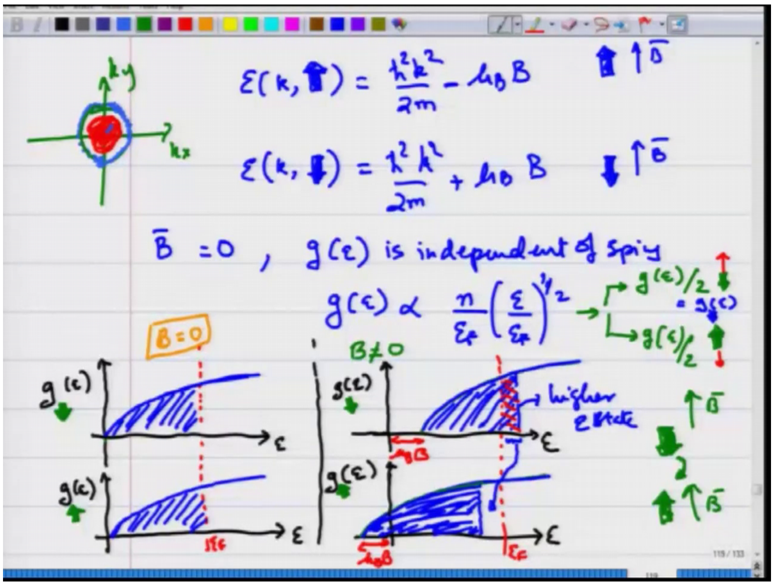
Introduction to Solid State Physics
Prof. Manoj K. Harbola
Prof. Satyajit Banerjee
Department of Physics
Indian Institute of Technology, Kanpur

Lecture – 26
Introduction to magnetism in metals Part – III

(Refer Slide Time: 00:19)



So, we had seen that the density of states in zero magnetic field is equally distributed between the magnetic moment opposite to the applied field direction and the magnetic moment of the electron in the direction of the magnetic field. Of course, there is no magnetic field at B is equal to 0, so they are exactly identical. But what happens to the energy when you switch on the magnetic field, when you switch on the magnetic field when B is not equal to 0 the energy get shifted. Now, the zero of the energy is starting from a lower value; for k is equal to 0, this is equal to 0, but the energy in a magnetic field is minus mu B. For an electron with magnetic moment parallel to the field direction and for an electron which is anti parallel magnetic moment anti parallel to the field direction it has a higher energy.

So, if you put k is equal to 0, the origin is now shifted ok. The electron at k is which was originally at energy equal to 0 is now starting with an energy which is moment anti-parallel has an higher energy, and this has a lower energy. So, they will get shifted and that is what we do. Total number of electrons still remains constant ok, but the energies are now shifted. So, we have to plot this density of states slightly differently.

So, let us do that. For the electron with magnetic moment anti-parallel to the direction of spins if k is equal to 0 the energy does not start from 0, but it starts from plus mu B. So, for this anti-parallel electron, the zero is actually from plus mu B. So, it starts from plus mu B. And for the electron whose magnetic moment is parallel to the applied field direction, the zero energy is not it for zero momentum, it is not at 0, but it is going to start from minus mu B. So, it is going to start from here. So, this is the density of states of the electron whose with magnetization in the magnetic field direction. So, this has a lower energy.

So, this distance is mu B into B, and this distance is mu B into B, but now it is shifted with respect to 0 is it has lower energy. So, for zero energy, it is starting from minus mu B this one, the one where it is parallel, magnetic moment is parallel to B, it is starting from minus mu B. And this one is starting from plus mu B. The Fermi energy is still here, it is unchanged. This was a Fermi energy; this is the Fermi energy.

But now, this set of electrons which have magnetic moment in the minus have all shifted in the positive direction. So, this it is equivalent to shifting all of this by plus mu B. So, what has happened is that these electrons have momentarily gone above the Fermi energy because you can imagine this just by moving it by shifting it by some amount. And similarly this for the electrons with magnetization parallel to the field direction, these have shifted down, you are not adding or removing any electrons from either you are just moving shifting these around by plus minus mu B. So, equivalently this much of electrons have shifted down. So, these electrons, these extra electrons have gone above the Fermi energy, and this much of electrons are below the Fermi energy by application of the magnetic field ok.

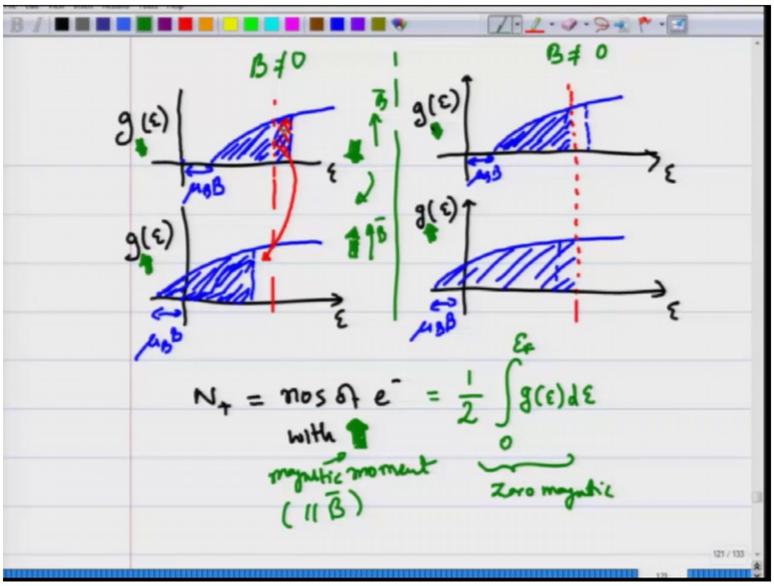
So, now, what will these, these are electrons at the higher energy state. They have higher energy state. How will they come to the ground state, and these are lower energy states these electrons will flip and come orient themselves and get align parallel to the magnetic field direction. So, only these extra electrons, these extra electrons which are sitting above the Fermi energy when you apply the magnetic field because of the shift in this energy which is generated by the magnetic field, these extra electrons which get excited above the Fermi energy are going to and which are actually aligned opposite to the field direction.

These electrons will actually flip and come to a direction which is parallel to the field direction; these are the magnetic moments which are align parallel to the field direction. So, these extra electrons will get generated parallel to the field direction, rest of the electrons do nothing, it is just these extra electrons which go from this higher energy state and get two vacant states which are available. This is the only way vacant states are generated and so they go from higher energy state to the lower energy state.

So, the electrons which were anti parallel to the field direction, the magnetic moments which are near the Fermi energy for those electrons which are near the Fermi energy which were anti-parallel to the field direction if this is the field direction these were the ones which are in the higher energy state, they flip and go into a direction which is parallel to the field.

So, this is through this process, and these are the electrons which are generated near the Fermi energy which are contributing to the magnetization. These are getting aligned in the field direction. And it is only these electrons which are getting aligned this process of moving from here to here is causing the alignment of the magnetic field.

(Refer Slide Time: 07:31)



So, if I just re express this feature once again that you have this is the density of states for the magnetization oriented opposite to the magnetic field. And this is the density of states for those electrons oriented parallel to the density of parallel to the direction of the magnetic field ok. And this is your Fermi level. So, the electrons some set of electrons get shifted above the Fermi level, and some set of electrons and equivalent number of electrons are shifted down below the Fermi level. And these electrons are the ones which will flip and go from this state to this state.

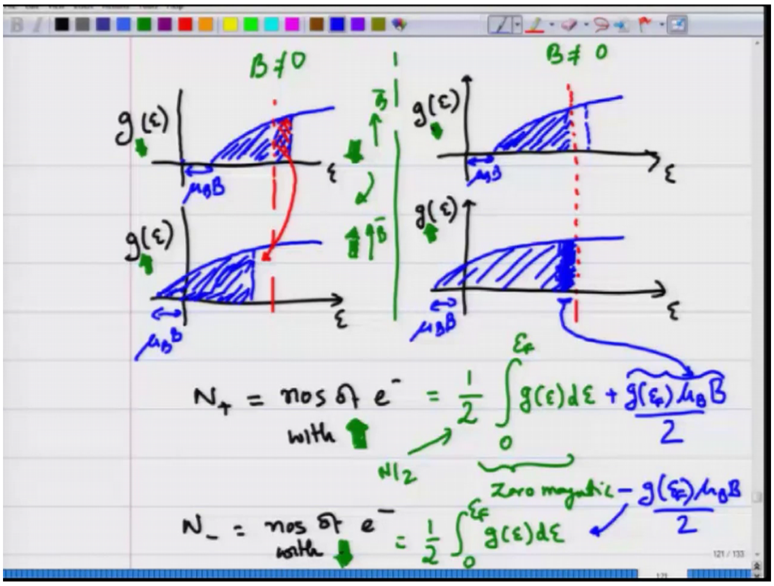
And as a result in a magnetic field, this is all in a nonzero magnetic field. So, very soon after this flip has occurred namely those moments which are aligned opposite to the magnetic field direction will flip and go into a state which is align parallel to the field direction. This flipping process is only for those electrons which are near the Fermi level. So, we very soon you will enter into a state which is this in a magnetic field.

Let me draw that. So, these extra set of electrons have now moved from here and occupied this. This extra set of electrons have gone from there and occupied this entire space. So, so after this flipping has occurred, after this flipping is over, these extra electrons which were present here have left this place and moved here, so that the Fermi energy level remains at the same location.

Otherwise, momentarily these electrons since they had occupied higher energy, the Fermi level locally for these electrons was higher than the Fermi level for this. Now, to maintain equilibrium again you can think that these electrons will flip, so that these electrons are moving from here from the higher energy state to the vacant lower energy states, so that the Fermi level again comes down and becomes equal ok. And this process of flipping where the oppositely oriented magnetization aligns in the magnetic field direction gives rise to the net magnetization inside the sample.

So, how many are these electrons? Let us calculate n plus is the number of electrons with magnetization aligned in the field direction namely magnetization up. My field is always in the plus up direction. So, n plus represents the number of electrons with magnetization or magnetic moment, this is the magnetic moment parallel to magnetic field, this will be equal to half zero to E f g of E d E with respect to the number of electrons which are present with respect to the number of electrons which are present in zero magnetic field. This is in zero magnetic field. The number of electrons with magnetic moment up with respect to this, this is the extra set of electrons; and that extra set of electrons is nothing else, but since only those electrons which are near the Fermi surface are contributing to this excitation.

(Refer Slide Time: 13:31)

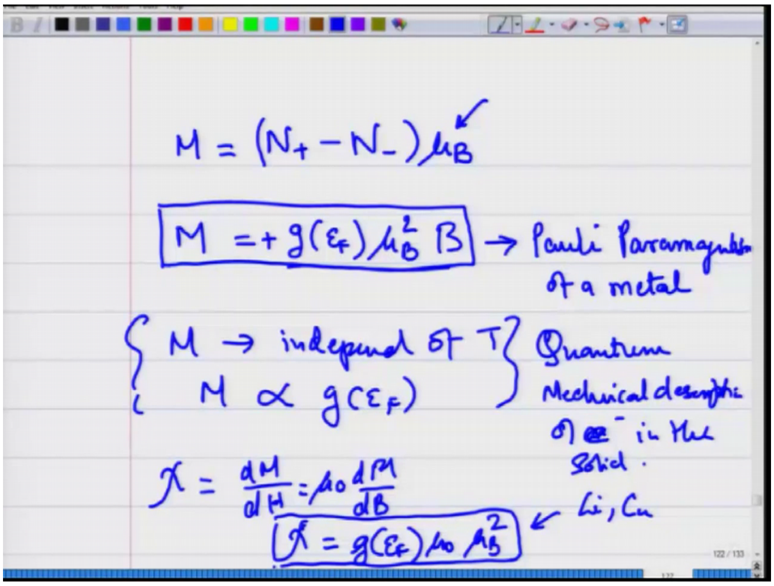


So, I will write this additional term as whatever is the extra, this extra is coming from the density of states, these extra electrons which are excited above the Fermi energy, these are the extra set of electrons which is half the density of states into mu B into B, because half is because spin half and spin down particles they are equally distributed. So, density of states into this energy interval divided by 2 will give you this extra set of electrons which are coming and adding out here.

And similarly this many number of electrons are being lost from the density of states for the down electrons with magnetic moment down. So, n minus is the number of electrons with magnetization pointing downwards with magnetization which is pointing in the downward direction with respect to the magnetic field. And this is equal to again half 0 to E f g of E d E, this is nothing else but n by 2. Similarly, this is also n by 2, the total number of electrons all of this is at zero temperature. So, this is the total number of electrons.

And with respect to the total number of electrons, this many number of electrons is lost, so that is going to be minus of g of E f density of states at the Fermi energy into the energy interval which is because they have all moved by mu B into B divided by 2. This factor 2 again is first, it is equal number of particles; it is equal for spinoff and spin down particles ok. So, this is your energy which is coming out here sorry I have written it slightly this is this.

(Refer Slide Time: 15:39)



Now if you want to get the total magnetization, the total magnetization which you get is N plus minus N minus into mu B. Each electron has a magnetic moment mu B, this is the magnetization of each electron. And each and this is the number of electrons with magnetization parallel to the field direction, this is the number of electrons with magnetic moment opposite to the field direction.

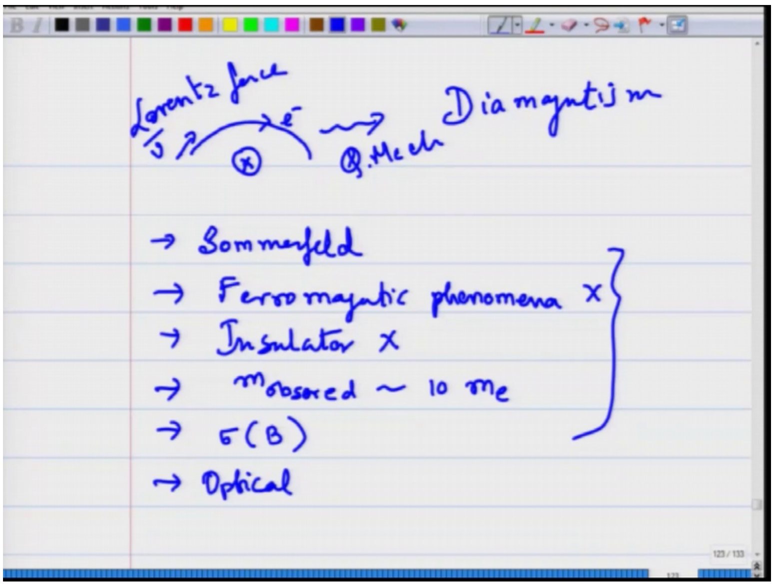
And if you use the earlier expression, it is very easy to show that the first term cancels out. This is nothing else, but N by 2, N by 2 for both the electrons. And this will give you the density of states at the Fermi energy into mu B square B; this is your total magnetization, the net magnetization. And the net magnetization contribution is coming from these electrons which had higher energy de exciting and coming to this lower energy state which is associated with the moments flipping.

The magnetic moments which were opposite to the magnetic field direction flip and go in the lower energy state. This is the lower energy state with the magnetic moment is parallel to the field direction and that happens in a when you apply a magnetic field. And these are the extra electrons which get aligned in the field direction which gives to the net magnetization, which gives you the net magnetization. And this is what is called as Pauli paramagnetism of a metal.

The magnetization is independent of temperature the magnetization is proportional to the density of states at the Fermi level ok. And the entire process is based on quantum mechanical description of electrons in the solid. It cannot be explained through any classical picture you have to invoke the quantum mechanical description. The magnetization is positive ok.

And if you, if you want, if you wish you can also find the susceptibility which is d M by d H which is mu naught d M by d B, and this will give you the susceptibility of a metal will be for a paramagnetic metal will be mu naught which is the permittivity into Bohr magnetron square ok, this will be your Pauli paramagnetic susceptibility. One can also describe in all of these description and this is what has been found for very simple metals like lithium and copper, the magnetic susceptibility if you measure it turns out to be somewhat close to these values. So, there is a degree of confirmation that the system you can actually describe it using this quantum mechanical description.

(Refer Slide Time: 19:11)



However, in all of these descriptions, I have not considered that in a magnetic field the trajectory of electrons also can become circular ok. And this leads to if you do a quantum mechanical description of this, then this will explain the phenomena of diamagnetism ok. So, I will not get into this, but you will include if you would take into account the effect of the Lorentz force which acts on the electron placed in a magnetic field, then the electron will take a curve trajectory for a given velocity, this electron will take some curve trajectory and this leads to diamagnetic phenomena.

So, paramagnetism is getting magnetize diamagnetism opposes the application of magnetic field ferromagnetism is a little bit more complicated. It is much more involved. And you cannot really explain it based on just the Sommerfeld's model. And this finally, leads us to the drawbacks of the Sommerfeld's model.

So, some of the drawbacks is that ferromagnetic phenomena cannot be explained through Sommerfeld's model. Insulators cannot be explained, how do you get insulators inside for a given material, how does it become insulating cannot be explained that the electron observed electron mass inside the solid can be 10 times the mass of the electron free electron mass is another drawback of the Sommerfeld's model. The magnetic field dependence of the conductivity of the material not the Hall Effect, but just the longitudinal conductivity or the resistivity and its dependence on magnetic field is not explained through the Sommerfeld's formalism.

So, while the Sommerfeld's formalism had many successes which we saw in terms of understanding the behaviour of the conductivity, the Fermi velocity, the concept of a Fermi energy, density of spades, specific heat of the electron gas, thermal conductivity, paramagnetism, it has its drawbacks and some of these drawbacks will be.

This is only some of the drawbacks another drawback is the optical behaviour of the metal, what is the optical response of the metal at different frequencies, it absorbs some frequencies, does not absorb some frequencies, these also are not completely explained by the Sommerfeld's theory. So, with this we sort of conclude our discussion of the Sommerfeld's description of an electron in a solid.