Introduction to Solid State Physics
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Lecture - 11
Introduction to Sommerfeld's Theory of electrons in a metal Part-1

We have till now looked at how you can understand the motion of an electron through a solid. And the first understanding was given by Drude, where he considered the electron as classic objective and considered the kinetic theory of gases to describe the electron inside the solid inside the metals specifically.

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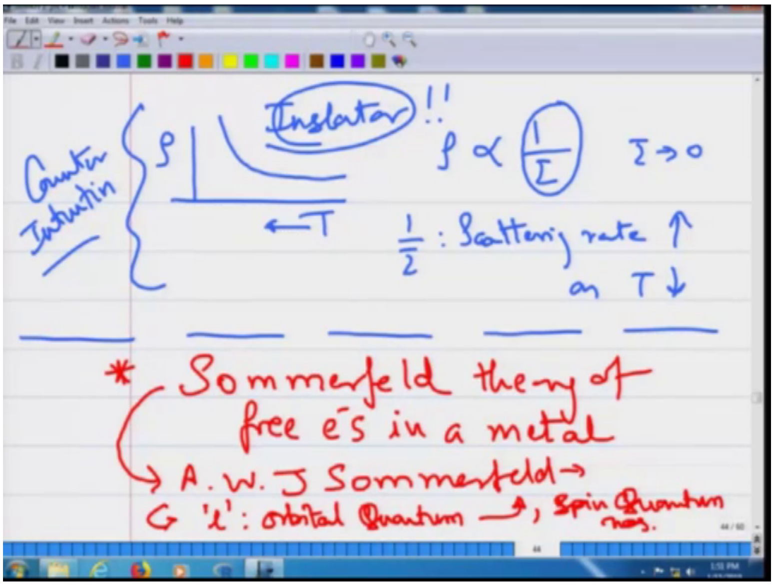

So, while Drude has its successes there are some major inadequacies within the Drude's theory. So, how do we go ahead from here? So, the way to go ahead from here is the next topic that we will look at the Sommerfeld's theory of free electrons in a metal or the Sommerfeld's model of free electrons in a metal. So, this is what we will now start discussing. So, what was the one of the main ingredients of the Drude's theory? One of the main ingredients of the Drude's theory was that, you have classical electrons inside the system. And it was well known since J J Thomson discovers after the discovery of the electron by J J Thomson, that very soon it was realized that electrons are with the development of quantum mechanics and so on that electrons have quantum particles, so they can really behave like classical objects ok.

And Sommerfeld was A. W. J Sommerfeld was a very famous physicist who had contribute its significantly in quantum mechanics he was the first one to propose the orbital quantum number which actually determines the shape of the orbital. So, this came from Sommerfeld he also gave the spin quantum number. He understood that the electron as the quantum particle. So, when you are describing a electrons in a metal you certainly cannot consider them as classical.

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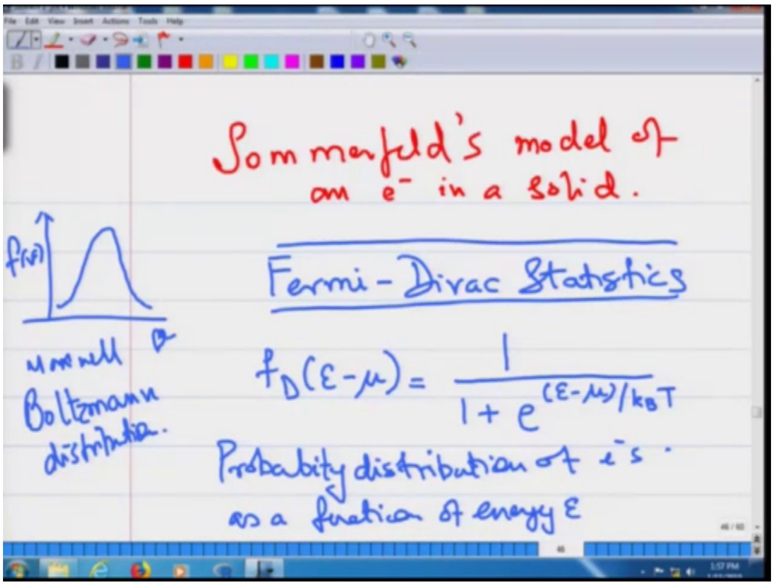

So, we look at Sommerfeld's model of an electron in a solid. Now, if you recall that the Drude's model had basically a Maxwell distribution of electrons inside the solid the Drudes model if the probability distribution of the electron as a function of velocity has got this Gaussian type shape. And this was the Maxwell-Boltzmann distribution Maxwell Boltzmann distribution.

Now, when Sommerfeld started thinking about the behavior of electron inside the solid, during that time it turned out or it was already available that the electron follows quantum statistics. And by that time the quantum statistics of particles was well-known you have fermions and bosons which have very different quantum statistics. And the probability distribution of the electron does not follow the Maxwell Boltzmann, but it has a very different statistics. This was realized immediately by Sommerfeld, and he proposed that electrons inside the metal do not follow a Maxwellian-Boltzmann type distribution, but the energy distribution of the electrons inside the solid follows the Fermi-Dirac statistics.

So, he immediately realized that electrons are quantum particles inside the metal, and therefore the statistics or the energy distribution of these electrons which are present inside the metal will follow the Fermi-Dirac statistics. So, what is the Fermi-Dirac distribution? The Fermi-Dirac distribution to recall is given by the Fermi-Dirac distribution for a particle with an energy e is given by this expression 1 by 1 plus e raised to e minus mu by k B T. This is the probability distribution of electrons as a function of energy e of the electrons.

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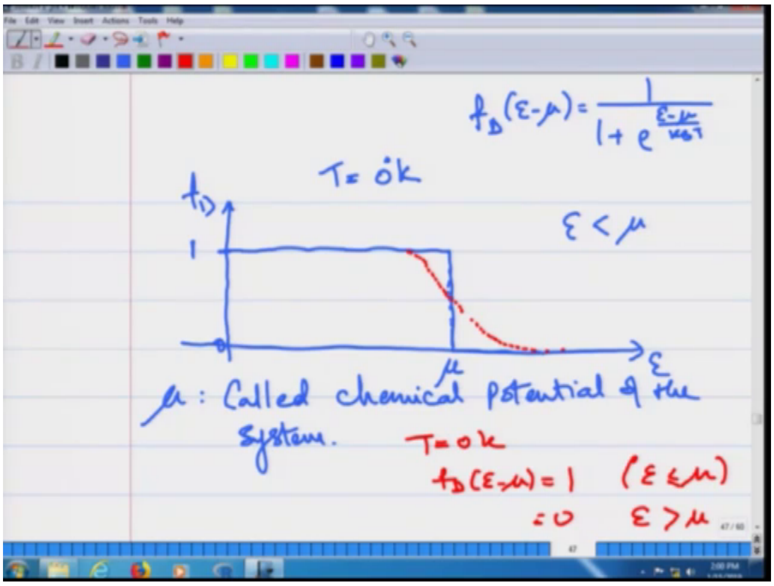

If we sketch it out, then Fermi-Dirac distribution looks like this. This is the Fermi function of e; this is the probability. And let us look at the distribution at a temperature T is equal to 0 Kelvin. What this tells you is that at 0 Kelvin, the probability of occupying states of energy e is one for the electrons up to an energy which is mu after which the probability beyond which the probability becomes 0. So, this is 0 out here.

So, for all energies which are less than mu at temperature T equal to 0 for all energies which are less than or equal to mu, they all are occupied with probability 1. There is 100 percentage occupation probability of those states, so you have different states inside the metal or inside the material energy states. And the occupation of those energy states for energies less than mu is 1. And for the energies greater than mu, it is 0, that is given by this expression. k B is the Boltzmann's constant; T is the temperature. And mu is called the chemical potential of the system. So, the electrons rather than having the a Gaussian type of distribution the electrons the energy distribution of the electrons follows a statistics which is the Fermi-Dirac statistics which is this as I show you here.

Now, if you increase the temperature this is at 0 Kelvin ok, if you increase the temperature then what is going to happen is that, you will find that this distribution changes slightly as a function of temperature and there is a tail which develops. So, we can understand it in slight more details by giving you a specific example and a specific picture and that is the following that if you have let us consider the picture of a particle confined in a box of length L.

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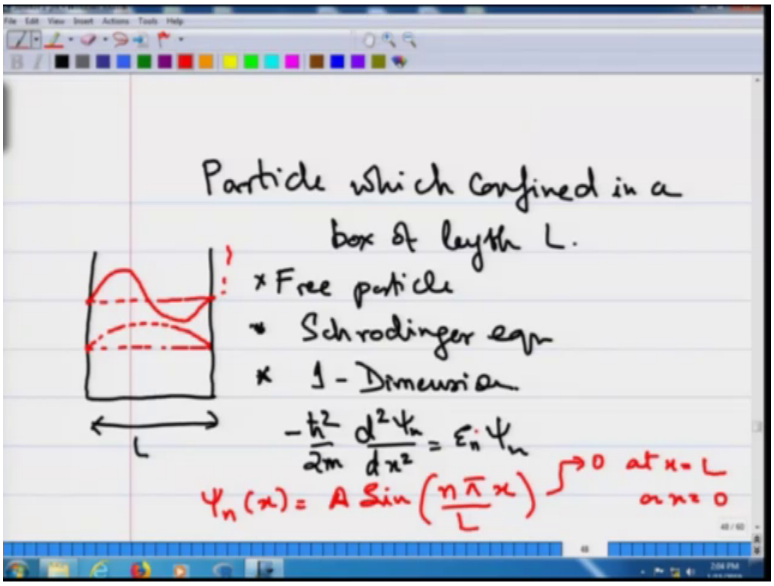

A particle which is confined in a box of length L. So, you have a box of length L and the particle is confined in this box. The particle is a free particle. And now the particle obeys Schrodinger's equation and we are going to consider one dimension. So, the one dimension form of the Schrodinger's equation is minus h cross square by 2 m del square sin by del x square is equal to E psi, where psi n is the nature of the wave function which you can fit in this box. This is the energy of the particle E n is the energy Eigen state of the particle.

And we know the solution of this problem; you have worked out the solution for this problem often. The ground state energy is where the wave function has is like this. The next exited state has a wave function which has a node in the center and then you get all the higher energy states. But all of them have the wave function becoming 0 at the edges of the sample at the ends of the samples you have the wave function becoming 0.

So, the solution for this wave function is A sine so that at x equal to l at x equal to l or x equal to 0, the wave function becomes 0. This becomes 0 at x equal to l or x equal to 0. At the two corners, it is 0 ok. And if you write down the energy of this state, if you take the second derivative of this expression you take the second derivative d square psi n by d x square and you put it in here, you will get your energy of the system.

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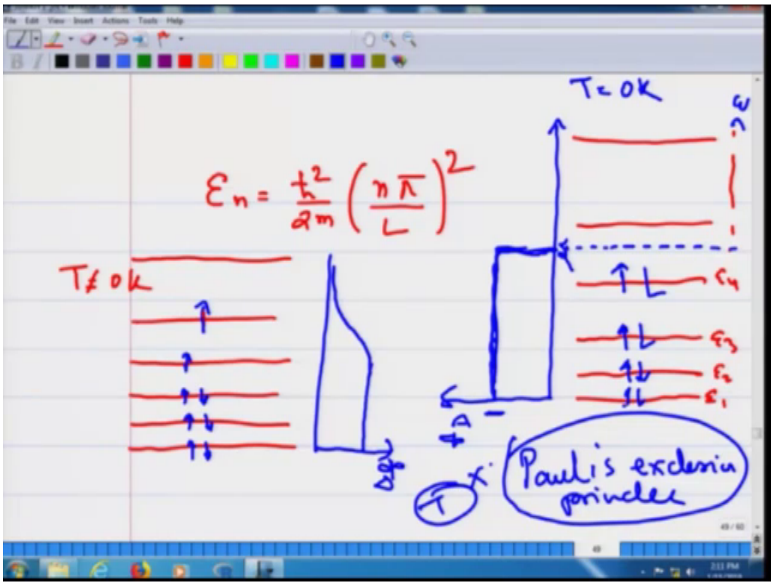

And energy Eigen states will turn out to be h cross square by 2 m n pi by L the whole square ok. You will directly get it by taking the second derivative putting it in here. You will see that immediately and putting this value out here, and the second derivative out here, you will get your energy Eigen states. So, these are the energy states that are available in the system.

So, let us draw these energy states. This is your ground state, the first exited state, second exited state and so on ok. So, these are your energy levels e 1, e 2, e 3, e 4 and so on. Now, so these are the energy states which are available for a particle you put inside this box. And now what we will do is that we will start distributing the particles in this box.

So, what will have you have spin half particles. And as you will follow Pauli's exclusion principle the Pauli's exclusion principle will actually put for you two particles per state this is one electron and other electron one spin up one spin down. So, you will first put two particles here, then the next particle will go here and that the next will go here, and you will start filling up. You will start filling up the state as you go along ok. So, as you put more and more particles, you will start filling up the states.

Now, what does the Fermi-Dirac distribution state, the Fermi-Dirac distribution states as that see along this axis our energy is varying along this axis along, this axis our energy is varying. Here we have the Fermi-Dirac distribution. What does it state, this states that the probability of occupying all these states is going to be fixed up to a certain value ok, and that value suppose we are looking at all of this at temperature T equal to 0 Kelvin.

The Fermi distribution is going to say that the probability of occupying e 1, e 2, e 3, e 4 is going to be 1 up to a certain value and then it is going to fall, it is going to become 0 at this energy mu and above it is all 0. So, the particle is going to occupy up to this point and above it all is empty. So, this is what means about the distribution is that when you start filling up the particles, you will start filling up the particles up to the chemical potential mu. And this is at t equal to 0, and then above them it is going to be empty. So, there is an energy important energy scale in the problem which is sitting at the boundary between completely filled state and completely empty states. The same thing will change a little if you include temperature into the problem.

If you look at the temperature which is not equal to 0 Kelvin, some higher temperature then what will happen is that things will change slightly. So, these are the same states that you have inside the system. Now, you are working with a finite temperature ok. And if you now start looking at the distribution, then you start putting in two particles, but near about this point you give enough thermal energy, that one particle comes here and another particles goes here another particles goes. So, they start occupying higher energy states. A particle which was originally occupying only these states now starts occupying the unoccupied states.

And now if you look at the Fermi-Dirac distribution of this system, then you will find that it is completely occupied, but then as you go higher you start getting higher energy states. Here I am just showing one for an example, but for lot of particles you will start redistributing the particles around this high energy point and you will start creating a distribution, where there will be some states will become vacant. For example, here you now have a vacancy and there are higher states which start getting occupied. The particles are distributing across available energy states following Pauli's exclusion principle, where temperature is now playing no role.

In this distribution, in the Maxwell's distribution, you had temperature playing an important role, where you have particles distributing depending on the temperature. But now you have particles which are distributing amongst available energy states e 1, e 2, e 3, e 4, the Eigen states of the system following this quantum mechanical principle of Pauli's exclusion. And the distribution of these particles is governed by the Fermi-Dirac distribution. So, with these concepts, we will try and see how to apply it to understand the properties of a metal.