Introduction to Solid State Physics
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Lecture – 13
Introduction to Sommerfeld's Theory of electrons in a metal Part – III

We had begun looking at the Sommerfeld's picture of how an electron moves through the solid and the important thing with Sommerfeld's realized was that the electron does not really follow the kinetic theory of gases. Its distribution is not like a Maxwell Boltzmann distribution, but the distribution follows quantum statistics. Inside the solid you have states energy states which are available and the electron fills up these energy states and the way they these states are filled up is governed by the Fermi Dirac distribution.

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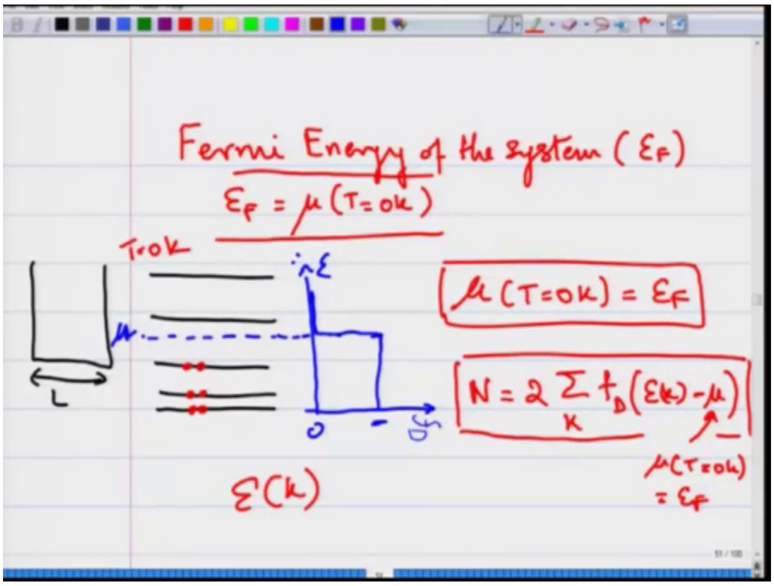

So, there is a chemical potential mu up to which the states are occupied with our occupancy or with a probability of 1, and above it becomes 0. The 0 temperature chemical potential is an energy level which is close to the maximum energy which the particles have at 0 temperature and that is what we call as the Fermi energy. And the general way to determine your chemical potential is that, if you count all the number of states which are occupied, then these are all the states which are occupied up to the chemical potential multiplied by two particles per state, then the total number of particles you will get the total number of particles. So, if you solve this equation you will get your chemical potential.

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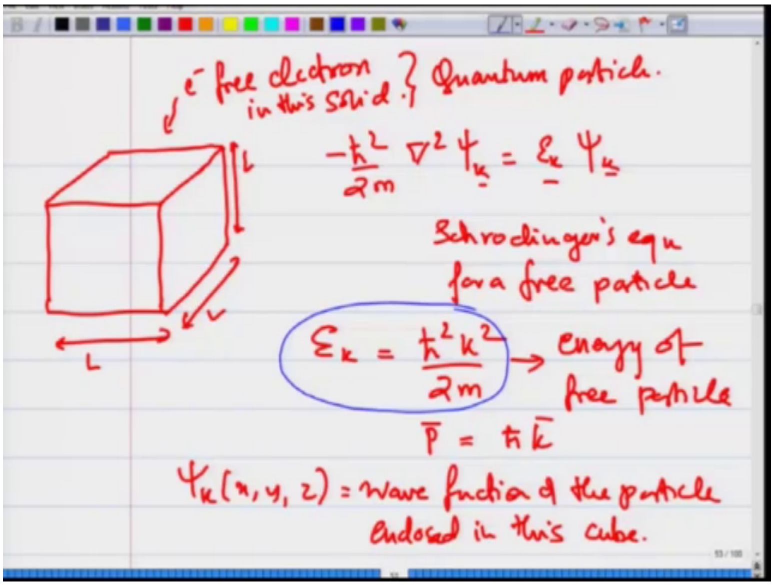

So, we consider that the solid is a cube of sides of length L we had seen this already and you write down the Schrodinger’s equation for the free electron, whose energy is given by this E k is equal to h cross square k square by 2 m and psi k is the wave function of the electron.

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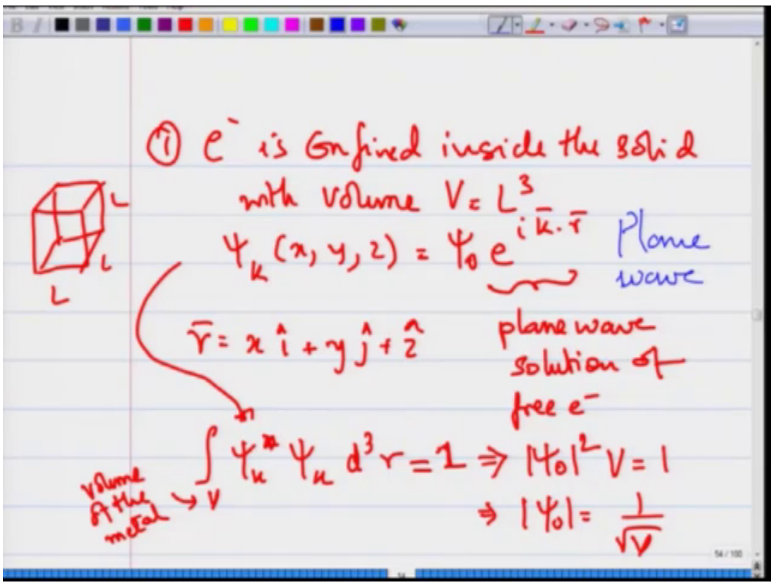

Being free electron we consider it like a plane wave. So, this is your plane wave solution for the free electron, which is moving through the solid. And you consider the solid has a finite volume, you normalize it and you get the constant the amplitude which is 1 by square root of V for this wave function and then you have to put some boundary conditions on this wave function.

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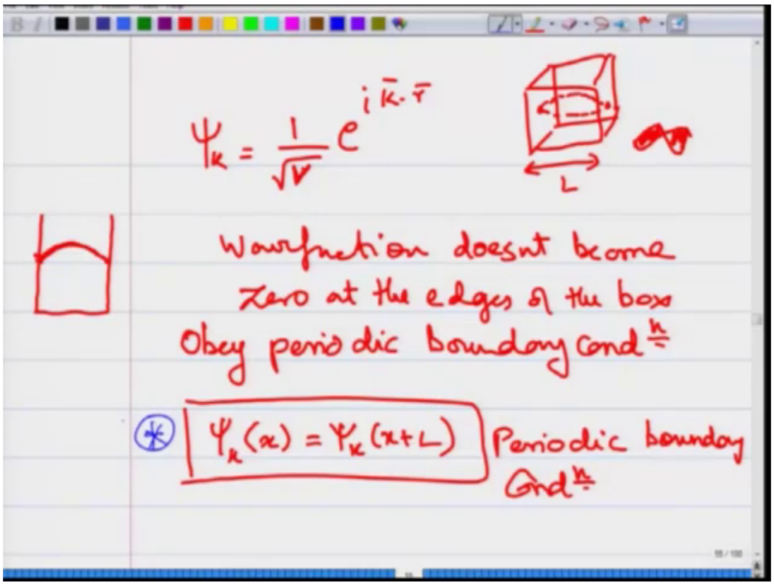

Now, is it that all momentum states because you want to generate the energy states which are available inside the solid, using the plane wave, you want to generate what are the energy states available are there a continuum of states or are there some finite discrete set of states which are available. And for that some of will used the periodic boundary conditions.

So, in your problem of just a single electron inside a box you consider that the wave function becomes 0 at the two edges, but this has a problem because it gives rise to standing waves inside the solid which you want a propagating mode, you want electrons to actually propagate through the solid you do not want them to be static and you do not want them to be standing wave type solutions.

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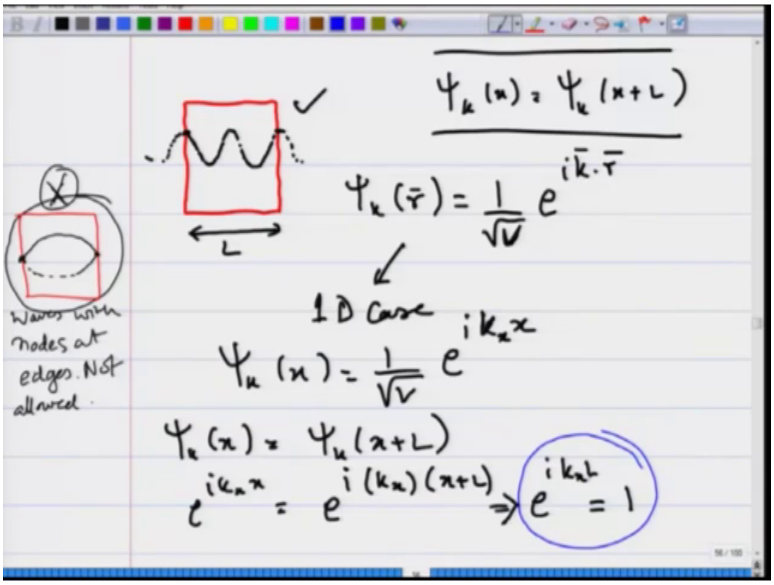

So, what Sommerfeld considered was this very important periodic boundary condition. That the wave function at x is equal to the wave function at x plus L and this leads to solutions which are of the travelling wave form where this is not a solution where you have nodes, but instead this is a solution, so you do not have nodes at the edges of the sample. And if you use this sort of a periodic boundary condition in one dimension then you will get a condition on your wave function which is e raised to i k into L is equal to 1 which gives you a condition on your momentum, that the momenta is going to be integral multiples of 2 pi by L.

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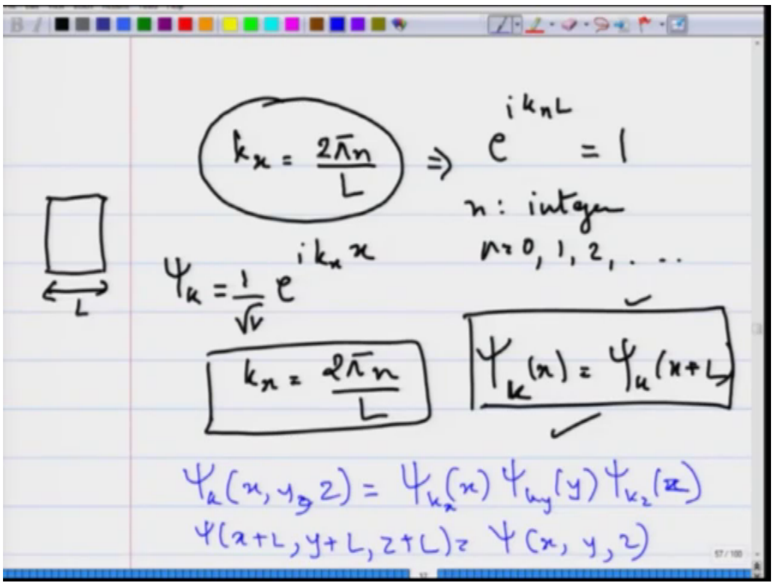

The net wave function psi k x, y, z is k psi k y y, psi k z. So, these are the mementos in the x, y and z direction, ok. So, we write the net wave function as the product of these wave functions and then you can show that you can write down the periodic boundary condition as psi x plus L, y plus L and z plus L is equal to psi x, y and z. Namely, there is a periodic boundary condition along the x direction along the y direction as well as along the z direction. And all of this leads to the condition on k x, k y and k z. So, let us write down the conditions on k.

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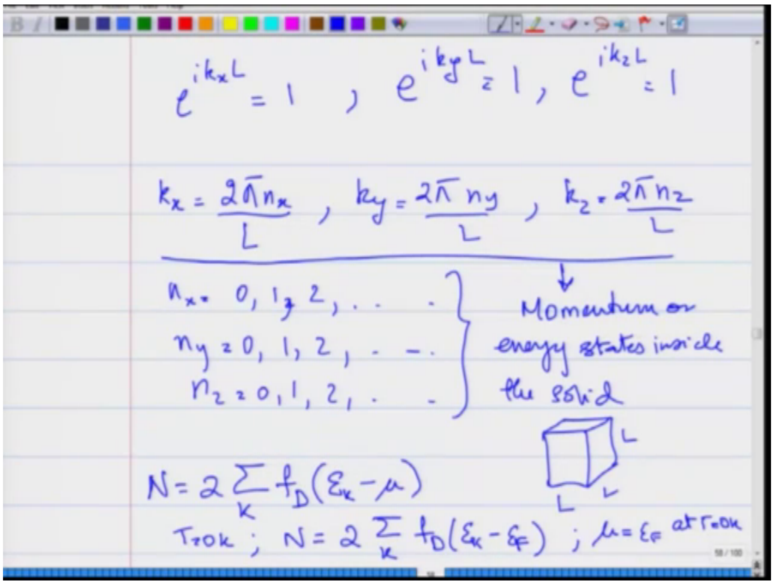

So, you will have e raise to i k x into L we consider that the cube is symmetric it has a side of length L, length, width and height are all of length L, and so if you use the periodic boundary conditions you will have this e raise to i k y into L will be 1 and e raise to i, k z into L will be 1. And this will put conditions on k x, these will give you your momentum states or the energy states, k y is equal to 2 pi n y over L and k z is equal to 2 pi and z over L. These are your different momentum states, where n x is 0, 1, 2 and so on; n y is 0, 1, 2; n z is equal to 0, 1, 2 and so on. So, these are your different momentum states.

So, you generate your momentum or energy states inside the solid which is a cube with sides of length L. And now the states that you generate you will have to start filling up. So, can we use this expression, N is equal to summation of all the momentum states this is the Fermi Dirac distribution e k minus mu, which at 0 temperature at t equal to 0

Kelvin this expression N is equal to twice of summation of all momentum states f D, E k minus E F as mu is equal to E F at 0 Kelvin. So, can we use this expression to evaluate what is the fermi energy of the system for this solid?

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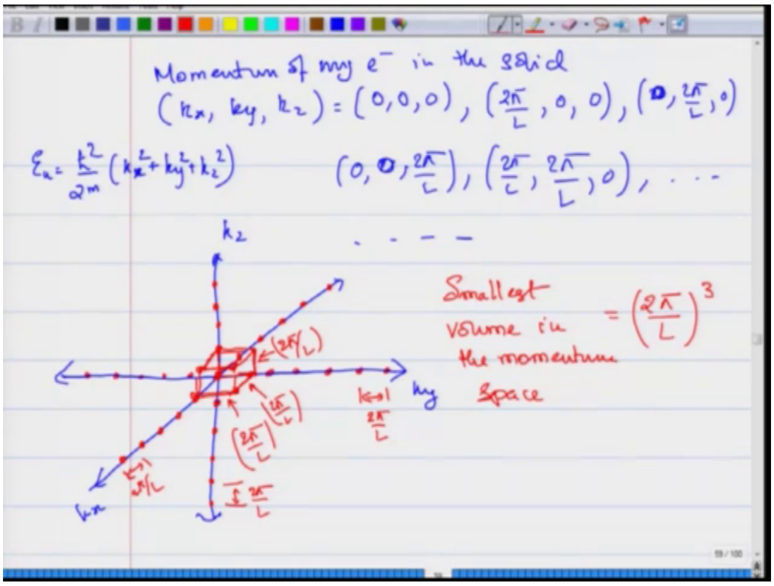

If I look at the momentum of my electron in the solid it has momentous k x, k y and k z. And these will be typically for n x n y you know you will have 0, 0, 0; you will have 2 pi by L 0 0; 0 2 pi by L 0; 0 0 2 pi by L, 2 pi by L 2 pi by L 0. And like this you will have a discrete set of states you will have a discrete set of momentum states inside the solid, each of them have their own kinetic energy which is given by E k is equal to h cross square by 2 m, k x, k x square plus k y square plus k z square. So, from these discrete momenta states you will get discrete energy states which will then be filled up based on Fermi Dirac distribution.

So, now, let us look at this momentum space k x, k y and k z, this is the 0 momentum state which is the 0, 0, 0 and every subsequent state either in the k y direction or in the k z direction or in the k x direction all the momentum states are spaced by a distance of 2 pi by L. These are the discrete momentum states or the energy states in the momentums. These are the discrete momentum states that we have we are drawing here which we have obtained just now and the spacing between any two points in either direction is either 2 pi by L.

And you can draw what is the smallest. So, there is a point in the k x, k y plane which will have 2 pi by L, this is this point which is in the k x, k y plane which will be this similarly there will be another point here, and you will have another point here, and you will have another point here. So, you will have one such cube like this there are no momentum states which are available. So, the smallest volume in the momentum space is equal to 2 pi by L the whole cube. This is the smallest volume because this distance has 2 pi by L this is 2 pi by L and similarly this length is 2 pi by L, ok.

You can show for yourself that this is the smallest volume. Below this volume there is no states which are available.