Introduction to Solid State Physics
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Lecture – 14
Fermi energy and Fermi sphere Part – I

For the solid using periodic boundary conditions we obtain the k x, k y, k z states. And these are discrete states which correspond to discrete energies inside the solid.

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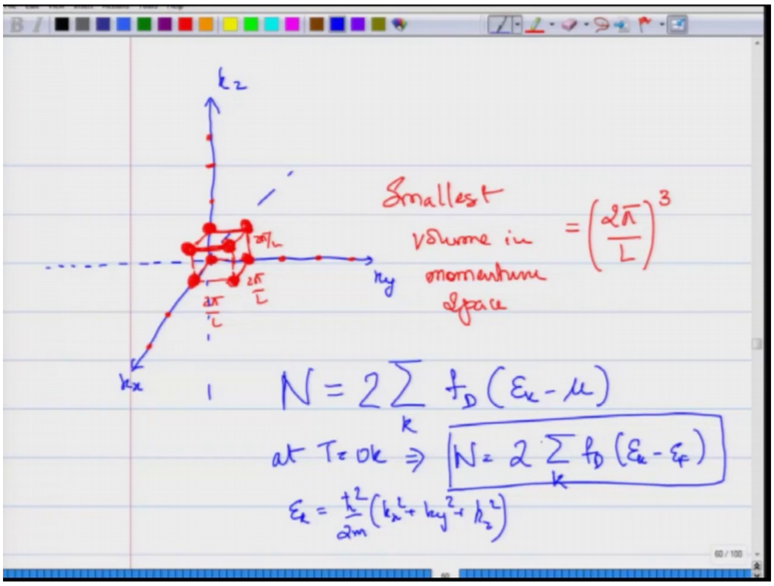

And we showed that these discrete states have a spacing of 2 pi by L between them. So, here is the plot where you show in the momentum space where you have plotted k x, k y, k z there are the discrete energy states which are or momentum states which are shown out here. And each of these momentum states in either directions or space by 2 pi by L.

And so you can find out what is the minimum volume in this momentum space where you will have about one momentum per such cube. So, this gives us the smallest volume in the momentum space. Below this volume you had not states which are available because the states are only discretely spaced by 2 pi by L. Now let us try and go back to our counting that we have done.

Our counting statistics involved that if I count the total number of occupied states in the system where at T equal to 0 Kelvin. This equation is nothing else, but twice of summation were at T equals to 0 Kelvin, this equation nothing else but twise of summation over all the k states the Fermi Dirac distribution E k minus mu. Where E k is nothing else, but h cross square by 2 m k x square plus k y square plus k z square.

Let me solve this instead of mu we will be replace it with a Fermi energy. So, if I count the total number of occupied states at 0 temperature, I will be able to calculate what is my Fermi energy. And if you see there is a summation over k. Namely I sum over all these k that are available in my space, in my momentum space if I sum over each of this I count 1 2 3 4 5 6 7 8 all these discrete states if I sum over them.

And consider those which are occupied then I should be getting the total number of particles inside the system ok. So, given that the volume in the k space the smallest volume in the k space is 2 pi by L the whole cube. Because this is the volume of this cube and there is one state per such volume.

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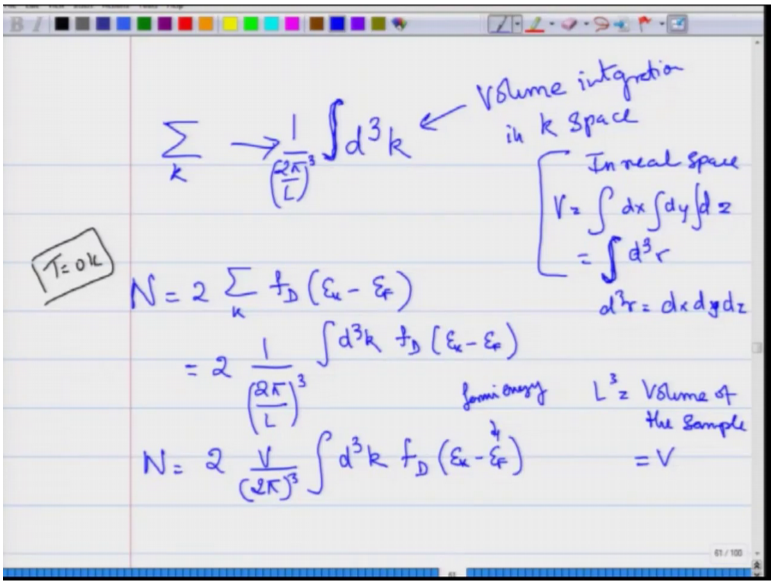

Then the summation over all the k states which exist this summation I can convert it into an integral; where I will take an integral over the entire volume in the momentum space divided by 2 pi by L the whole cube. This is the smallest volume where there is one state per such volume. So, you have the entire volume in case space divided by the smallest volume which is occupied by a single state.

This will again if I take this continuous integral from summation if I go to this integral formulation. Then I will again count the total number of states. This is nothing else, but the volume integration in k space. If you recall in real space the volume is integral of dx integral of dy integral of dz which you can write it as integral of d cube r where d cube r in Cartesian coordinates is d cube r which is equal to dx dy and dz.

And this is we write it in this volume integration is written like this. So, instead of doing the volume integral in real space we are doing a volume integration in k space. Hence this N goes over which was twice of summation k fD E K minus E F can be rewritten as twice 1 by 2 pi by L the whole cube integral d cube k Fermi Dirac E K minus E F.

I am doing all the calculations at temperature T equal to 0 Kelvin ok. So, I take the integral over the entire volume divided by this factor. And L cube is nothing else, but the volume of the sample which is V. So, N is equal to 2 into V by 2 pi the whole cube integral of d cube k Fermi Dirac E K minus E F this is your for Fermi energy ok. Now, since we are doing at 0 temperature all our calculations T is equal to 0 Kelvin.

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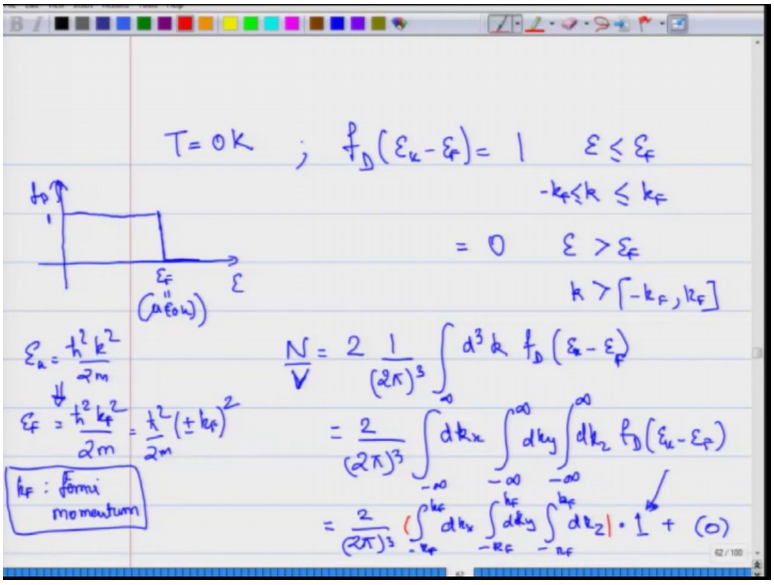

What does the Fermi Dirac distribution say E K minus E F should be equal to 1 for all energies which are less than or equal to E F. And it is equal to 0 for all energies which are greater than E F. If you recall the Fermi Dirac distribution the f D distribution as a function of energy it was 1 at 0 temperature it was 1 up to the Fermi energy which is the chemical potential at 0 Kelvin and then it is become 0 afterwards.

So, it is 1 and then it falls to 0. Now, the energy also you know that the momentum is h cross square k square by 2 m. So, the Fermi energy is also h cross square k f square by 2 m. Where k F is called as the Fermi momentum. Which is really the maximum momentum which is possible inside the system. So, if energy is if E F is going to be equal to 1 for energy less than or equal to E F. This is going to be valid for all momentum states which are greater than minus k F less than or equal to plus k fF ok.

Because the Fermi energy is proportional to the square of the momentum. And therefore, plus minus k F will satisfy this because it is. So, this is also equal to h cross square by 2 m plus minus k F the whole square. And for those momentums which are outside the range of plus minus k F the Fermi Dirac distribution will be equal to 0.

So, the momentums which are between plus minus k F those will have 100 percent occupancy because that is what the Fermi Dirac distribution says. You have already got the energy states and you are filling up those energy states up to plus minus k F momentum. And so your expression for the total number of particles I take my volume downstairs from this side. So, this is 1 over 2 pi the whole cube integral of d cube k times f D E K minus E.

Here I was taking the integral from minus infinity to plus infinity for k x going from minus infinity to plus infinity, ky going from minus infinity to plus infinity and kz going from minus infinity to plus infinity ok. So, I let me write it as this 2 pi the whole cube integral d kx going from minus infinity to plus infinity integral minus infinity to plus infinity d ky integral minus infinity to plus infinity d kz Fermi Dirac E K minus E F ok.

But you know that the Fermi Dirac distribution is going to be one only between plus minus kF and it is going to be 0 elsewhere. So, I can convert this again into 2 by 2 pi cube integral minus kFto plus kF, d kx integral minus kF to plus kF d ky integral minus kF to plus kF d kz into 1. Because in this range your Fermi Dirac distribution will give you one and for anything which is outside that range it is going to be 0. So, the other term is going to be 0 ok. For any things which are outside this range these integrals will have 0 because this quantity become 0.

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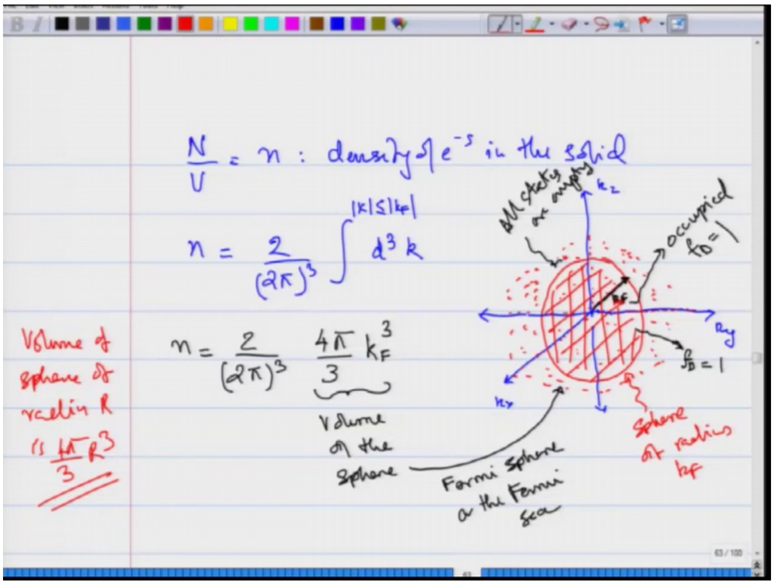

So, then it is again N by V is the density of electrons in the solid. So, the density of electrons is 2 by 2 pi cube integral d cube k. So, my earlier expression which showed that my kx is going from minus kF ky is going from minus kF and kz. This is nothing else this term this term out here is nothing else, but the volume integration. And the volume integration is being done over a k which is up to kF ok.

So, for all momentums which are within this length kF will be contributing to this integral ok. This is the volume integral and for all the momentum states which are within this length if you take the length from the origin. If they are within that then they will contribute to this volume integral. And the way to think about it is that in the momentum space; we are going to consider a sphere; kx ky and kz.

We have a sphere and there are momentum states which are avail on this sphere you know you have momentum states which are on this as well as in between you know you have momentum states which are present in between outside and so on. But there is a sphere whose radius is kF this is the sphere of radius k F. For all the momentum states which are present within this sphere of radius kF for all momentum states which are enclosed within this radius kF they are the ones which will have a non zero contribution because they are fully occupied.

The Fermi Dirac distribution for all the states which are within this sphere of radius kF they have occupancy 1 probability of occupancy is 1. So, I can rewrite n as 2 by 2 pi cube n is the density into 4 pi by 3 k F the whole cube this is the volume of the sphere I just described.

And this is called as the Fermi sphere or the also called as the Fermi Sea namely inside this sphere all the states are fully occupied all the states inside the sphere are fully occupied with the Fermi Dirac distribution equal to 1. And outside the sea sphere all states are empty. So, the surface of the sphere encloses or distinguishes between occupied states and vacant states.

There are states all around which are present there are discrete states which are all around. The difference is that the states which are outside the fermi sphere are vacant and those which are enclosed inside the Fermi sphere are fully occupied. And this is the volume of that Fermi sphere which is nothing else, but the volume of a sphere of radius R a volume of a sphere of radius R is 4 pi by 3 R cube. This you already know and now since we are looking at the momentum space it is 4 pi by kF cube.