Introduction to Solid State Physics
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Lecture - 03
Postulates of Drude's Theory

In my last lecture we had seen that solids can show vast variation in their electrical property, namely the conductivity or the resistivity can vary by vast magnitudes. And insulator can have very high values of resistivity whereas, a metal can have very low values of resistivity. There could be semiconductors which have intermediate values of resistivity, they have very distinct temperature dependences and there could be materials like superconductors which actually loose resistivity below a certain temperature. All of them are carrying electrons, they have electrons at the charge carriers, but yet they have fast variation in their properties.

Before we look at some of the properties of these other materials like insulator, semiconductor, superconductors, part of which we will be covering in this course, let us go back to metals and let us try and understand how does an electron move through this metal, how does it conduct electricity, and can we explain its conductance properties. So, as I said metal has positively charged ion cores surrounded by a sea of electrons, as shown here.

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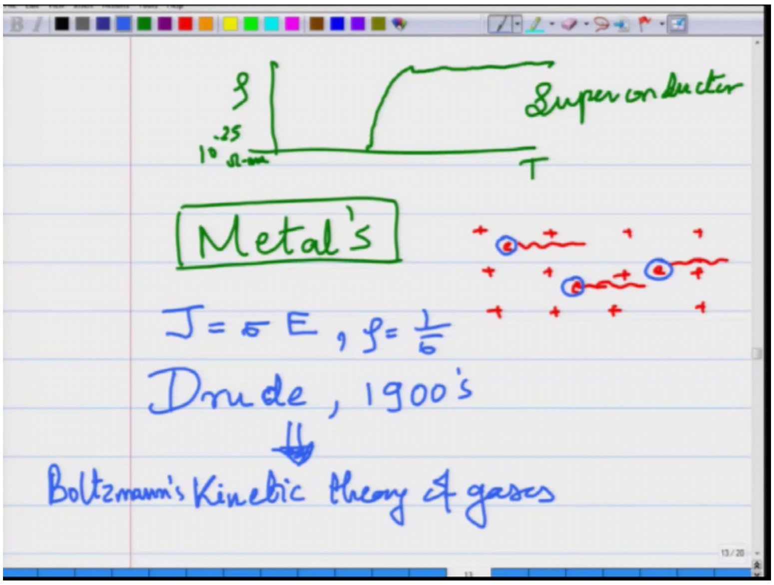

You have electron which are moving all through the lattice of positive ion cores, and this is the typical structure of a metal, ok. And this metal will have a conductivity sigma or a resistivity rho which is 1 upon sigma and J is equal to sigma times the electric field. Now, let us try and understand the conductivity, how does it arise and where from does it arise.

Here came Drude with his model in 1900s, and he wanted to explain from where from does conductivity or resistivity arise inside a metal, and how do electrons give rise to this property of resistivity or conductivity. And his idea was simple, he realized that he could invoke a model which is very similar to the kinetic theory of gases, to explain the behavior of metal electrons inside a metal. So, we now start discussing about the Drude's model.

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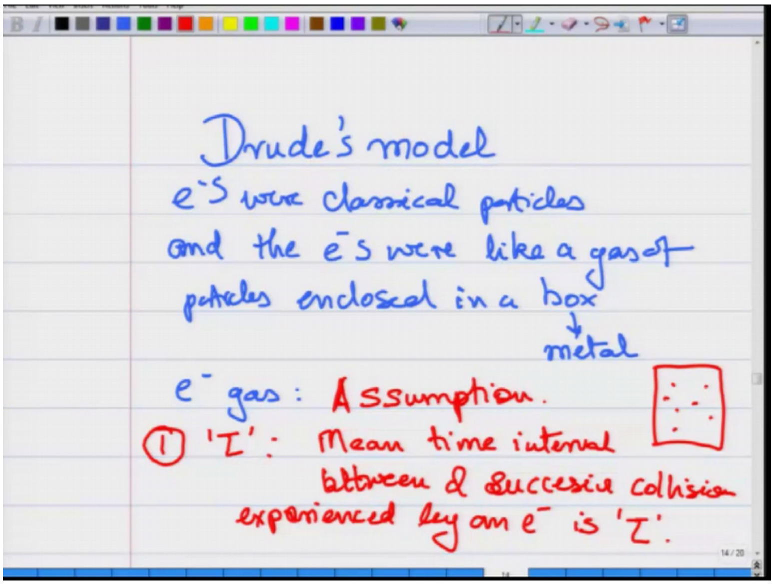

And Drude suggested that the behavior of electrons inside the metal is very much similar to a gas of electrons inside the metal. It is just like a gas of electrons inside a box. He considered that electrons were classical, and these electrons were like a gas of particles enclosed in a box, the box being the metal in which the electrons are present.

Now, to describe the properties of this electron gas he came up with a few assumptions which are the cornerstones of this Drude's model. So, let us discuss the first assumption of Drude's model. The first assumption says that in this gas of electrons, you have a box in which you have these electrons, now it is the picture is almost similar to that of kinetic theory of gases, but instead of atoms now inside the metal you have a gas of electrons.

And his first assumption was that, these electrons encounter a collision typically after a time tau. These electrons are not continuously moving that if they start moving, they are continuously moving, but they typically encounter a collision. The mean time interval between two successive collisions experienced by an electron is tau. So, the electrons undergo collisions, and there is a time scale tau which is associated with theses collisions.

And this was the first assumption that he made. That, this electrons are not completely moving with, if you have an infinite metal it does not mean that the electron moves over infinite distances; the electron does undergo a collision and the time interval between two successive collisions is tau.

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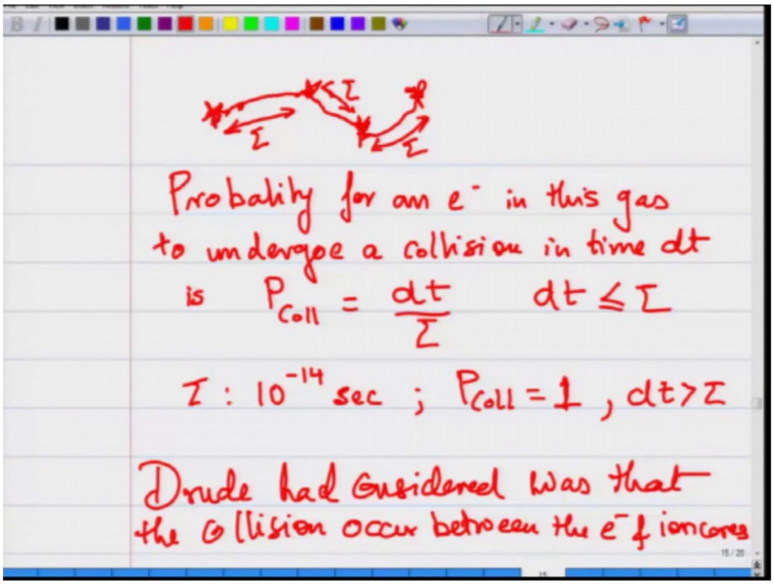

So, there is one collision which occurs the electron emerges, and then there is another collision it experiences, and then moves on, and experience is another collision in its path and then again moves on an experience another collision. The typical time interval between all of this collisions is tau. So, he suggested that the probability for an electron in this gas to undergo a collision in time dt is P of collisions is equal to dt divided by tau, where tau is the mean time interval between the collision and dt is the time interval within which you are going to see whether there is a finite probability of the electron undergoing a collision or not. And this is for dt less than or equal to tau.

Typically, the time scales of tau are 10 raise to minus 14 seconds. We will come to that how does this come about later on this course, but typically that probability of collision that we are speaking of that in a time interval tau whether there is a finite probability of collision is when this dt is going to be less than 10 raise to minus 14 seconds. You are really looking at very small time intervals and within that small time interval you are asking is there a finite probability that the electron will undergo a collision, ok.

And what Drude's have considered was the collisions occur between the electrons and the iron cores. So, the collisions are occurring between the electrons and iron cores, this was the assumption of Drude that this is where the collisions are occurring. And the typical time interval mean average time interval there is of course, of distribution of times, but the typical average time interval between two successive collisions is tau. The probability of the collision in time dt, if dt is less than tau is dt by tau. And the probability of collision is 1, if dt becomes greater than tau.

Namely, if t is 10 raise to minus 14 seconds and dt is like 1 second if you are looking at a 1 second interval and asking what is the probability that the electron in the metal has undergone a collision, it is guaranteed that the electron has undergone collisions not one, but a large number of collisions.

But if you are asking that, that what is the probability that in 10 raise to minus 13 seconds after the beginning of, after the electron started to move or in a finite time interval of 10 raise to minus 13 seconds what is the probability of the electron undergoing collision then that is a finite value.

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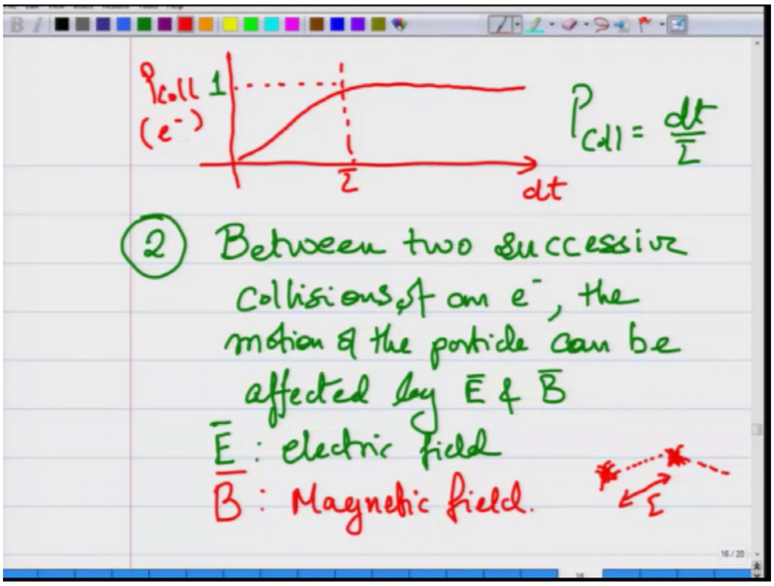

So, if you look at the probability of the collisions of the electron as a function of this dt time interval, where beyond tau the value saturates to 1 because it is 100 pursuant guarantee there will be a collision. And for time intervals which are less than tau the probability of collision is dt by tau.

So, you can see that the gas of electrons that you have is now being considered almost similar to the kinetic theory of gases where we are considering collisions of the electrons, as they are moving through the lattice of atoms they are undergoing collision. And there is a mean time interval between two successive collisions which is tau. So, this was the first important assumption of Drude.

Now, let us move to the second assumption. The second assumption states that, between two successive collisions of an electron the motion of the particle can be affected by E and B; E being the electric field, B being the magnetic field. So, when the particle is moving between two successive collisions, the electron has encountered one collision and it is moving and it encounters another collision, then it is only in this time interval tau between two successive collisions that the effect of electric field and magnetic field can really change the motion of this particle.

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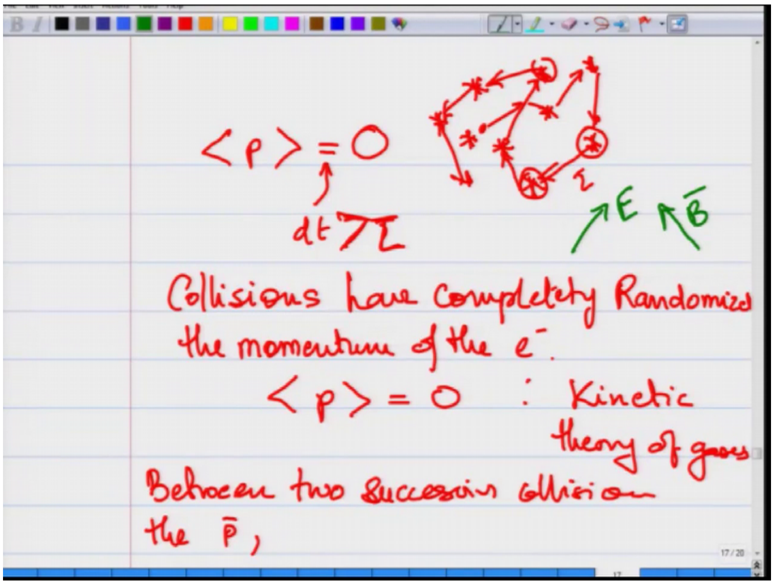

What is an outcome? What is the natural outcome of this thing about collisions? The natural outcome of this thing of collisions is or the second assumption is that the net average momentum of the particle of the electron in this electron gas over long times, time intervals which are greater than tau is 0. Because the electron as it moves between successive collisions, as it encounters a collision it is going to move randomly, it is going to get scattered randomly. And in this random way of scattering as it moves from one scattering to another, if you find the average net momentum of this electron over a reasonably long-time scale it is 0, because the collisions have completely randomized the momentum of the electron.

The collisions have completely randomized the momentum of the electron, so the net average momentum of the electron over a long-time interval turns out to be 0, and this is again very similar to the kinetic theory of gases which you have studied. Just like the kinetic theory of gases here also the collision are playing the role of randomizing the momentum.

So, if you take a long-time interval and you look at the average momentum of the electron it is going to be 0, but between two successive collisions the particle has a finite definite momentum. So between these two successive collisions as it is moving from this point to this point it is undergoing a collision at these two points, but in this time interval between the two successive collisions the particle is moving in a straight line with a finite momentum P, ok. And it is only between these two successive collisions that if I apply any electric field E or a magnetic field B then the particles motion will get affected between these two successive collision and this is the most important part of the second assumption of Drude.

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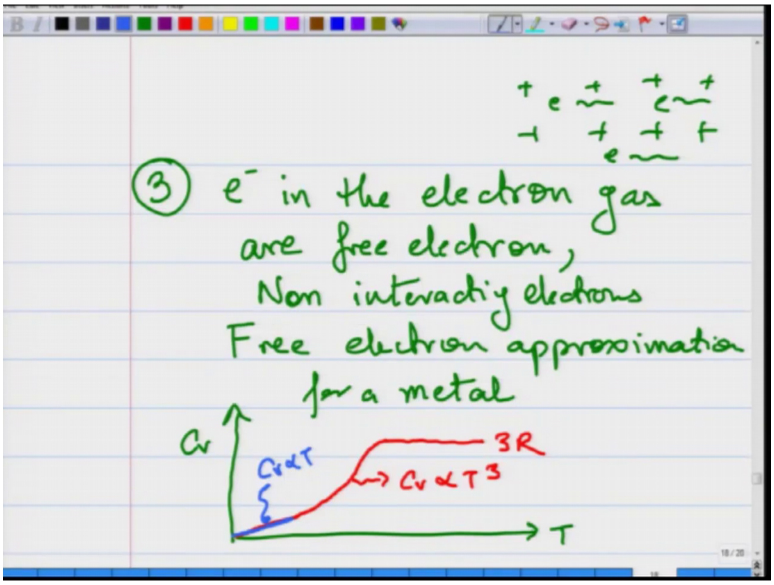

The third assumption is that and which is quite a drastic assumption is that these electrons in the electron gas are free electrons. Namely, they are non-interacting electrons.

Just like we had discussed that for the kinetic theory of gases, you will consider the gas is made up of particles which are like hard spheres and they have only kinetic energy.

There is no potential energy because they have no interaction between them. Here too he made a very drastic sort of an assumption that, within a metal where you have this positive ion cores and you have these electrons which are wondering, you have the sea of electrons which are wandering across this matrix of ions which are placed inside the metal that this gas of electrons are basically free electron, each electron is independent, and they are non-interacting.

And this was a very important assumption because you do not it simplifies a lot of thing because you do not have to consider the electron electron interaction any longer, and you can almost read this, electrons as free particles and do have a with a lot of complications, which come because of interactions. Because the electron is not only interacting with another electron, the electron could be interacting with the lattice. And all these interactions play an important role. When you look at other conducting with the properties you will find that later on electron electron interaction become important. Electron ion interaction become the extremely important, how the electron is moving through this lattice of ions which are sort of attracting giving a week attractive potential to the electron is very important. And they actually change the motion of the electron and lead to various affects.

Those are important, but in the Drude's theory you will find that for a metal he considered that these electrons are completely free, just like an ideal gas electrons are not interacting. And, it turns out this assumption is actually quite correct, a lot of metals and lot of the properties of metals you can actually do away with these interactions and you can explain a lot of the features of the conductivity of the metals by considering that the electrons in the metal are free. So, this is the free electron approximation, this is called as the free electron approximation for a metal.

And what is the signature of this? Is there any experimental signature which actually can show that electrons inside a metal are free; they actually behave like free electrons and non-interacting electron? The proof of that is something which I had already shown you in my earlier lecture. The experimental proof of it is that if you measure the specific heat as a function of temperature, then I had shown you that at high temperatures it is roughly constant at 3 R.

As you lower the temperature the curvature changes as the specific heat becomes proportional to T cube, and at low temperatures there is a regime where the specific heat is behaving linearly with temperature. This region where the specific heat behaves linearly with temperature can be explained only if you consider electrons inside the metal are behaving like free particles, non-interacting particles. You know that electrons have coulombic repulsion and so on, but inside a metal if you consider the gas of electrons has completely free electrons and if you try and derive the specific heat of these electrons you will find a linear temperature dependence. And the observation of this linear temperature dependence of the specific heat is one of the very important proof to show that the electrons inside a metal behave as free electrons.

And we will discuss this why does it come out linear in few you later lectures. But for now you can take this as an important experimental evidence that electrons inside a metal are free.