Introduction to Solid State Physics
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Lecture – 15
Fermi energy and Fermi sphere Part – II

We had seen going about calculating the total number of particles using the expression for the Fermi Dirac distribution through which we wanted to get a way to calculate the Fermi energy.

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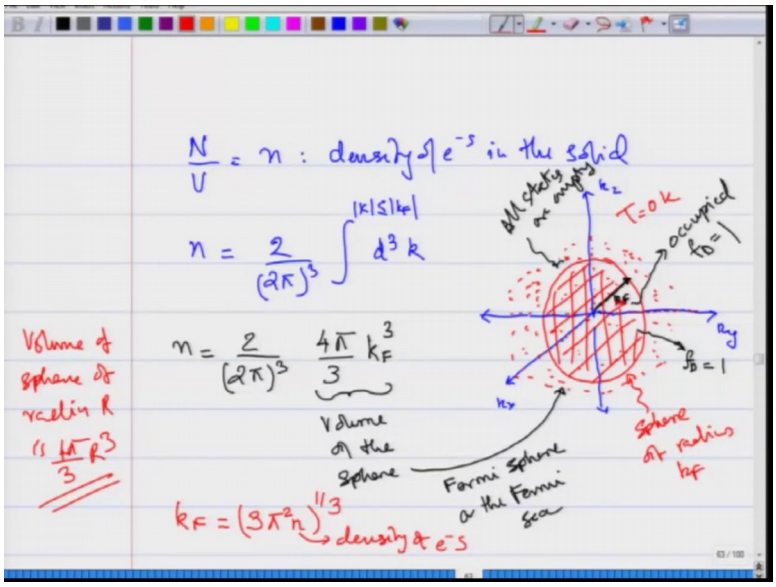

And there we use the idea that states up to the Fermi energy or the Fermi momentum k F are occupied. And anything or any states which are outside that are unavailable because the Fermi Dirac distribution governs that. And so we come across this idea that we construct a Fermi sphere and all states which are within this Fermi sphere are completely occupied.

You have a number of states, you start putting in electrons they start occupying the state's 2 electrons per state. And this is occupied until you reach the boundary of the sphere whose radius is k F in momentum space. And anything which is outside that are completely vacant, they are completely unoccupied. This is all at temperature T is equal to 0 Kelvin.

So, we define something called as a Fermi sphere or the Fermi sea which is at the surface of this sphere. So, you have a sea of electrons at the surface of a sphere whose momentum is k F, the sphere separates states which are occupied below it. And for momentum states which are above it are unoccupied. And so therefore, our integral is non-zero only for momentum states which are less than or equal to k F they are completely occupied. And so this integral will be basically the volume of the sphere ok. And so you can now use this to evaluate your Fermi, you can just rewrite this and you will get your Fermi momentum as or the Fermi wave vector is 3 pi square n raised to 1 by 3.

This is your Fermi momentum where n is now the density of electrons. The Fermi momentum or the Fermi wave vector the size of this vector which is the boundary between occupied and unoccupied states. The maximum momentum of the particle is governed by the particle number or the density of particles that you have inside the solid.

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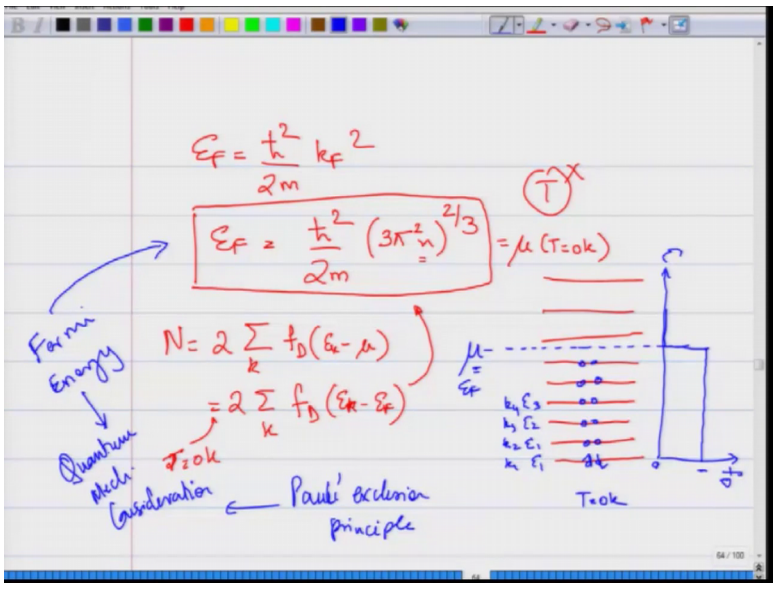

And this also gives you the Fermi energy now, because you know that the Fermi energy is h cross square by 2 m k F square. So, now, the Fermi energy is h cross square by 2 m 3 pie square n raised to 2 by 3. This is nothing else, but the chemical potential at T equal to 0 Kelvin. So, starting with this expression for counting of momentum states following the occupied and unoccupied states at T equal to 0 Kelvin.

We get an expression for the Fermi energy and this is related to, not to the temperature this is the energy of very close to what is the maximum energy which the particles have inside the system. So, the energy of the particles which are on the Fermi say sea, which are on the surface of the sphere that energy is the Fermi energy. And it is proportional to the density of electrons temperature has no role to play in determining the kinetic energy of these particles which are sitting on the Fermi sea.

Quantum mechanics has fully governed getting this energy scale inside the problem. And you can understand it, it basically arises out of Paulis exclusion principle that you have discrete set of states ok. And as you start filling up this discrete set of states you start filling up with particles as you keep on adding particles. There is 2 particles per state spin up and spin down particles you keep on adding these particles. And so they keep on going to higher and higher energy states, this is energy E1, E2, E3 each of them has momentum k 1 k 2 k 3 k 4 these are discrete momentum states which we have already shown.

And this way you keep on adding more and more particles and they start occupying higher and higher energy states until you exhaust your particles. All of this we are of course, speaking at T equal to 0 Kelvin and this is where your chemical potential is going to sit, which is your Fermi energy which is close to what is the maximum energy the particles will have. And as you know your Fermi Dirac distribution is going to be this.

So, your Fermi Dirac distribution will be occupancy 1 until you reach, this is your energy E. So, until you reach your Fermi energy the occupancy is going to be 1 and then of course, it drops to 0. So, all states are fully occupied and this gives you the maximum energy which has to be proportional to the number of particles. More the number of particles higher you can go up in energy. Lower the number of particles lower will be the location of the Fermi energy.

So, this tells you that the average momentum or the average energy of the system of particles or the electrons average energy of the electrons inside the solid because of quantization of the states and Fermi Dirac statistics, Pauli's exclusion principle plays a very important role is not governed by temperature rather it is governed by the total number of particles. So, this is the concept of a Fermi energy which clearly comes out because of quantum mechanical considerations, essentially Pauli's exclusion principle

plays a very important role to give rise to this concept of the Fermi energy which is close to the maximum energy which the electrons can have.

And it also gives rise we come across this concept of your Fermi sphere and the Fermi surface. This surface is called the Fermi surface the solid surface of the sphere which is the boundary between occupied states and vacant states. This Fermi surface, this surface is called the Fermi surface or the electrons which are on this surface and moving with a momentum k F is called the Fermi sea.

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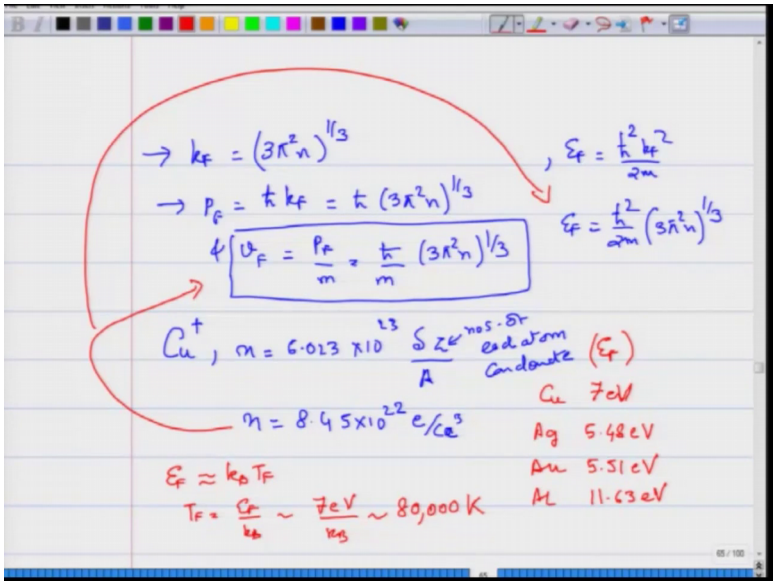

So, k F is 3 pi square n raised to 1 over 3. So, your actual Fermi momentum is h cross k F which is h cross 3 pi square n raise to 1 by 3. And therefore, you can get the Fermi velocity which is P F over m which will be h cross over m 3 pi square n raise to 1 over 3. You can also obtain your Fermi velocity, your Fermi energy anyway is h cross square k F square by 2 m. So, that is h cross square by 2 m 3 pie square n raised to 1 by 3.

So, this is the energy of the particles which are sitting on the Fermi surface and the velocity of those particles or the momenta are given by these expressions and they are controlled by the density. So, let us do an calculation how large is this Fermi energy and how large is the Fermi velocity of these particles, how high do they turn out. So, for a copper metal, for a metal of copper we know that you know you can calculate the density as the Avogadro's number times the density of the electrons into Z divided by A this is the atomic mass this is the density and this is the number of electrons each atom can

donate. And using this density of copper we know is 8.45 into 10 to the power of 22 electrons per centimeter cube. And it has also been measured to be in this range. And if you use this density and you calculate the Fermi energy of the system the Fermi energy for materials like copper.

So, you can put this in here and you can also put it in here and you can calculate the Fermi energy. And you can do the same thing for other metals and the typical values of Fermi energies that you will get let me write them down. This turns out to be about 7 electron volts this is about 5.48 electron volts 5.51 e V 11.63 e V. So, the maximum energy or close to the maximum energy the electrons can have inside a metal because of these quantum mechanical considerations turns out to be far larger than what can be generated by thermal energy.

These are of the order of few 10 of electron volts, since copper it is 7 electron volts, in silver it is 5.4, 5.5 silver and gold have similar sort of Fermi energies of 5.5 aluminum has 11. So, these are very large energy scales which are being generated inside the solid. Primarily because again I repeat it is because of quantum mechanical considerations.

If we were to estimate how much is this temperature which corresponds to this much of energy and try to seek and thermal energy produce it. Then there is a very simple way of doing it that we may write that the Fermi energy suppose we could write it as equal to some temperature which can give an energy which is equal to Fermi energy. Then if thermal fluctuations are giving you this energy we write KbT as equal to this Fermi energy; T is the Fermi temperature the equivalent temperature which can generate an energy of this order.

So, TF is nothing else, but Fermi energy by Kb. And for the heck of it if you just put seven electron volts out here divided by Kb put it in the right units you will get this as something like 80,000 Kelvin which is actually far larger than what the material the material will vaporize at 80,000 Kelvin. So, this temperature that is associated with this sort of energy this far larger than what is given by the actual temperature of the system.

The metal you will maintain it as 300 Kelvin or so, but the Fermi energy far larger than the temperature at which the metal is being held at. It is not related to the thermal fluctuations inside the system and that is the concept of the Fermi energy.

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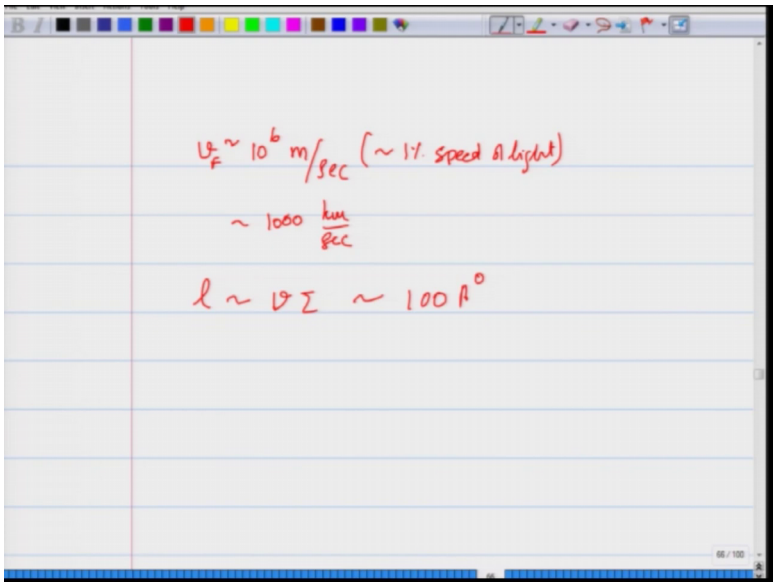

If you calculate the Fermi velocity again in the same way using the earlier formulas it will turn out to be in the range of 10 raise to 6 meters per second which is about one percent of the speed of light. It is roughly about 1000 kilometers per second ok. This is the typical velocities you will get Fermi velocities which are of this order for the different materials, 10 raise to 6 to 10 raise to 7 meters per second square and meters per second and these are very large velocities again.

So, the electrons which are sitting on the Fermi surface which are sitting on the Fermi sea are moving with very large velocities. And this is the basic difference that comes out from in some of his theory compared to Drude's theory. And now your mean free path which is velocity into the collision times will increase to about 100 angstroms. Because your velocities have now gone up by 1 to 2 orders of magnitude.

So, your mean free paths have become much longer than the lattice spacing's which were 1 to 10 angstroms, now they have become much larger. And so summer field was able to explain why electrons inside the solid typically are associated with large velocities. It is because of fundamentally because of these considerations which we have discussed.