Introduction to Solid State Physics
Prof. Manoj K. Harbola
Prof. Satyajit Banerjee
Department of Physics
Indian Institute of Technology, Kanpur

Lecture – 19
Understanding the electronic contribution to the specific heat of a solid Part-I

We had seen in the last lecture, how the conduction process inside a solid was described by Sommerfeld and he described in terms of a drift of the electrons.

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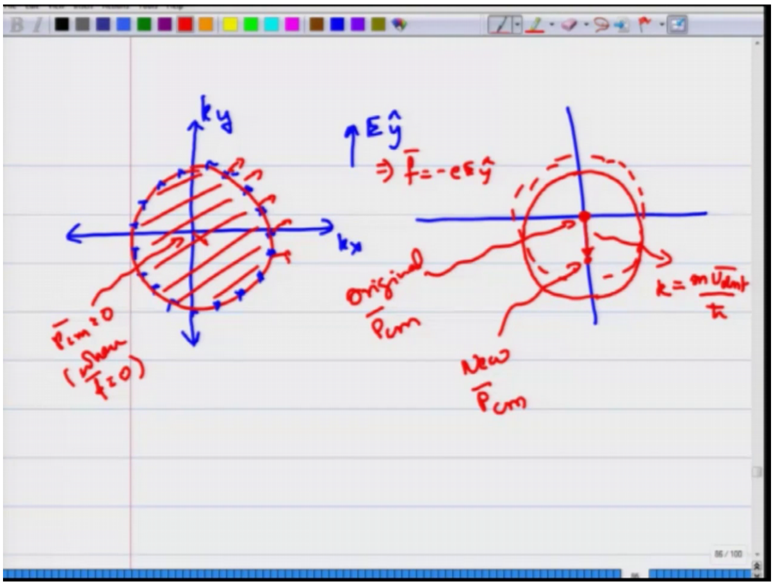

Sommerfeld model gave rise to the concept of a Fermi surface, where all the states below the Fermi surface are completely occupied and only the states which are sitting on the Fermi surface the occupied states and electrons which are sitting on top of the Fermi surface, on the Fermi surface and near the Fermi surface they are the ones which can undergo excitation because there are momentum states which are available for these particles to actually undergo an excitation. There are unoccupied states around the Fermi surface. Whereas, for electrons sitting deep inside these electrons cannot move into any other state because there is no available states, quantum mechanically it is unallowed those are occupied states.

So, only these electrons on the Fermi surface participate in any conduction process or any process relating to excitation of electrons. So, when you apply an electric field or you try to give momentum to the system Sommerfeld said, that the only way you can give or generate momentum in the system is to cause all these particles on the Fermi surface to start moving. If you apply a field in the y-direction they will move in the minus y-direction, but only those electrons which are sitting in the Fermi surface, these are the electrons all these set of electrons will immediately start responding to the electric field because they have states available to go.

But, what is important to take forward is that for any excitation which these electrons are going to experience if you apply an electric field, magnetic field or apply any radiation to the system or apply temperature to the system the electrons get excited, but the Sommerfeld's picture is that only those electrons which are sitting on the Fermi surface will participate in the process. With this concept in mind let us now look at another property of the solid and try and calculate that property.

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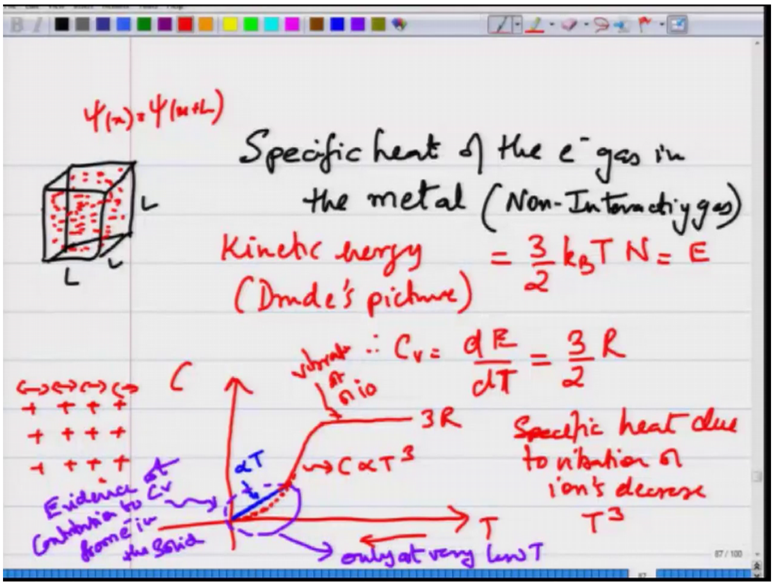

Specific heat of the e^- gas in the metal (Non-Interacting gas)

Kinetic energy (Drude's picture) $= \frac{3}{2} k_B T N = E$

$\therefore C_v = \frac{dE}{dT} = \frac{3}{2} R$

Specific heat due to vibration of ion's decrease T^3

Evidence of contribution to C_v from e^- in the solid

And, that is, let us relook at the specific heat of the electron gas in the metal. It is still a non interacting gas, but within the Sommerfeld's model what is the specific heat of this electron gas inside the metal? So, again you have a metal which has sides of length L and you have the gas of electrons which are present inside this metal and they are drifting as plane waves on which Sommerfeld's applied periodic boundary conditions. These are the electrons inside the metal; they are plane wave states with periodic boundary conditions applied to them. For example, in the x-direction psi x is equal to psi x plus L if you recall which gave us the states, the quantized states which are available for the electrons.

Now, if you consider the electron gas as an classical gas then they have only kinetic energy and from the Drude's picture or model of these classical gases which have this Maxwell Boltzmann distribution their energy in a 3-dimensional solid is 3 by 2 k T into N, where N is the total number of electrons and from that this is the total energy of the system and therefore, the specific heat of this electron gas which is dE by dT the change in energy total energy as a function of temperature dE by dT the derivative of the change in the total energy as a function of temperature is 3 by 2 R. So, this is the energy of the electron gas inside the solid.

However, when people started measuring the specific heat of the electron gas as a function of temperature, first of all the specific heat of the metal at high temperature was not 3 by 2 R, but was 3R. This is something that you will understand later on. This is coming from the ions which are present inside the solid we have not talked about the ions, but those ions are vibrating as you supply heat to the system they absorb energy and they start vibrating and as they start vibrating there is a change in the energy of the system so, that governs the specific heat of the system, ok.

So, there is a contribution to the specific heat which is coming from the ions which are present inside the solid. The positive ions which are present inside the solid, from the vibrations of these ions these ions start picking up energy and they start vibrating and this vibration has got some temperature dependence which goes down as a specific heat changes as T cube. As you lower the temperature of the system, the vibrations become weaker and weaker and there is a specific way with this vibrations decays down and the way it decays down is T cube.

But, as you go to zero temperature the specific heat of the solid finally, vanishes. The specific heat due to vibration of ions decreases as T cube. So, you would have expected that it should have gone down in this manner. The specific heat of the solid should have gone down in this manner, but as you go to lower and lower temperatures the vibrations decrease and you think that the specific heat of the solid which is coming from the vibration of the ions is going to decay as T cube. But, what you find is that below a certain temperature the specific heat instead of decaying and going to 0 as T cube actually begins to change linearly.

And, this linear regime where the specific heat behaves linearly with temperature rather than going down and vanishing to 0 as T cube, this was shown to be evidence of contribution to specific heat from electrons in the solid. The presence of the electrons in the solid actually contribute to the specific heat and the contribution of these electrons to specific heat is shows this linear temperature dependence.

So, this linear temperature dependence has nothing to do with the vibrations of the solid. The heat that you give into the system the energy of the system does not change because you have these solid atoms which are picking up the energy and vibrating and contributing to the specific heat, rather it is the energy which is being picked up by the electrons inside the solid when you give heat to the system that is contributing to the specific heat. You see it is effect only at very low temperatures. Remember that this is only at very low temperatures.

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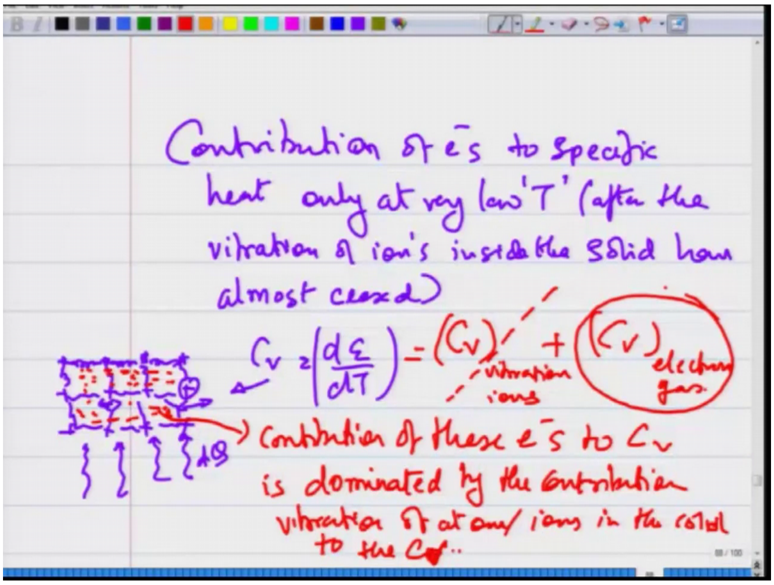

Contribution of $\bar{e}$'s to specific heat only at very low 'T' (after the vibration of ion's inside the solid have almost ceased)

$C_v = \left(\frac{d\varepsilon}{dT}\right) = (C_v)_{\text{vibration ions}} + \boxed{(C_v)_{\text{electron gas}}}$

Contribution of these $\bar{e}$'s to C_v is dominated by the contribution vibration of atoms/ions in the solid to the C_v..

So, one sees the contribution of electrons to specific heat only at very low temperatures after the vibration of ions inside the solid have almost ceased. So, only after the vibration of ions which are present inside the solid have completely ceased only then you see the contribution of electrons. And, this is important because the ions are these positive ions are taken like as if they are connecting with springs.

So, when you give heat to the system if you give heat energy to the system dQ, these ions will start vibrating. All these ions will start vibrating and shaking. So, they will pick up the energy their energy will increase and there will be a specific heat which is the change in the energy where is a function of temperature which is being governed by the vibration of these ions inside the solid. So, although you will have electrons which are present inside this solid, the contribution of these electrons to specific heat is dominated by the contribution of vibration of atoms or ions in the solid to the specific heat.

So, C v has two contributions, C v has one contribution from vibration of ions in the solid and one contribution from the electron gas. And, only at low temperature when you can eliminate the effect of this vibration of ions do you start seeing the effect of this electron gas and this is what is shown here is that this temperature dependence of the specific heat is all due to the vibration of ions.

This is all related to the vibration of ions and only below at very low temperatures which are well below 4 Kelvin and so on very close to absolute 0, if you do the measurements or specific heat where the ions have almost become rigid they are no longer fluctuating and vibrating about, there you start seeing the effect of the specific heat of the electron gas inside the solid because you have eliminated the effect of specific heat of vibrations.

The thermal energies are too low to cause vibration of these heavy objects, only the electron gas responds to these vibrations.

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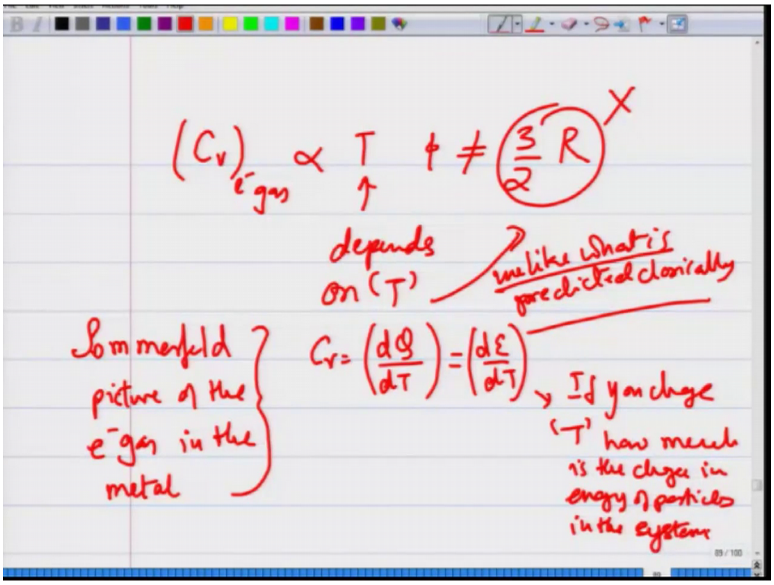

$$(C_v)_{e^-gas} \propto \underset{\uparrow}{T} \neq \boxed{\frac{3}{2}R} \; X$$

depends on 'T'

unlike what is predicted classically

Sommerfeld picture of the e^- gas in the metal

$$C_v = \left(\frac{dQ}{dT}\right) = \left(\frac{dE}{dT}\right)$$

If you change 'T' how much is the change in energy of particles in the system

And, what you see is that the specific heat of the electron gas is proportional to temperature and is not 3 by 2 R. It is clearly depends on temperature unlike what is predicted classically.

So, now, how do we understand this specific heat of the electron gas and how does it develop a temperature dependence? Let us try and understand that and that comes from again Sommerfeld's picture of the electron gas in the metal. What is specific heat? Specific heat measures the when you supply a dQ amount of heat to the system the change in temperature or if you change the temperature of the system how much is the change in the energy of the particles inside the system. If you change temperature, how much is the change in energy of particles in the system.

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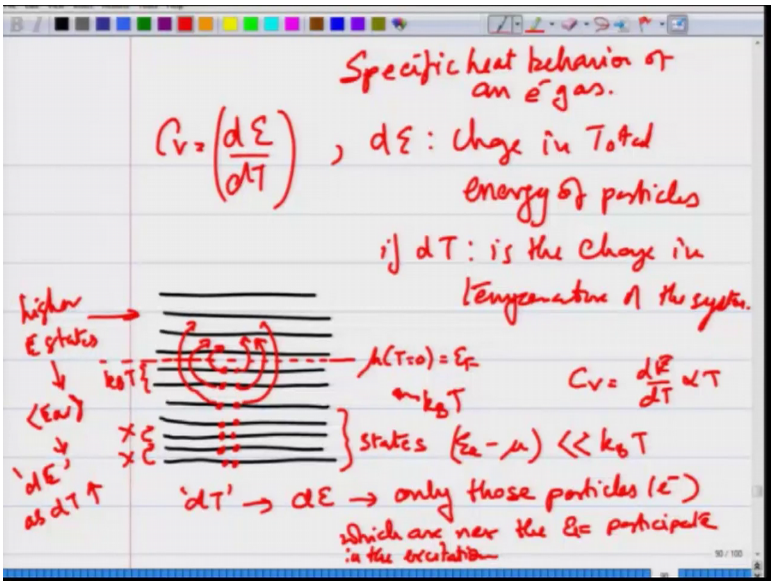

Specific heat behavior of an e^- gas.

$C_v = \left(\frac{d\mathcal{E}}{dT}\right)$, $d\mathcal{E}$: Chage in Total energy of particles if dT : is the Change in temperature of the system.

$C_v = \frac{d\mathcal{E}}{dT} \propto T$

'dT' → $d\mathcal{E}$ → only those particles (e^-) which are near the $\mathcal{E}_F$ participate in the excitation

So, C v is the change in energy when you change the temperature of the system by dT; d E is the change in total energy of particles if dT is the change in temperature of the system.

So, where from is the specific heat coming in the presence in a solid and that is very easy to understand. If you consider the Sommerfeld's picture, the Sommerfeld's picture speaks of energy states, by periodic boundary conditions we have energy states inside the solid, and these energy states are occupied the filling is by Pauli's exclusion principle. So, depending on the total number of particles inside the solid you will fill up a set number of states, and this is where they are Fermi energy or the chemical potential at 0 temperature is the chemical potential at 0 temperature is the Fermi energy which is close to the maximum energy the particles have.

So, the particles have keep on occupying states until they are very close to the Fermi energy of the system the boundary between the occupied and unoccupied states. Now, in this situation if you apply a k B T if you apply heat to the system if you supply heat and k B T to the system states whose E k minus mu is much much less than k B T, states whose energy E k from the Fermi energy if this distance of these states from the Fermi energy is much much less than k B T then k B T will not be able to cause any excitation of these particles.

Finally, if you change the temperature to change the total energy you have to cause some excitation they have to go to higher energy states. But, these excitations for the states which are very far away from the Fermi energy or deep inside the system away from the Fermi energy. These states are completely occupied. You cannot cause any excitation for these states only. The states which are within k B T of the Fermi energy these states can be excited and taken to higher energy states and only these are the ones which can absorb energy and can change.

So, because of excitation of particles which are around the Fermi energy when you change the temperature, the total energy of the particle changes and only those particles or electrons which are near the Fermi energy participate in the excitation. Higher the temperature more will be the number of particles then you can cause to excite because this k B T then starts increasing, because as you increase the temperature you start accessing more and more states which are deeper inside the Fermi energy to actually excite to higher and higher states inside the system. And, therefore, the specific heat becomes a function of temperature.

The specific heat which is the rate of change of energy as a function of temperature becomes proportional to temperature, because as you increase the energy you start accessing more and more states below the Fermi level which can be excited to above the Fermi level. So, they absorb energy and they go to higher energy states and as they go to higher energy states the mean average energy of the system increases and.

So, there is a total increase in the dE there is a net increase in the energy as dT increases. So, as a function of temperature your specific heat will increase because you can cause more and more excitations into the higher energy states and this is the basic understanding of the specific heat behavior of an electron gas within the Sommerfeld's model.

We will next show how do you get this linear temperature dependence of the specific heat of the gas.