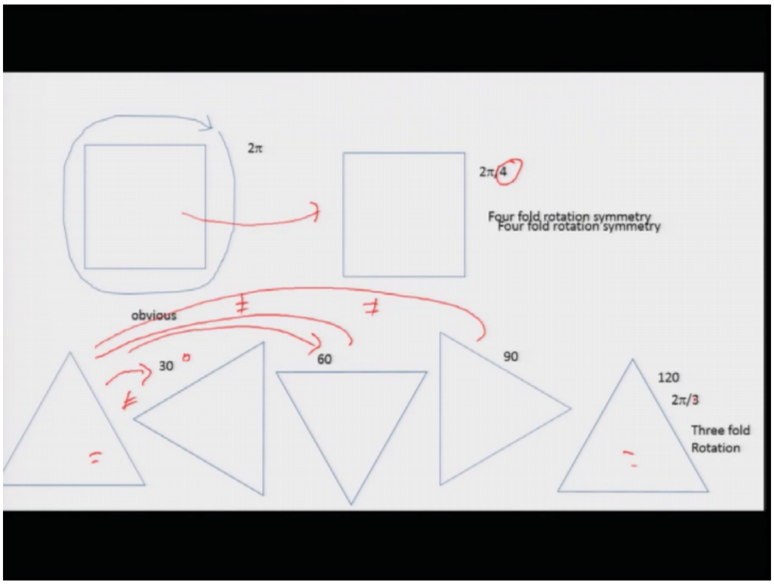

So, therefore, for example, this squared lattice if I rotate it by 2 pi, 2 pi is the full 360 degree 2 two pi by 4 will leave this square will be taken by 2 pi by 4 or pi by 2 rotation will be taken in into a shape which will remain invariant. So, we say that this four represents a 4 fold rotation symmetry.

Similarly, if I have this sort of a triangle if I rotate it by 30 degrees it becomes a shape which is not equal to this, if I rotate it by 60 degrees it is certainly not equal to this if I rotate it by 90 degrees these two shapes are not equal, but if I rotate it by 120 degrees I get back the same shape these two shapes are exactly identical. So, again I can take out this shape and I can keep it aside and say that this shape is going to be invariant under a three fold rotation two pi by three rotation.

So, there are certain rotations which will keep such lattices invariant and such lattices which remain invariant under rotations, under specific types of rotations will be kept aside as special lattices which follow some symmetry operation which, remain invariant under a symmetry operation. And not all lattices obey symmetry operations they will remain invariant ok, they will vary as you do a rotation, but there are certain types of lattices which will remain invariant. ok.

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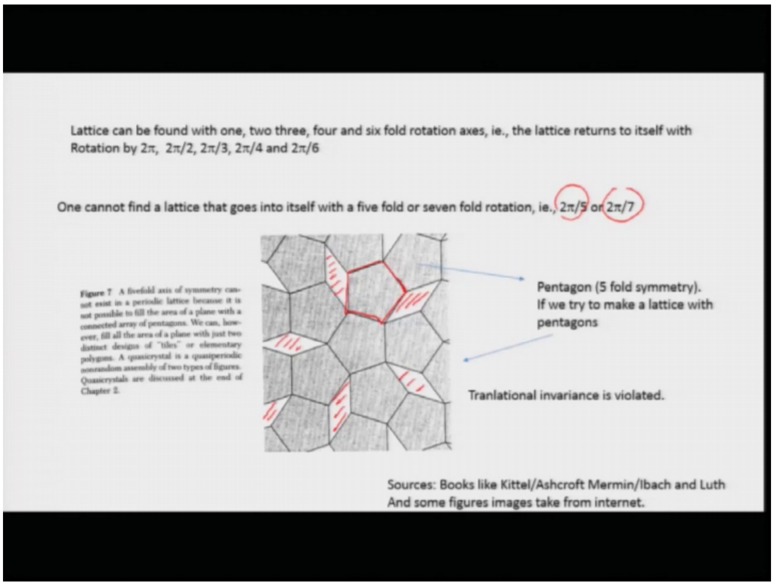

So, you can find lattices which can be classified as remaining invariant under one two three four, five or six fold rotations namely they are invariant under 2 pi 2 pi by 2, 2 pi by 3, 2 pi by 4 and 1 pi by 6 rotations. So, you can classify those lattices as remaining invariant under those rotations and it is surprising actually that you cannot find lattices which have five fold or seven fold rotations ok. For example, here is a lattice which is made up of pentagons, this particular structure is invariant under two pi by five rotation, but this if you try to make a lattice using by repeating these pentagons you will not get a area which is completely covered by the lattice. You can see that there are these areas which are remaining vacant which are remaining with voids.

. So, a lattice which has 2 pi by 5 symmetry or five fold rotation symmetry or a structure which has 2 pi by five fold symmetry you cannot make it into a lattice by using it. Similarly for seven fold symmetry they do not cover the space without leaving gaps inside the system. So, these are symmetry operations which are used for classifying lattices.

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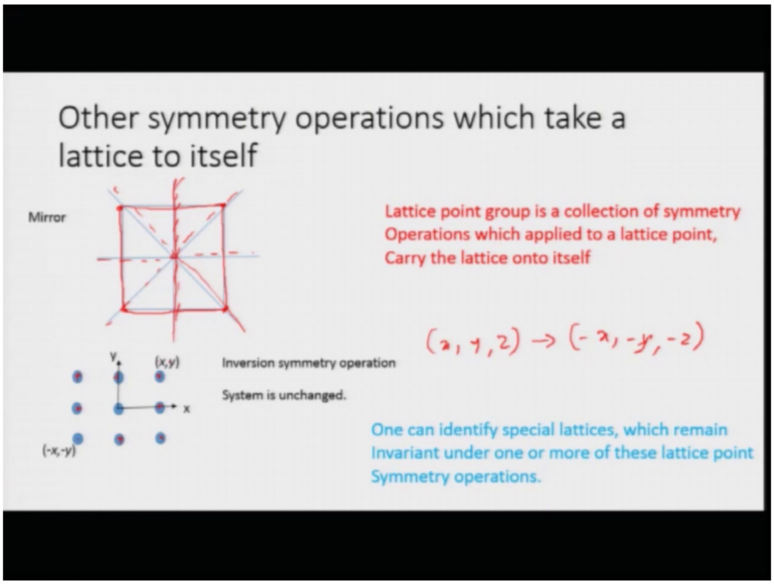

And there are other important symmetry operations for example, mirror symmetry; if you take the square lattice of points and if I take the square lattice of points and I reflect, I put a mirror along this line which I show you here, if I put a mirror along this line then I will see exactly the same lattice, similarly if I put a mirror along this line or along this line or along this line all points will be taken to each other and the lattice will remain unchanged.

So lattices which obey mirror symmetry can again be classified separately, those lattices in which the points go on to themselves if I do a mirror operation and there are different types of mirror operations I can do. For example, in the square I can put the mirror at different location, specific locations and I will find that for mirrors placed along these locations the lattice points will go over onto themselves.

The structure that you will get after the mirror operation will look exactly identical as the original lattice. So, we say that lattice follows some sort of a mirror symmetry; you also have inversion symmetry where about a point you can go and you can actually take each point of the lattice to and replace it by minus x and minus y for example, in this point suppose I take this point and I replace the coordinates by minus x minus y I will find that there is another point which is sitting here.

. So, this point by inversion symmetry; so, you can take x y z of each lattice point and replace it by minus x minus y and minus z and you can do it for every point and you will find inversion symmetries obeyed for certain lattices. So, every point can go onto every other point and then the system actually obeys inversion symmetry ok.

So, there are these different types of symmetry operations that you can do on a lattice and you will get certain lattices which will remain invariant under those operations. The lattice that you get back after certain types of symmetry operations like rotation by some 2 1 2 3 6 1 2 3 4 or 6 fold rotation they will lead it back into itself.

Similarly, mirror symmetry operations certain types of them will lead some types of lattices which will remain invariant inversion symmetry will lead to some types of lattices being invariant and. So, from the entire type all the different types of lattices that you can have the Bravais lattices that you can have, you can classify certain lattices out of them which will obey or which will remain invariant under some of these symmetry operation.

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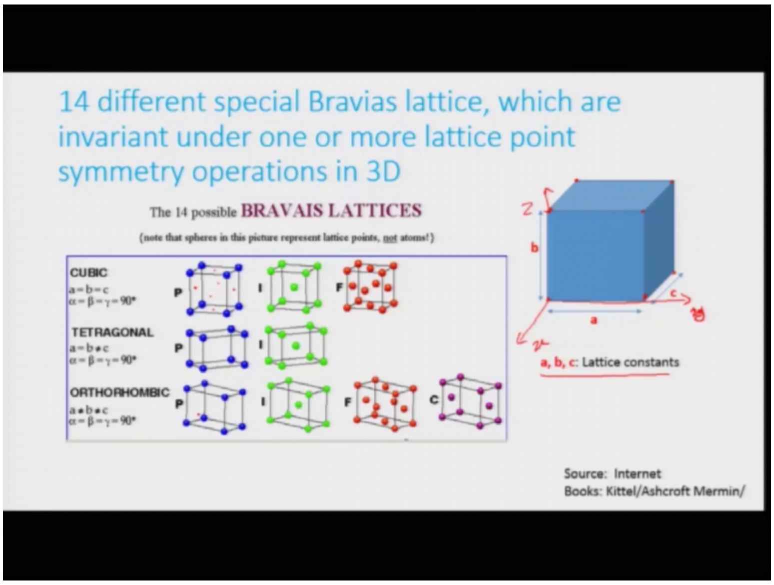

And this is the basis of 15 possible different Bravais lattices that out of the large number of different lattices that you can have one can identify out of those many different types of possibilities fourteen specific Bravais lattices, where these lattices will remain invariant under some particular symmetry operation, the operations which I have told you.

And these 14 different lattices can be classified as follows, I will not go over the details of them the pictures are already shown here ok, but there are certain terminologies which I will like to introduce here, If I have a cubic lattice then these lengths between these points along the different directions this a, b and c. So, the distance along say this is the let us say this is the y axis this is the x axis and this is the z axis ok. This distance between two successive lattice points is given as a, the distance between two successive lattice points say along the x direction is c and the distance between two successive lattice points along the z direction is b and these are called as lattice constants.

. So, you can measure the lattice constants along certain translation fundamental translation vectors, here I have just shown you this simple perpendicular directions ok. You can measure the lattice constants also along certain translation vectors, but these are the distance between two successive points along different translation directions in your lattice and these are called as lattice constants. So, amongst the 14 different Bravais lattices which remain invariant under one or more symmetry operations those are for example, you have the cubic class where, this is completely cubic a is equal to b is equal to c and the angles between the different axis are 90 degrees.

And you have different varieties, one in which you have a cubic system, one in which there is one lattice point in the center of the cube and the third in which there is a lattice points on each of the faces of the cube. There is one lattice point here one on that face, one on this face, one on this face, one on this face, one on this face.

So, this is different similarly you can have tetragonal two types of systems where a equal to b, two directions are the same c is not the same ok. So, you can define and you can have one variant of it which is again going to remain invariant under so, one or more of these lattice operations orthorhombic a not equal to b not equal to c, but at the same time the angles are all 90 degrees.

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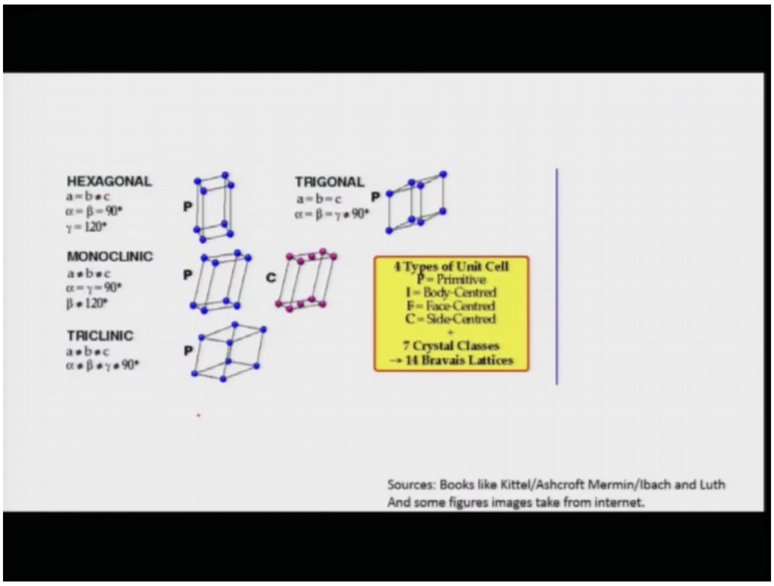

So, with this you can define variety of classes of 14 different Bravais lattices hexagonal, monoclinic, triclinic and their different types within this you can have different types. And so, these will form 14 different types of crystal lattices; 14 different types of crystal lattices which follow which remain invariant under some symmetry operations.

So now, we will look at what are the different types of lattice structures or crystal structures are possible. Let us try and look at some of those crystal structures, their characteristics and study some of the tools required to understand those structures.