# Introduction to Quantum Mechanics

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## Lecture - 08
## Radiation as a collection of particles called photons

In the previous lecture, what we have seen is that the quantum hypothesis.

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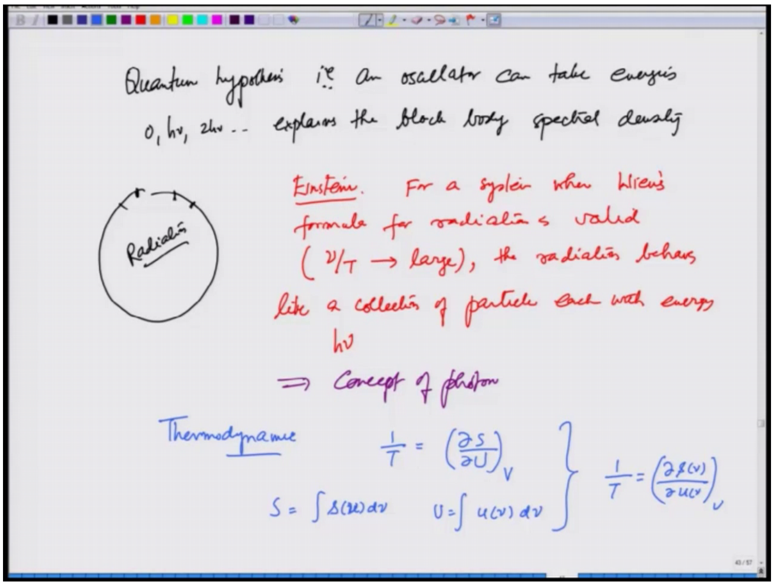


That is an oscillator can take energies 0 h nu 2 h nu and so on explains the black body spectral density. So, this is only the oscillator. So, what we had assumed that there is a cavity and these oscillators take only quantize value, but at the same time, with treating the radiation as if it is continuous, this seems to be an inconsistency. And that is where Einstein came in and he said may be radiation should also be treated as this continuous containing particles of energy h nu and he went on to show this I want to do it for you, because this is a beautiful piece of scientific investigation.

So, why; what Einstein says is that for a system where Wien's formula for radiation is valid and that is a resume where nu over T is large, when this is valid, the radiation behaves like a collection of particles each with energy h nu and that gives the concept of photon and we want to see; how it did; that use a little bit of thermodynamics, right. So, thermodynamics says that 1 over T is equal to d S by d U at constant volume.

Now, what Einstein considers is that S is also made up of a spectral s nu d nu. This is spectral distribution of the entropy just like U is given by U nu d nu and without proof, I am now going to claim that 1 over T is also equal to d s nu over d U nu at constant volume. So, even the partial whatever the spectral density of the entropy is one of the internal energy is there partial derivative gives you 1 over T.

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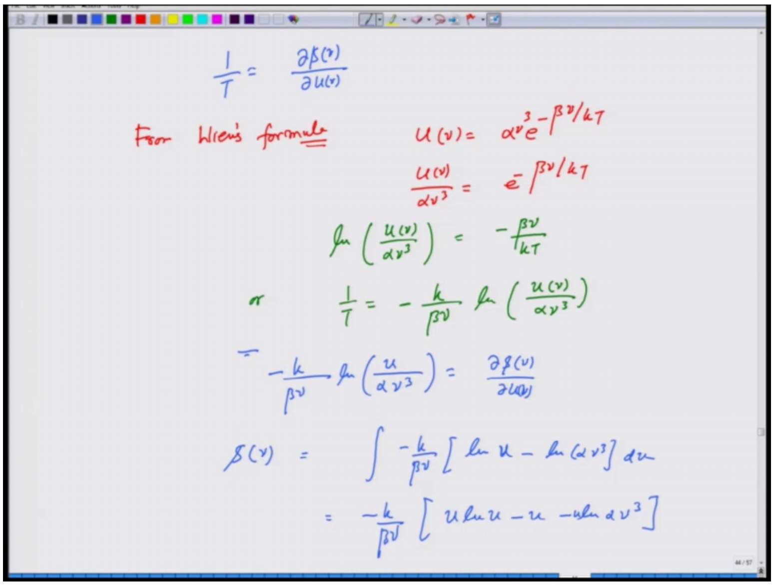


So, what we have is one over T equals d s nu over d U nu. Now from Wien's formula, we have u nu is equal to sum alpha e raised to minus beta nu over k T and there is a nu cubed. So, u nu over alpha nu cubed is equal to e raised to minus beta nu over k T take log of both sides and you get log of u nu over alpha nu cubed is equal to minus beta nu over k T or 1 over T is equal to k over beta nu log of u nu over alpha nu cubed.

And therefore, I have 1 over T which is minus k over beta nu log of u over alpha nu cubed equals d s nu over d U nu, we integrate this and we get s nu; small s nu is equal to integral minus k over beta nu log nu minus log of alpha nu cubed d of u which is minus k over beta nu u log u minus u minus log of u log of alpha nu cubed let me simplify this.

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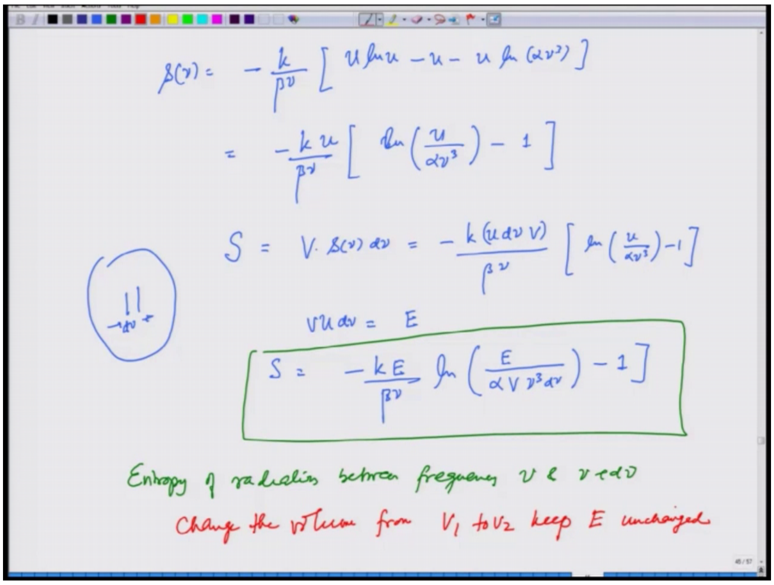


So, let me see; what we got. We have got s nu equals minus k over beta nu k over beta nu inside, I get u log of u minus u minus u log of alpha nu cubed which I can write as minus k over beta nu u, I can take out log of u over alpha nu cubed minus 1. This is s nu, this is the entropy density.

So, now I consider in this cavity a small interval of d nu. So, the entropy of the entire cavity is going to be volume times s nu d nu which I can write as minus k u d nu times V over beta nu log of u over alpha nu cubed minus 1. Now notice that u d nu times V is the energy contain between u and the nu. So, S is equal to minus k times the energy over beta nu log of I can write this as u d nu times V again. So, energy over alpha volume nu cubed d nu minus one this is the entropy of radiation. So, I can write this entropy of radiation between frequency nu and nu plus d nu, in volume V. Now consider change the volume from V 1 to V 2 keeping E unchanged keeping the energy of the system unchanged.

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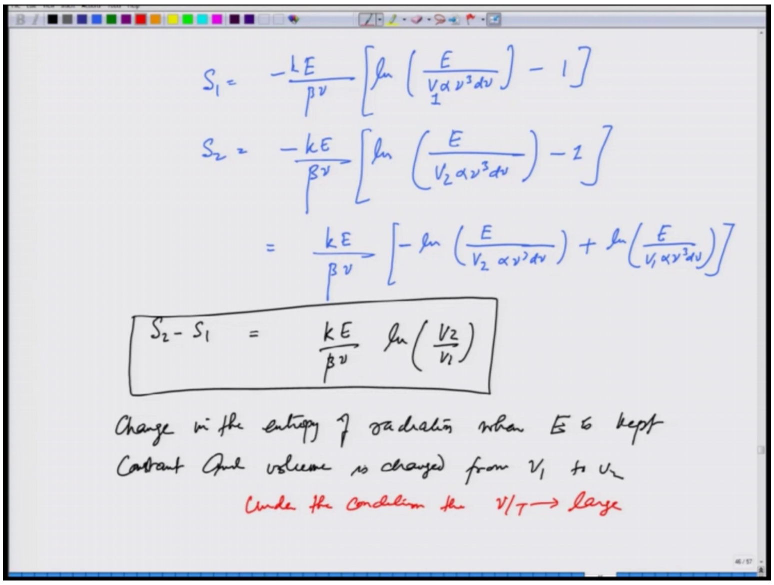


Then you are going to get. So, S 1 is going to be minus k e over beta nu log of E over V alpha nu cubed d nu minus one this is V 1, S 2 is going to be minus k E over beta nu log of E is remaining unchanged, V 2 alpha nu cubed d nu minus 1 is going to be k E over beta nu log of E over V 2 minus log of E over V 2 alpha nu cube d nu plus log of E over V 1 alpha nu cube d nu.

So, this gives S 2 minus S 1 is equal to k E over beta nu log of V 2 over V 1. So, this is the change in the entropy of radiation when E is kept constant and volume is changed from V 1 to V 2 and this is also under the condition that nu over T is large. So, T is small.

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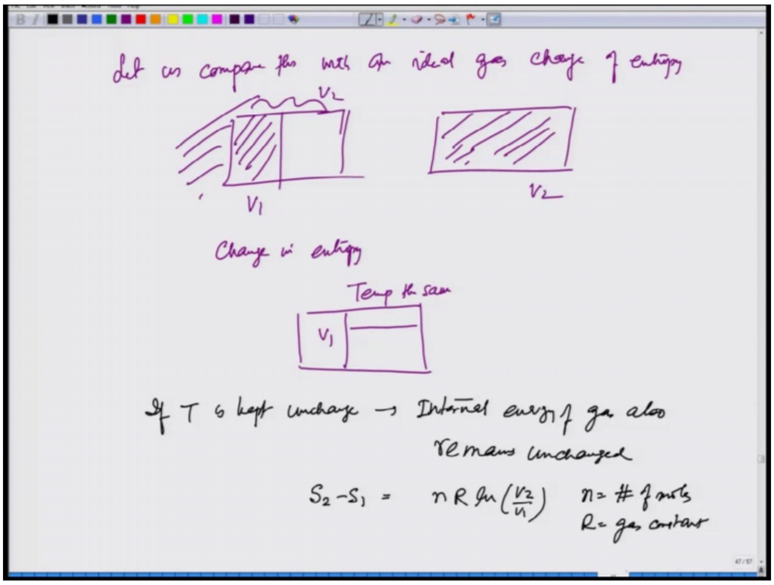


Now, let us compare this with an ideal gas change of entropy, if I take an ideal gas fill 1 half or 1 portion of a container which is isolated from surroundings right of V 1 and this whole volume is V 2 and puncture this wall.

When I puncture this wall, what happens is the entire container gets filled and there is a change in entropy and this change in entropy can we calculate very easily what we do is push this gas back to volume V 1 keeping the temperature the same. If the temperature is kept the same if T is kept unchanged the internal energy of gas also remains unchanged and I leave it as an exercise for you to show that; in this case, if we calculate the entropy change you get S 2 minus S 1 to be equal to n R log of V 2 minus V 1 V 2 over V 1 where n is the number of mols and R is gas constant.

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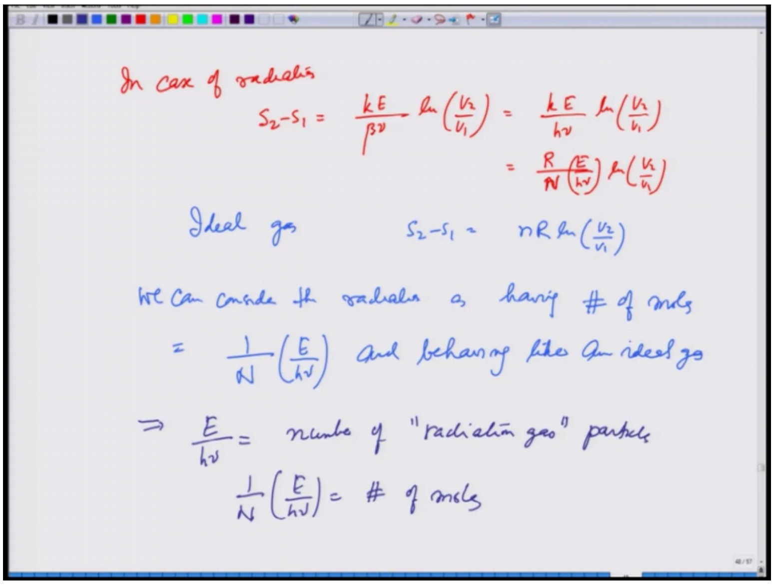


So, what we have learnt is that in case of radiation the change in entropy S 2 minus S 1 come out to be k E over beta nu log of V 2 over V 1, we know the value of beta is h. So, this is k E over h nu log of V 2 over V 1 which I can write as k is nothing, but R over n, n is the Avogadro number E over h nu log of V 2 over V 1 and in gases S 2 minus S 1 comes out to be n R log of V 2 over V 1.

So, this means, we can consider the radiation as having number of mols equals 1 over n R over h nu, right number of mols equals this much and behaving like an ideal gas. So, this also implies then immediately that E over h nu is equal to the number of radiation gas particles. So, that one over n; n is Avogadro number E over h nu becomes number of mols.

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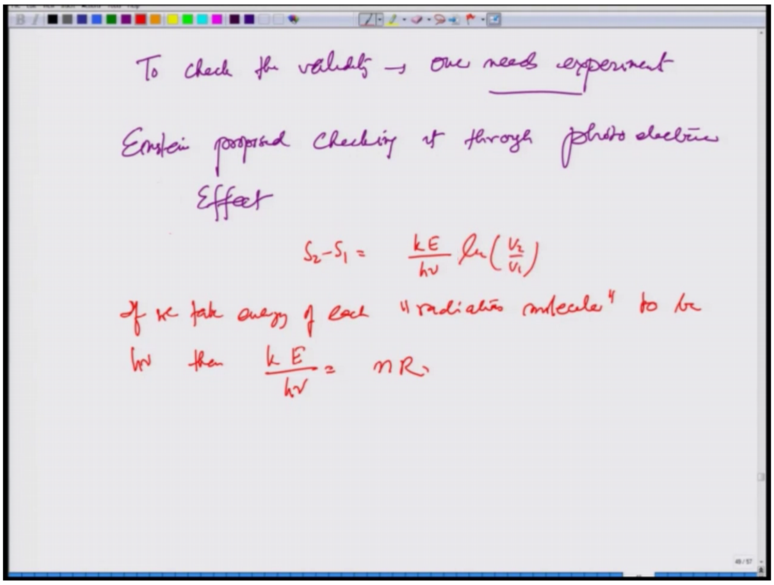


So, this analogy between entropy change of the radiation when it is allowed to changes volume without change in the energy and ideal gas gives you an idea that it can be treated like collection of molecules, but that is just a showing it theoretical to check the validity one needs experiments. So, for that Einstein proposed checking it through photo electric effect. So, let me this give you again that S 2 minus S 1 come out to be k E over h nu log of V 2 over V 1 and if we take energy of each radiation molecule to be h nu then k E over h nu comes out to be n R.

Just like ideal gas and that is what gives Einstein the idea that I can think of this as a collection of particles with each having energy h nu and n becomes n E by h k E by h nu becomes n R and then through photoelectric effect, it is checked and everything comes out to be correct, I am not going to do photoelectric effect here because you studied it many many times in your 12th grade and so on. So, this is what gave the idea of radiation; also having this party particle nature of h nu and experimental evidence was photoelectric effect. He also applied it to some other effects; call the; you know when the radiation goes out, radiation hits an atom and goes out, its frequency changes such that the wavelength increases is frequency goes down. So, it is given part of his energy through the atom.

What we will do next is again apply Einstein ideas; Einstein's to study the low temperature behavior of solids to see that the ideas of quantization of an oscillator can also be applied to mechanical oscillators.