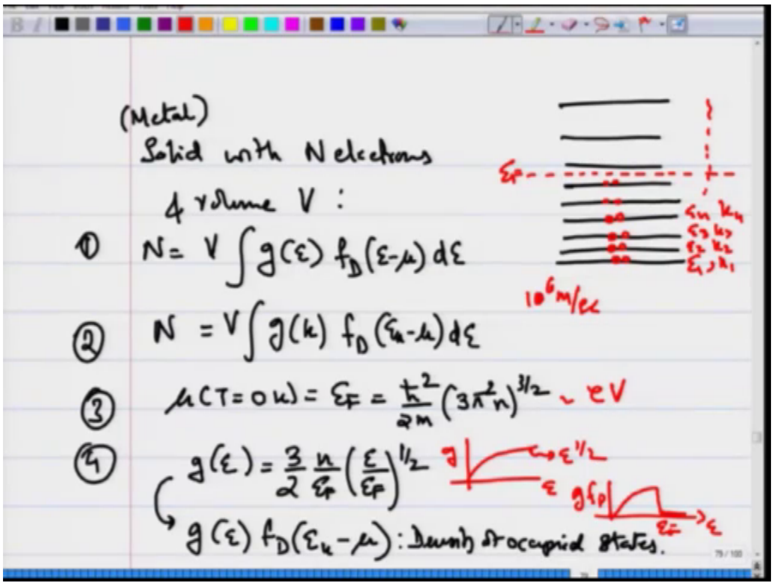
Introduction to Solid States Physics
Prof. Manoj K Harbola
Prof. Satyajit Banerjee
Department of Physics
Indian Institute of Technology, Kanpur

Lecture – 18
Summary and Discussion of Sommerfeld's Model

In Sommerfeld's model; so we have seen the Sommerfeld's model and unlike the Drude's theory which considered the electrons as classical particles, the Sommerfeld's model considers the electrons as quantum objects which follow quantum statistics. The distribution is not like Maxwell Boltzmann distribution and for the electrons inside the solid using periodic boundary conditions you get the electrons which are distributed across quantum states.

(Refer Slide Time: 00:47)



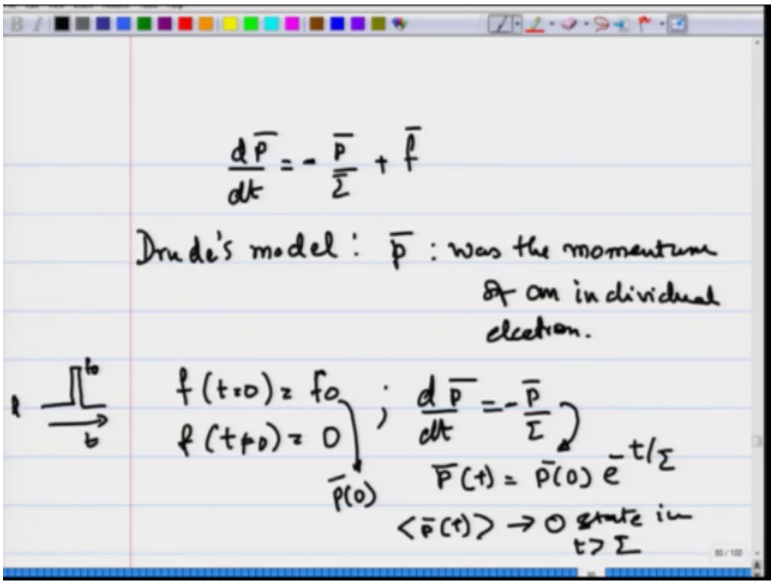
And these states could be labeled by energy E 1 E 2 E 3 E 4 and so on or moments as k 1 k 2 k 3 k 4 and so on; as you go along. And with this idea you can calculate the total number of particles which are there inside the system. The total number of particles for a sample of volume V is equal to V integrated over the density of states into the probability of occupying a given state integrated over all energies ok. This gave us an idea or a way to actually calculate what is the chemical potential of the system at 0 temperature.

And the chemical potential of the system at 0 temperature is the Fermi energy and this Fermi energy comes out not to be a function of the temperature. Temperature has no role to play; the Fermi energy is quite high at 0 temperature of the system is quite high and it depends on the density of electrons because of Pauli's explosion principle each state is occupied by only 2 electrons. So, the electrons every electron that you add to the system keeps on filling up until you reach N number of electrons.

And this N number of electrons actually takes you to a very high energy state which is the Fermi energy which is typically of the order of few in electron volts. And so the particles which have the highest energy are traveling with pretty high velocities of the order of 10 raised to 6 meters per second we have looked at all of this. And because of this distribution and the electrons are occupying different states, we have the concept of density of states. Namely how many states are available per unit interval of energy per unit volume at a given energy E. And we showed that it is proportional to E raised to half the density of states as a function of energy goes as E raised to half.

And there is a related concept that out of these states how many are occupied? We multiplied by the Fermi Dirac distribution and you will get the density of occupied states which at 0 temperature will look something like this; the density of occupied states as a function of energy where. So, these are the concepts which actually govern the behavior of electrons inside a solid, the quantum mechanical behavior of electrons. Now within this concept how do you describe the conductivity of the electrons inside the solid?

(Refer Slide Time: 04:00)



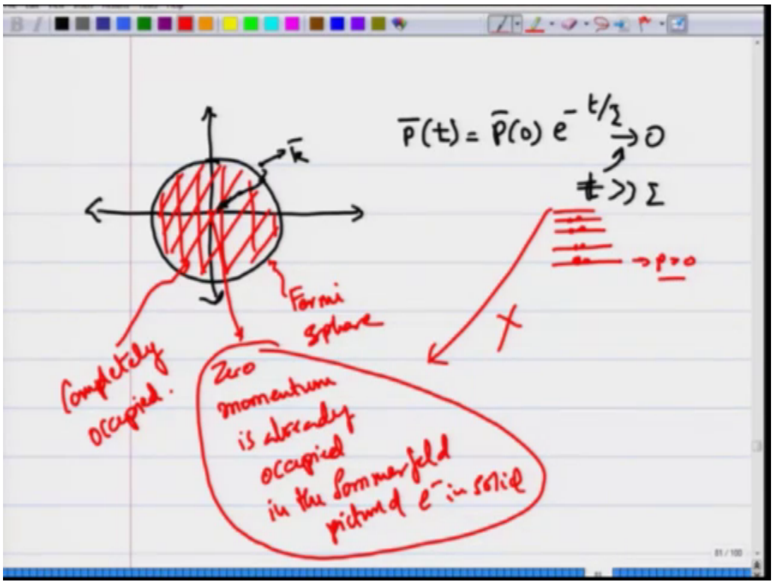
So, if you go back to the Drude's model; Drude's equation was dp by dt is equal to minus p by tau plus the force which is acting on the electron inside the solid. And f is could be coming from an electric field, it could be coming from a magnetic field or both electric and magnetic field applied to the system simultaneously. But in the Drude's model of the conductivity of the electron; the p was the momentum of an individual electron; this is the momentum of an individual electron.

However, the moment you start talking of individual electrons within the Sommerfeld's model of the solid; you land up into problems. And the reason simple reason being that if you take f is equal to 0; so you give initially some at time t equal to 0; you give some initial force f naught and at subsequent time you make it equal to 0. So, let us write it that you give a pulse or you give a force to the system. So, f at time t equal to 0 is equal to f 0 and at any subsequent time; t not equal to 0 the force is equal to 0. So, it is just like a pulse of force that you have given to the system f as a function of time; it has a finite value f 0 which is shown here and then it becomes 0.

So, in this pulse of force if you look at the equation of motion dp by dt after the force. So, initially because of f 0; there is a certain momentum which is given to the particles p 0. So, each individual particle gains some momentum p 0 and your equation of motion and any later time can be written as p of t is equal to p 0 e raised to minus t by tau. This is the equation of motion which; whose solution is this tau is the scattering time in the problem. So, therefore, if you give an electron some momentum the electron scatters and very soon comes down to the average momentum of the electron comes down to the zero momentum state. So, the average momentum of the electron actually comes to the 0 state in time greater than tau.

So, in a time greater than tau the electron undergoes collisions; it has initially been given p naught amount of momentum by the force, but very soon it undergoes collision and comes to the zero momentum state.

(Refer Slide Time: 07:03)

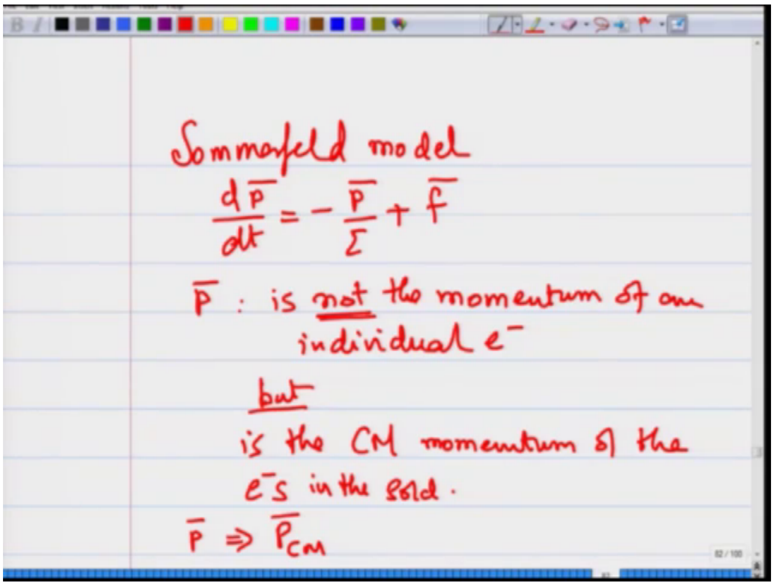


However, this is a problem in quantum mechanics because you know there is a Fermi surface which I have already described ok. So, for any electron which has acquired some momentum, after collisions the equation is saying that the momentum in a long time will actually tend to 0 as t tends to so; that means, the particle will very soon try to go towards the zero momentum state.

But what do we have quantum mechanically? Quantum mechanically as per Sommerfeld's model we know that all the states within the Fermi sphere, what are they? All these states are completely occupied. So, although some electron acquires a momentum after scattering the zero momentum state is not available to that individual electron because it is already occupied; the zero momentum state is already occupied. The zero momentum state is already occupied in the Sommerfeld's picture of electron in solid.

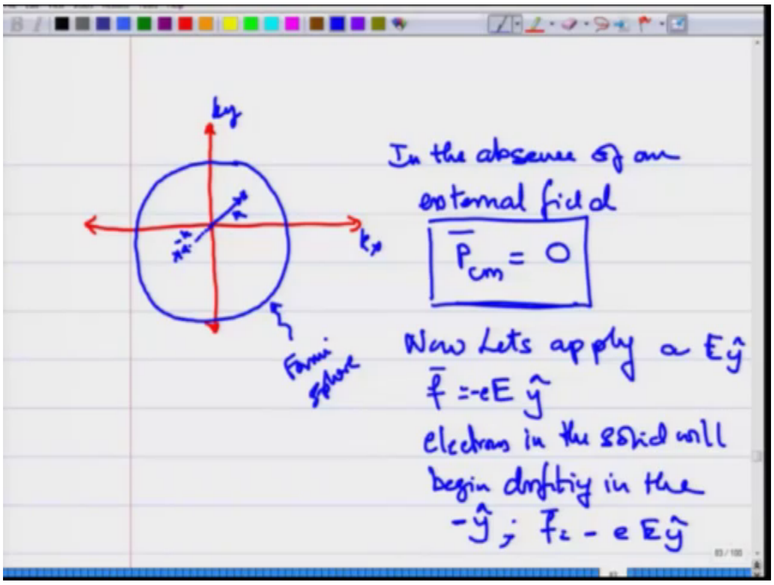
So, this leads to a problem; if you are discussing the equation of motion as corresponding to the motion of an individual electron, then the Drude's equation tells that electron should approach a zero momentum state, but the zero momentum state is already occupied. You have all these states and the zero momentum state and the states are filled by quantum mechanics. So, the zero momentum state is already sitting occupied, there is no way to put an electron there scatter it and bring it to the zero momentum state; so that is a problem. How does one look at this equation within the purview of Sommerfeld's model.

(Refer Slide Time: 09:25)



So, what Sommerfeld's considered was that the equation which Drude had written down; the sort of classical like equation which Drude had written down for the electrons inside the solid is correct with the difference that this equation is still ok, is reasonably correct within the Sommerfeld's model also. However, the p is not the momentum of an individual electron; is not the momentum of an individual electron, but is the center of mass momentum of the electrons in the solid. So, this p in this equation should actually correspond to the center of mass momentum of the electrons in this solid and how are from the Sommerfeld's picture.

(Refer Slide Time: 10:51)



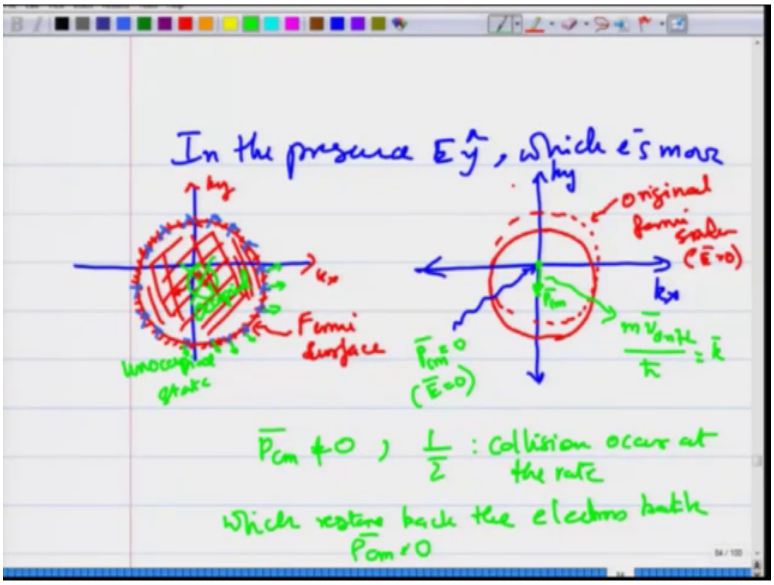
So, let me draw this; this is your Fermi sphere; just for simplicity I am showing it in a 2 dimensional plane, it is like a ball but at the moment I am just showing it as a circle where within the circle everything is occupied. Whatever is bounded within this circular region is all occupied states and whatever is above it is unoccupied states. These are kx and ky and the center of mass ok; so particles for any particle for any ok. So, these are all occupied states which are sitting within this; for any particle with a momentum k in this within this sphere, you can always find another state with equal and opposite momentum.

You can find another state within this which has equal and opposite momentum and you can do this for any of the states inside the system. So, in the absence of any external field; the total center of mass momentum of the system is 0. This is the total center of mass momentum and this is consistent with the equation that dp by dt is equal to minus p by tau; that p by tau will come into very soon, but in the absence of any external force acting on the system, the system is retains its perfectly circular momentum distribution which corresponds to the total center of mass momentum being equal to 0.

Now let us apply a force f which is an electric field; which let us say we apply it along the y direction. The moment we apply an electric field in the y direction what will happen to the electrons? The electrons will start drifting the electrons in the solid will begin drifting in the minus y direction. Because the experience of force the force on the electrons is actually minus e times the electric field. So, let us apply an electric field in the y direction and the force acting on the electron in the minus y is minus e times y cap ok.

So, the force which acts on the electron is in the minus y direction; so now electrons start drifting in the minus y direction. And as a result what happens is that the center of mass momentum, now start shifting down ok. Because there is a net flow of electrons in the y direction, but which electrons start moving in the y direction ok? When you give a momentum to the system which are the electrons; which are given this as the occupied set of electrons out of these electrons which are those electrons which are going to start moving?

(Refer Slide Time: 14:41)



So, in the presence of the electric field E which electrons move, if you look at the Fermi sphere only those electrons which are sitting on the Fermi surface; only these electrons which are sitting on the Fermi surface will be available for movement. Because when you apply electric field they will gain some momentum and they will start moving, but can they move to any state which is inside? These are all occupied states remember that these are all occupied states.

So, these electrons cannot move; these electrons which are present inside cannot move to any other momentum state. The Fermi energy is sitting far away it is pretty large energy scale of few electron volts ok. So, if you apply a reasonable electric field; you do not expect that the electrons which are sitting far away from the Fermi sphere to actually start moving in this region because they have no momentum available momentum states to move into.

Only the electrons which are sitting on this surface only these electrons which are sitting on this surface at the boundary between occupied and unoccupied states, all these electrons which are occupying states near the Fermi surface are the ones which will start moving because there are vacant momentum states which are available. These are all occupied states and outside are unoccupied states.

So, the electrons which are deep inside cannot move; their movement is completely forbidden because there are no available unavailable states for them to move into. But these electrons can move into states which are still lying vacant above the Fermi energy. And as a result the movement will be only of those electrons which are present on the Fermi sphere.

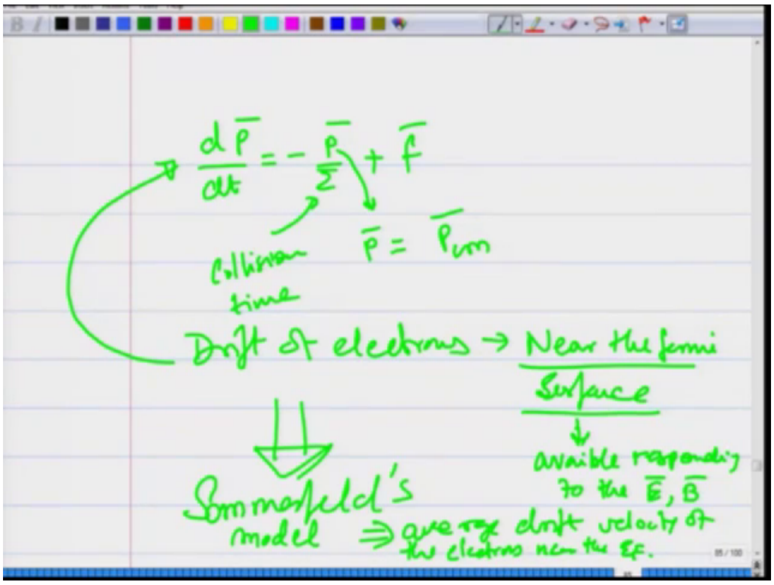
So, there is a net motion of these electrons; the electrons which are sitting on the Fermi surface, they start responding to the electric field and there is a net movement of the electrons. And this is shown in this way that because of the electric field; this was the original Fermi sphere, original Fermi sphere with when electric field was 0. And in the presence of an electric field the Fermi sphere now moves a little bit down in the minus there is a momentum which is along the minus ky direction I am exaggerating it, but this is the momentum in kx; ky this is in the minus ky direction.

So, now the Fermi sphere shifts downwards. Originally the center of mass momentum was here; originally if you look at the center of mass momentum P CM was 0 when the electric field was 0. Because of the electric field the Fermi sphere because of the net drift of electrons, there is a net drift of electrons in the minus ky direction because I have applied an electric field in the plus y direction there is a movement in the minus ky direction.

There is going to be a net drift of the electrons downward. So, now, the center of mass P CM this is the new position of the center of mass which develops and this is m v drift divided by h cross; this is the momentum. If you wish the momentum vector which defines by how much is the drift or the change in the position of the center of mass. So, the center of mass now shifts and it is representing a change in the center of mass momentum. So, when you apply an electric field these electrons start drifting and there is a net drift which develops which is defined as the change in the center of mass.

And the moment there is a change in the center of mass; very soon these electrons will want to go back to the original p is equal to center of mass 0 state. And so the moment you have distorted it from the p is equal to 0 state, collisions are occurring at the rate of 1 by tau; collisions occur at the rate of 1 by tau which tries to restore back the momentum. So, the collisions occur at the rate of 1 by tau. So, when the center of mass momentum is no longer 0 because of an applied force; the collisions start happening which tries to restore back the electrons back to P CM equal to 0.

(Refer Slide Time: 20:52)



And this is described by dp by dt is equal to minus P by tau plus the force where this P is the center of mass momentum. So, tau is still the collision time scale; the time between 2 successive collisions, but now instead of describing the motion or the scattering and the behaviors of one single electron; we are talking of a drift of electrons which are the electrons, which are near the Fermi surface. Only these electrons are available for responding to the forces or electric field or drive; only the electrons which are near the Fermi surface are the ones which can respond to the drive or the force.

Because, there are available momentum states near it into which they can go into and the moment this happens the center of mass momentum changes. And the change in the center of mass momentum is counteracted by scattering which tries to restore back the momentum back to its 0 center of mass position. And finally, the system achieves some sort of a equilibrium drift velocity and that drift velocity is calculated by this equation of motion.

So, although the equation of motion is quite similar to the Drude's model; the interpretation is very different. And this is what comes out of the Sommerfeld's model that there is an average drift velocity of the electrons near the Fermi energy. And this is how one interprets the behavior of conductivity in a metal; the conductive in it in the metal is happening through electrons, which are near the Fermi surface which are those electrons which are sitting near the Fermi surface.

When you apply a force to the system these electrons start drifting and these electrons the drift of these electrons is counteracted by collisions which tries to bring back the center of mass momentum to 0. And ultimately they acquire a net finite drift velocity which contributes to the net current inside the system. And ultimately the expressions are almost similar for conductivity because the equation remains almost the same, but the interpretation is quite different is very different and this is consistent with quantum mechanics, introduced into the motion of electrons.