Introduction to Solid State Physics
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Lecture – 25
Introduction to magnetism in metals Part-II

We had begun of our discussion on magnetism.

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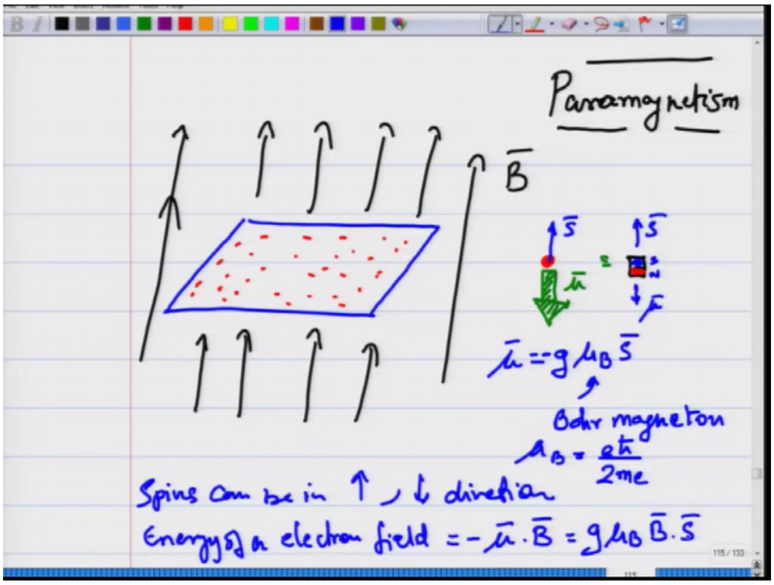

$\bar{\mu} = -g\mu_B \bar{S}$

$\mu_B = \frac{e\hbar}{2m_e}$

Spins can be in ↑, ↓ direction

Energy of a electron field $= -\bar{\mu}\cdot\bar{B} = g\mu_B \bar{B}\cdot\bar{S}$

And, we wanted to discuss or explain the phenomena of paramagnetism using Somerfield’s theory. And what is paramagnetism? Paramagnetism is when a material gets magnetised when placed in an external magnetic field. So, here you have a solid within which you have electrons, you have some metal with electrons inside it and you are placing it in a magnetic field. So, how does this material develop a magnetization that is the question that we are going to ask.

So, fundamentally how does a material develop magnetization, it comes from each electron. So, if you recall from your studies of quantum mechanics that each electron has a spin associated with it which can be pointing up or down and along with this spin you can have also a magnetic moment which is associated with this spin. So, each electron not only has a spin associated with it, it also has a magnetic moment. Each electron you can think of like a small bar magnet.

So, it is like a bar magnet which has a north pole and a south pole. So, the north pole and this is the south pole. So, this is the south pole and this is the north pole where this magnetization or the magnetic moment associated with each electron is opposite to the direction of the spin. So, spin is in this direction, the magnetic moment or the magnetization of this electron is opposite to the spin direction and mu you know is g mu B into S, where mu is the Bohr magneton, g is equal to minus the magnetic moment is minus g into mu B into S because, it is opposite to the direction of the spin and mu B is a h cross by the bore magnetron is e h cross by 2 m e.

So, this is your Bohr magneton and this is the expression for the magnetic moments. So, each electron in the metal is like a small bar magnet which you have placed in a magnetic field and there are both types of spins. Spins can be in up and down direction and what is the energy of a electron in a magnetic field, that is equal to minus mu dot B. If B is the magnetic field then the energy of the placing this magnetic moment in the magnetic field is minus mu dot B which is equal to nothing else, but you can just substituted it here it is g mu B B dot S the dot product of the magnetic field with the spin, ok.

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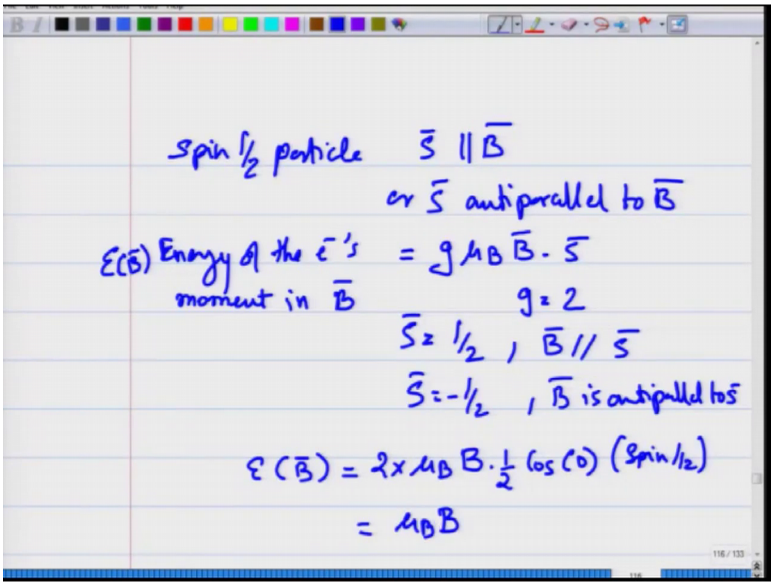

spin 1/2 particle $\bar{S} \parallel \bar{B}$

or $\bar{S}$ antiparallel to $\bar{B}$

$\mathcal{E}(\bar{B})$ Energy of the $\bar{e}$'s moment in $\bar{B}$ $= g \mu_B \bar{B} \cdot \bar{S}$

$g = 2$

$\bar{S} = 1/2$, $\bar{B} \parallel \bar{S}$

$\bar{S} = -1/2$, $\bar{B}$ is antiparallel to $\bar{S}$

$\mathcal{E}(\bar{B}) = 2 \times \mu_B B \cdot \frac{1}{2} \cos(0)$ (spin 1/2)

$= \mu_B B$

And so, for a spin half particle; for a spin half particle, the spin can be either aligned parallel to the magnetic field or spin can be anti parallel to B. So, the energy of the electrons moment in magnetic field can be g mu B B dot S, when S is half g for an electron is approximately 2 when S equal to half, for S is equal to half the magnetic field is parallel to the spin, for S is equal to minus half the magnetic field is anti parallel to spin.

So, let us substitute for spin half the energy. Let us write this as the energy of the magnetic moment in a magnetic field. The energy of the electron in a magnet field is 2 into mu B B into half cos 0 for spin half particle and this is equal to mu B B.

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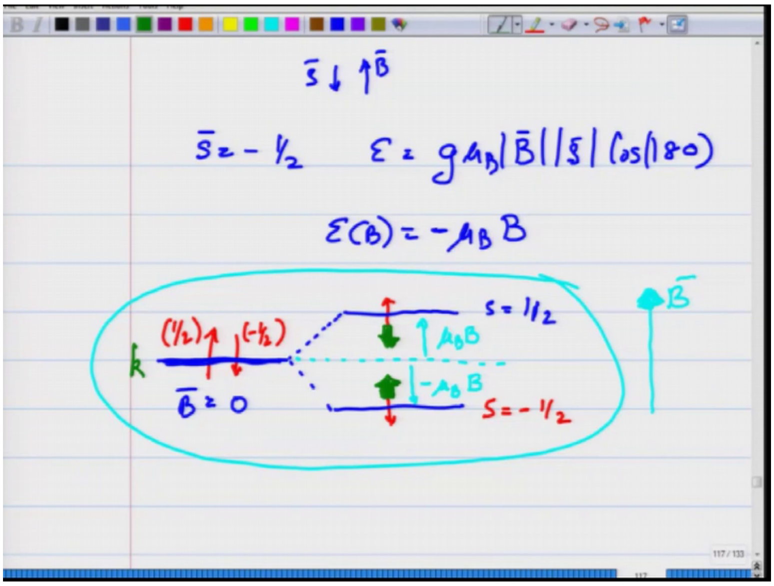

$$\bar{S} \downarrow \uparrow \bar{B}$$

$$\bar{S} = -1/2 \qquad \mathcal{E} = g\mu_B |\bar{B}||\bar{S}| \cos(180)$$

$$\mathcal{E}(B) = -\mu_B B$$

And, for an opposite spin particle for spin S is equal to minus half the energy is g mu B B the magnitude of the field S cos 180 because, the spin if B is the magnetic field the spin is in the minus half direction the opposite direction. So, B dot S is nothing else, but this will give you minus mu B into B.

So therefore, in the absence of a magnetic field for B is equal to 0, the spin plus half and minus half, the spin plus half and minus half were both occupying the same energy state. Suppose, k is the momentum of that state both of them were occupying the same energy state these are the two electrons. However, in the presence of a magnetic field now the energy is become very different. The spin half particle goes into the higher energy state and the spin S is equal to minus half goes into the lower energy state.

So, this is minus mu B in to B the Bohr magneton into B and this goes up by plus mu B into B and this is how an electron with spin degree of freedom or when you introduce the spin into the considerations this is what happens to the electron. Let us take this as the magnetic field direction B; B this is the direction of the magnetic field. If I draw the momentum of the particle, if I draw the magnetic moment of the particle for the spin half particle this is the direction of the magnetic moment. For the spin half particle the magnetic moment is opposite to the direction of the magnetic field it has a higher energy and for the spin down particle this is the direction of the magnetic moment which is in the direction of the magnetic field. So, it is parallel.

So, the magnetic moment which I have shown you here as dark green is for the spin minus half is actually parallel to the direction of the magnetic field and it has lower energy expected. But, the spin down particle its magnetic moment is oppositely aligned to the magnetic field direction. So, it has a lot of energy to maintain this configuration it requires a lot of energy. So, this is the difference that happens the moment you include the spin degree of freedom.

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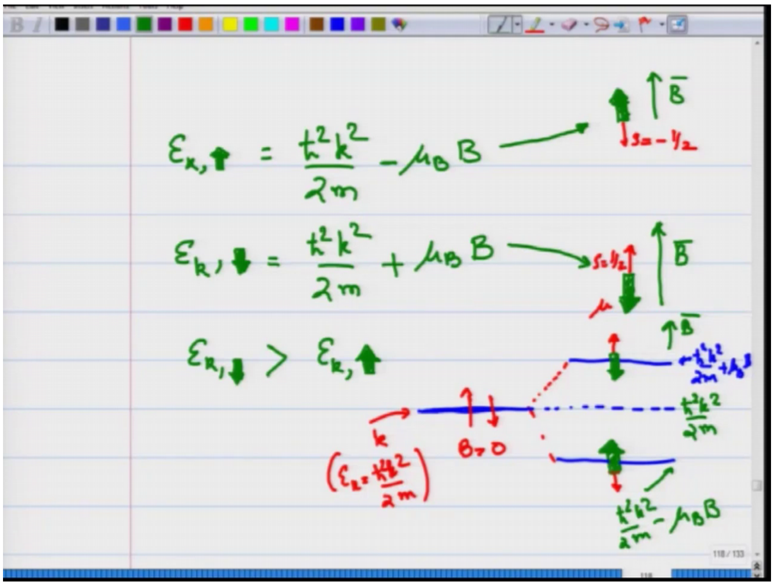

$$\varepsilon_{k,\uparrow} = \frac{\hbar^2 k^2}{2m} - \mu_B B$$

$$\varepsilon_{k,\downarrow} = \frac{\hbar^2 k^2}{2m} + \mu_B B$$

$$\varepsilon_{k,\downarrow} > \varepsilon_{k,\uparrow}$$

So, what is the energy of the particle and now, instead of talking in terms of spins I will start talking in terms of magnetic moments. You can do either ways, but let us write that if the particle is in a state k with magnetic moment spin up then this particle has energy. So, by spin up I mean or the magnetic moment is up direction it is parallel to B ok, namely its electron is in the spin S is equal to minus half. So, for this state its energy is lowered because, this spin is parallel to the magnetic field direction.

So, the energy for this is written as h cross square k square by 2m minus mu B into B; where B is the strength of the magnetic field that you have applied. And for the electron with momentum k, but in the magnetic moment down direction for the magnetic moment opposite to the direction of the magnetic field, the energy is h cross square k square by 2m plus mu B into B. This is for the case, this was for the case where the magnetic moment is parallel to the direction of the magnetic field. This is for the case where the magnetic moment is anti-parallel. The spin is the plus half case. The spin is plus half, this is the magnetic moment of the particle which is in the opposite to the direction of the magnetic field that has a higher energy. So, E k anti parallel to the B direction has energy which is greater than E k parallel to the B direction.

So this is exactly where we had drawn this picture that for a given k state is just restating this the k state for B equal to 0, this case state has an energy E k h cross square k square by 2m which is identical for both the spins because, then you do not have this term. But, the moment you apply a magnetic field this state splits with respect to h cross square k square by 2m. This is the higher energy state, this is the lower energy state with respect to this, this state has energy h cross square k square by 2m plus mu B into B. This is the spin up electron or the magnetic moment, this is a spin up electron or it has a magnetic moment which is pointing down opposite to the magnetic field direction if B is the direction in this.

And, similarly the down state has spin down, but the magnetic moment is pointing up and this has got an energy which is h cross square k square by 2m minus mu B into B and this is h cross square k square by 2m. So, with respect to this h cross square k square by 2m level, the up and down moments are shifted in a presence of a magnetic field. And, this we will try and analyse and this is true for all electrons inside the solid because all the electrons inside the solid are the small bar magnets which have a magnetic moment mu B, and it can be either in the direction of the magnetic moment or opposite so their energies are split.

So, now let us look at what happens to the behaviour of these electrons in a magnetic field. Now, certainly you know that when you place the system in a magnetic field some electrons gain energy some electrons are going to lose energy, ok.

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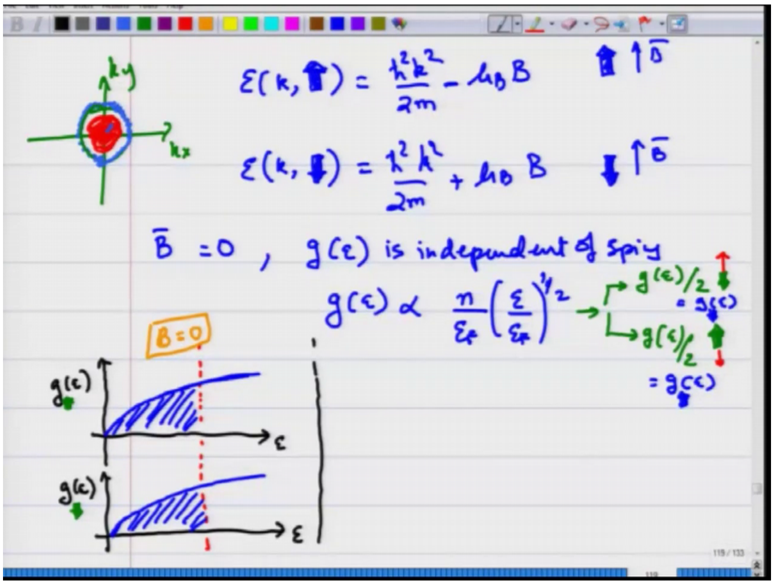

$$\varepsilon(k,\Uparrow) = \frac{\hbar^2 k^2}{2m} - \mu_B B \quad \Uparrow \uparrow \bar{B}$$

$$\varepsilon(k,\Downarrow) = \frac{\hbar^2 k^2}{2m} + \mu_B B \quad \Downarrow \uparrow \bar{B}$$

$\bar{B} = 0$, $g(\varepsilon)$ is independent of spin

$$g(\varepsilon) \propto \frac{n}{\varepsilon_F}\left(\frac{\varepsilon}{\varepsilon_F}\right)^{1/2} \rightarrow \begin{cases} g(\varepsilon)/2 = g_{\Downarrow}(\varepsilon) \\ g(\varepsilon)/2 = g_{\Uparrow}(\varepsilon) \end{cases}$$

So, the again from Somerfield's theory electrons which are very far away from the Fermi energy will not contribute because there are no states for them to go into. So, if you have a Fermi space k x, k y electrons which are very far away from electron, which are very far away from the Fermi surface these electrons will not be able to cause go to higher energy states or lower energy states because these are already completely occupied. Only those electrons which are present near the Fermi energy these are the electrons which will contribute to the entire process. So, these are the electrons which will contribute to getting excited about the Fermi energy are going below the Fermi energy.

So, we will make the following picture. So, let us use the following information that we have already derived E k for magnetization in the up direction or for a magnetic moment in the up direction. This is the magnetic moment, not the spin. I am describing everything in the magnetic moment up means that this is parallel to the magnetic field direction. My magnetic field is all already in this.

So, this means parallel magnetic moment is parallel which means it has a lower energy. This is h cross square k square by 2m minus mu B in to B magnetic moment parallel to B and E k energy k with magnetic moment anti parallel to B direction is h cross square k square by 2m plus mu B in to B here the magnetic moment is anti-parallel to B direction. So, this is how the energy states are going to shift, ok.

Now, if we look at the behaviour of the density of states for the spins which are either in the up or down state in the absence of a magnetic field for B is equal to 0, the density of states is independent of spins. If you recall that the density of states is proportional to density of electrons n by E f E by E f raise to half, ok. This was the density there is not no component which is related to the spins it just depends on the energy of the electron, ok.

And, so, there is no spin dependent. So, you can consider that the density of states for moments in the up direction. These are composed of this is identical because each state is occupied by two electrons; one spin up and spin down, or magnetic moment up and magnetic moment down, ok. So, this is exactly identical for. So, this is g E by 2 and this is g E by 2 for magnetic moment down and g E by 2 for magnetic moment up which is another words is for first spin up or spin down. It is exactly identical, that is what I wanted to say that the density of states is exactly identical for spin up and spin down electrons.

So, let us draw this density of states first in 0 magnetic field for spin up and spin down electrons. Function of energy, somewhere we have the Fermi energy, this is the density of states for the spin up electrons or one of them magnetic moment pointing up and this is magnetic moment pointing down and the density of states is going to go as E raise to half. These are all occupied states, this is the density of states for spin down, namely this is g of E magnetic moment down and this is the density of states magnetic moment up, ok. And, so, I have just plotted that. Both of them are going to go as E raise to half, and it is equally distributed, half number of particles spin up half number of particles spin down.

Now, how does this situation change when I switch on a magnetic field? This is all at for B equal to 0, this is when magnetic field is 0. Let us look at this in the next lecture when B is not equal to 0, how does this situation change.