

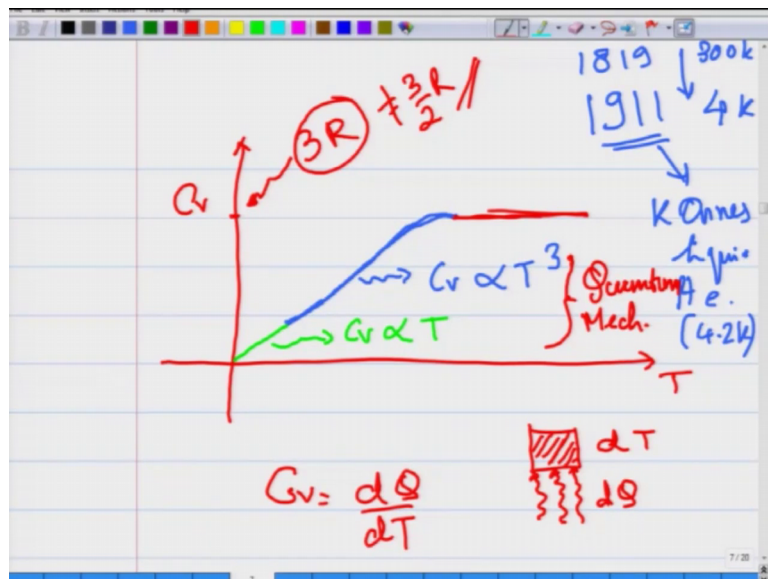
**Introduction to Solid State Physics**  
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**Lecture – 02**

**Introduction to Drude's Theory of electrons in a metal – Part II**

In the last lecture we saw that there was this kinetic theory of gases and we saw that the specific heat of these gases by taking the assumptions of the kinetic theory of gases. We could get a value of  $\frac{3}{2} R$  and the question there we posed is that now from gaseous phase if you bring the particles close together, you will get a solid and if you measure the specific heat of solids which was done by Dulong and Petit in 1819; then he obtained a specific heat not  $\frac{3}{2} R$ , but it was a value of  $3 R$  it was constant.

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And it was  $3 R$  not  $\frac{3}{2} R$ ; so, this was a first sign that something else is happening in the solid which is unlike that of a gas. And what is one of the most obvious things that you can think of? In kinetic theory, the particles are non interacting they are hard spheres, but when you bring the particles close together; they start interacting.

So, you have to bring in the role of interaction that was true and as we will see that that is important. But apart from that there were other things and other mysteries which came

up. Because at that point from 1819 onwards, people started finding ways to lower the temperature of the solid and you could go down by 1911; from 1819 to 1911 from room temperature of 300 K you could go down to 4 K; this was the year in which Kamerlingh Onnes liquefied helium.

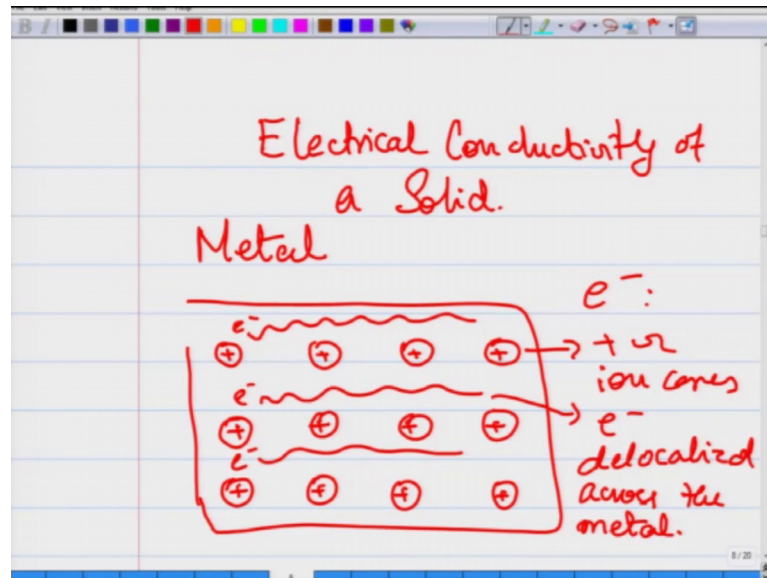
He obtained liquid helium which was at 4.2 Kelvin. So, you could go from room temperature down to 4 K a wide temperature range and people were interested what happens to the properties of the solid. For example, this thermal property which is the specific heat; if we measure it as a function of temperature does it remain constant at  $3R$  or does it vary? And what was found was that over a certain window of temperature; it is constant at  $3R$ , but then it starts varying below a certain temperature, it has a temperature dependence.

And this temperature dependence turns out to be cubic; the specific heat has a cubic temperature dependence. If you go to very low temperatures below 4 Kelvin and as you go lower and lower; which people were able to do then instead of a cubic temperature dependence, you start getting a linear temperature dependence the specific heat becomes linear. So, from a linear specific heat if you are starting from low temperature the specific heat is initially linear.

Then the specific heat develops a curve which has a cubic temperature dependence and then as it goes to high temperature; it becomes constant. So, the thermal property of the solid turns out to be very complex and we will look at all these different regimes in the course. But this is just to motivate you that various effects and phenomena are coming into the picture. And just to tell you that this temperature dependence of the specific heat that you see; you cannot explain it using classical theories. You cannot use, explain it using something like kinetic theory of gases where you consider classical particles.

You have to invoke quantum mechanics to understand these temperature dependences and that will be what will be covered in this course also. So, let us go forward from here; I have talked to you a little bit about and given you a motivation about what happens to thermal properties, when you go to a solid and you see unusual things happen; what about other properties.

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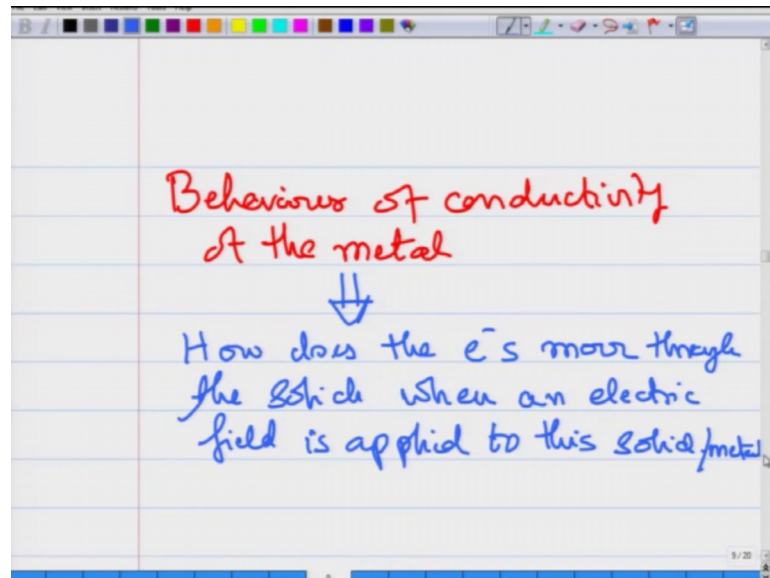


And here we come to the and we will spend some time and effort into trying and understand being the property of this solid; the electrical conductivity of a solid.

Now, a solid has atoms and we will essentially study a metal. So, what is a metal? A metal has atoms which are such that the outermost electron of the atom is; the outermost electron in that atom is loosely bound to that atom and it gives up that electron; the electron is free to wander around inside the solid. So, what you have are positive ion cores and there are these electrons which are wandering across the lattice. And this was sort of known and it was known that inside the metal; the charge conductors are charge carrying particles are electrons.

The metals have weakly bound electrons; the metal ions or the metal atoms have weakly bound electrons; these electrons are delocalized they are sort of not bound to the atom as such, but they spread out across the solid and they wander around the solid. So, this is the model of a metal you have positive ion cores and you have electrons which are delocalized across the metal.

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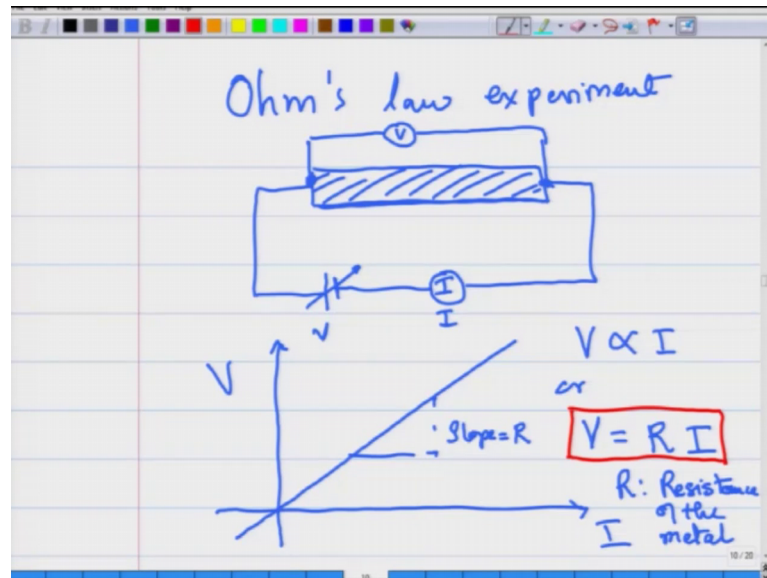


And people wanted to understand the behavior of conductivity of the metal; namely what they wanted to know was how does the electrons move through the solid, when an electric field is applied to this solid or a metal?

So, the question was that you have electrons inside the metal which are the charge carriers; how do these electrons actually move through the metal? Can we understand that? And this was one of the very important topics in the early 1800s and which continues; I mean part of the theories which were developed here approximately they are still valid and people still use it to a good amount.

And this was an important part of solid state physics to understand the behavior of electrons moving through the metal. How does it conduct electricity?

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So, the earliest experiments which wanted to study this was your very well known Ohm's law experiment. What was the Ohm's law experiment the Ohm's law experiment was quite reasonably simple. Say you take a piece of metal; it could be a piece of copper or whatever silver, aluminum whatever.

You connect to leads to the ends of this piece of metal; you connect it to a battery and when you connect it to a battery, you can change the voltage which is given by the battery and you can measure how much is the voltage drop across the metal and how much is the current I which is flowing through the metal.

As you change the voltage; as you apply the voltage to the metal, how does the current through the metal change? And what did Ohm's law find? Ohm's law found, Ohm's found that if you measure the voltage as a function of currents; then the behavior is linear. The voltage and a current are directly proportional to each other or the voltage is written as  $R$  into  $I$ , where  $R$  is the resistance of the metal; the slope of this curve is equal to  $R$ .

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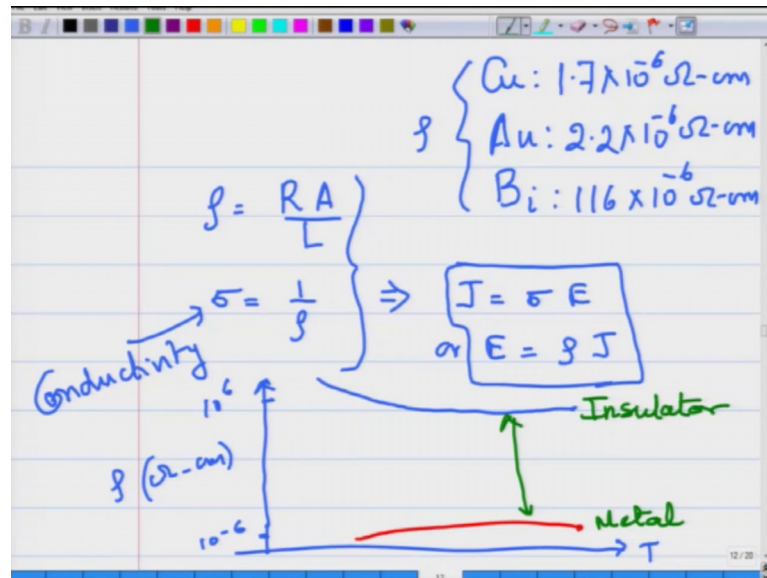
The image shows a digital whiteboard with handwritten notes in red and blue ink. At the top, the equation  $V = RI$  is written in red, with a red arrow pointing down from the  $I$  term. To the right, it says "E : electric field" and "J : Current density." Below this, the equation  $E = \frac{V}{L}$  is written in red, with a blue arrow labeled  $I$  pointing to the right above a red rectangle representing a wire. The width of the rectangle is labeled  $A$  and its length is labeled  $L$  with a red double-headed arrow below it. Below the  $E = \frac{V}{L}$  equation, the equation  $J = \frac{I}{A}$  is written in blue. Further down, the equation  $EL = RAJ$  is written in blue. Finally, the equation  $E = \left(\frac{RA}{L}\right)J \Rightarrow \rho = \text{Resistivity of the metal} = RA/L$  is written in blue.

Now, you can quickly sort of convert this Ohm's law expression  $V$  is equal to  $R$  into  $I$  and you can write it in terms of electric fields and current densities. If  $E$  is the electric field and  $J$  is the current density; then you can recast this expression in terms of electric field and current density; where electric field is volts divided by the length of the metal over which the voltage is being applied.

So, suppose you have a wire of length  $L$  if this is a wire of length  $L$  and  $V$  is the voltage applied across this wire then  $E$  is the electric field which is volts per unit length. And if the cross sectional area of this wire is  $A$  and if current  $I$  is flowing across this wire.

Then the current density  $J$  is defined as current per unit area of cross section; where  $A$  is the area of cross section of this wire. And now using the expressions for  $V$  and  $I$  in terms of  $E$  and  $J$ ; you can rewrite the Ohm's law as  $E$  into  $L$  is equal to  $R$  into  $A$  into  $J$  or  $E$  is equal to  $R A$  over  $L$ ; where  $A$  is the area of cross section of the wire,  $L$  is the length and this gives us a quantity which is called the resistivity of the metal or of a material where  $\rho$  is equal to resistance over  $L$ .

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This just comes from the earlier expression and we can define also the conductivity sigma which is 1 upon rho. So, you can rewrite J as sigma times E or E is rho of J.

And these are important parameters which measure it does not these quantities you have already removed the effect of the length and the area of cross section. So, you can compare materials; you can compare different materials although they can have different lengths and different areas of cross section. If you measure their conductivity or their resistivity then which is equal to R A divided by L. And this sigma which is 1 upon rho is the conductivity of the material of the metal. And the important thing about sigma and rho is that; they do not depend upon the geometry of the sample what is the area, what is the length you have already taken into account that effect.

So, you can start comparing different materials and looking at the behavior of this quantity. And it turns out that this resistivity can vary significantly; there is a huge variation in the resistivity. So, just let me give you an example; we can take a look at first the materials like copper which has a resistivity; copper has a resistivity of 1.7 into 10 to the power of minus 6 ohm centimeter. Gold has a resistivity of slightly higher; in fact, this is pure copper it has a very high; its resistivity is very low.

Gold has slightly higher resistivity and if you take let us say bismuth; then the resistivity can jump up typical values are of this order. That if you look at the resistivity of these

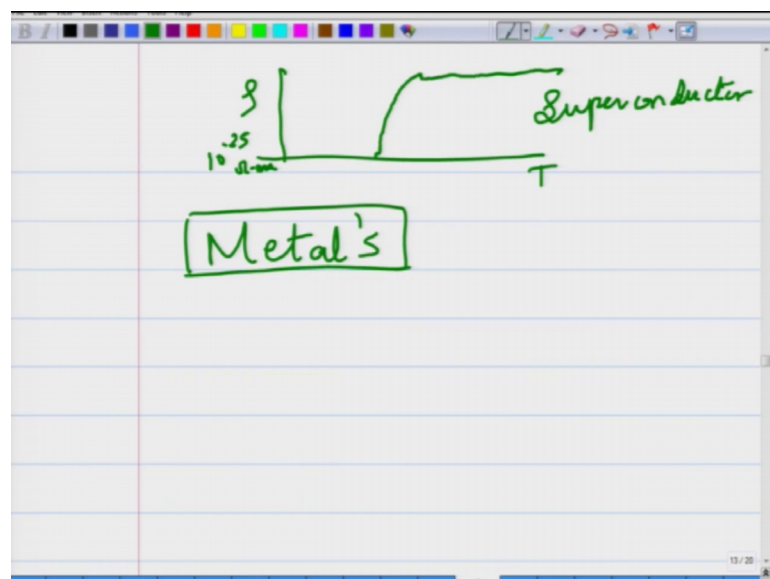
materials these different materials then there is a variation in the resistivity; these are as far as metals are concerned.

But if you go to insulators; then the resistivity could go orders of magnitude high. You can go from values as large as  $10^6$  and the metals will be at  $10^{-6}$ . So, this spans actually orders of magnitude if you go to an insulator; its resistivity could be sitting as  $10^6$  ohm centimeter. Whereas, if you take a metal the metal could be sitting at  $10^{-6}$  ohm centimeter.

And on this axis I have just drawn the temperature and on the temperature scale; the resistivity of an insulator and a resistivity of a metal can behave very differently. If you lower the temperature the resistivity of an insulator can actually show an increase; whereas, for a metal the resistivity can actually decrease. So, you have things like an insulator and you have things like a metal and there is a huge difference between the resistivity why is it so different?

What is it that makes the solids have such huge variations in the properties of the resistivity or even if you think if you want to think in terms of conductivity; they will be correspondingly the huge difference in conductivity. Why does this difference in conductivity come about? It is still an electron which is flowing through the material, but what makes it so different? Furthermore, towards the end of this course you will again take a look at the behavior of resistivity as a function of temperature.

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And there are certain materials whose resistivity could be constant or varying with temperature, but below a certain temperature the resistance drops to almost 0; below measurable limits. The resistivity actually falls below  $10^{-25}$  ohm centimeter and these are called as superconductors.

They are not metals they are not insulators, but they are fantastic conductors with very very low resistivity and the resistivity of these materials can be very low. So, you can have insulators, you can have metals, you can have superconductors, you can have semiconductors all of them have electrons as conductors, but yet the resistivity and the temperature dependence is very significantly.

You will study some of these as we go along, but let us come back to metals and try and understand the behavior of metals. And let us try and understand the conductivity of metals; the electrical conductivity of metals.