Introduction to Quantum Mechanics
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Lecture - 07
Black Body Radiation VII - Quantum hypothesis and Planck's distribution formula

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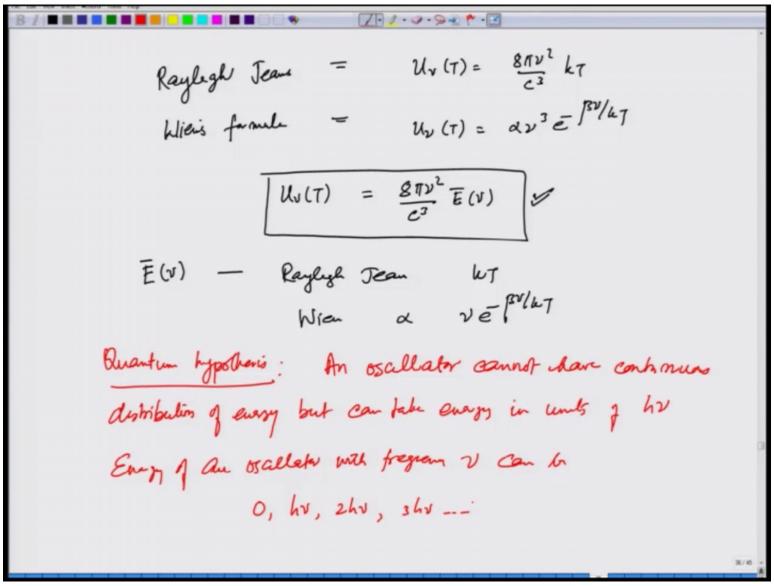

We have seen that there are 2 formula; Rayleigh Jean's and Wien's formula that give you radiation density; this gives you u nu T equals 8 pi nu square over C cubed k T and this gives you u nu T equals sum alpha nu cubed e raised to minus beta nu over k T and (Refer Time: 00:47) is somewhere in between because the curve does not fit both ends one limit or the other. And to resolve this and also u nu T should be of the form 8 pi nu square over C cube E bar nu. This is the correct formula.

This formula is derived in an exact manner. So, what we need is E bar nu which according to Rayleigh Jeans is k T and according to Wien is proportional to nu e raised to minus beta nu over k T and truth is somewhere in between. So, this is resolved by quantum hypothesis, it says that an oscillator cannot have continuous distribution of energy, but can take energy in units of h nu; h is some constant. So, energy of an oscillator with frequency nu can be 0 h nu, 2 h nu, 3 h nu and so on, but cannot be continuous classically we had continuous distribution, but now it cannot be continuous.

Let us see how this helps in getting the right formula or the plan's formula for correct formula for the distribution of spectral density of the radiation.

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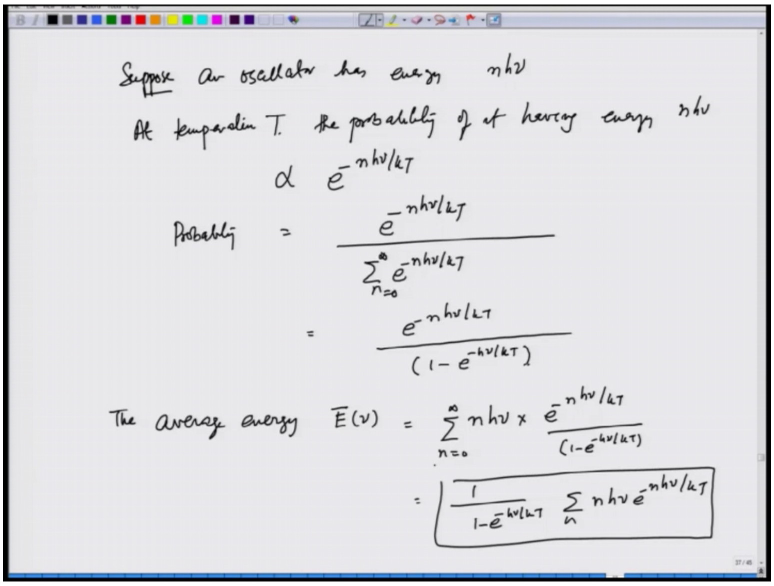

So, suppose an oscillator has energy n h nu and at temperature T, the probability of it having energy n h nu is proportional to e raised to minus n h nu over k T; that is the probability proportional to. So, the probability will be equal to e raised to minus n h nu over k T and I sum over all this n and e raised to minus n h nu over k T and it vary from 0 to infinity, energy could be all the way up to infinity I starting from 0. So, this is the probability because this is proportional to e raised to minus n h nu by k T, I normalized it by this and you can see that the net the probability of having any value is going to be one; some energy, right.

So, now this is the probability which I can write as e raised to minus n h nu over k T divided by I can add this as the geometric series which is going to be 1 minus e raised to minus h nu over k T. So, the average energy E nu; E bar nu is going to be energy n h nu times the probability of having that energy which is e raised to minus n h nu over k T over 1 minus e raised to minus h nu over k T summed over n, n equal 0 to infinity. So, I am going to write this as 1 over 1 minus e raised to minus h nu over k T sum over n n h nu e raised to minus n h nu over k T and have to evaluate this let us do that.

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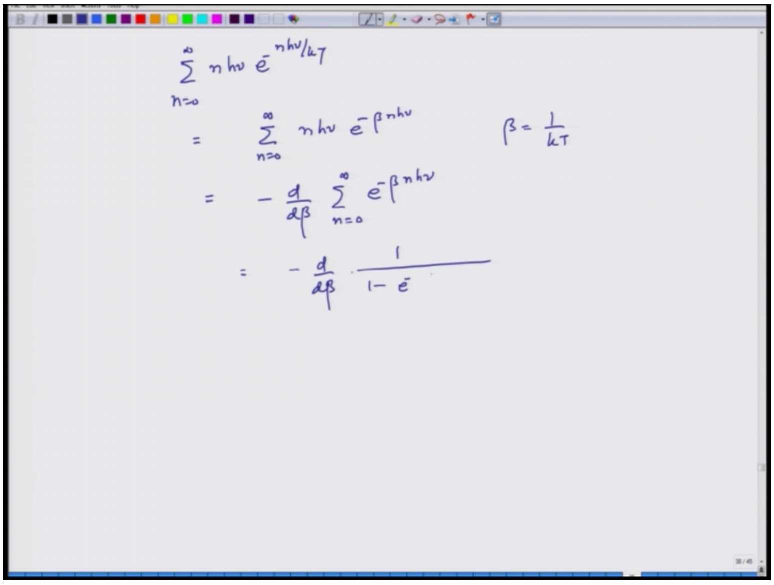

So, to calculate summation n h nu e raised to minus n h nu over k T n equal to 0 to infinity, I am going to write this as write k T equals beta. So, I am going to write this as summation n equals 0 to infinity n h nu e raised to minus beta n h nu where beta is 1 over k T and this can be written as minus d by d beta of summation n equals 0 to infinity e raised to minus beta n h nu, but this sum we just now did. So, this is minus d by d beta of 1 over 1 minus e raised to minus beta h nu even this energies we want to calculate the average energy of the oscillator with frequency nu.

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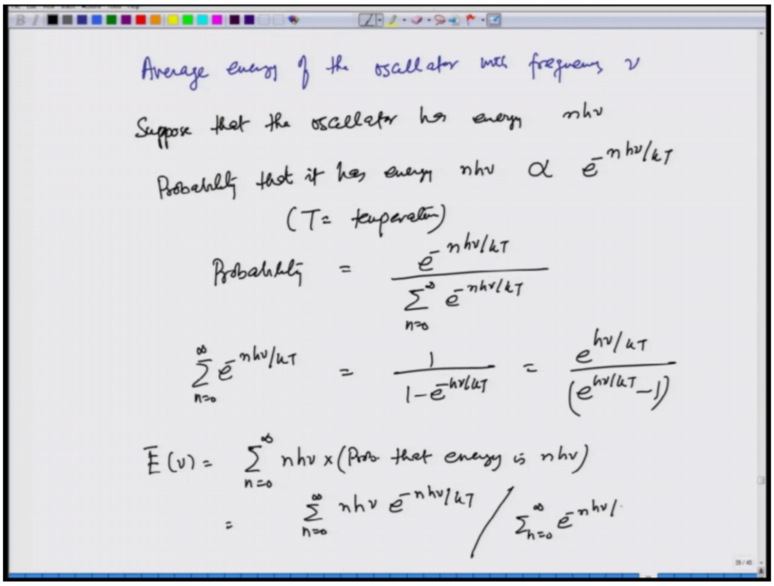

So, that let us suppose that the oscillator has energy n h nu then according to Boltzmann, the probability that it has energy n h nu, this proportional to e raised to minus n h nu over k T where T is the temperature.

And therefore, probability by itself is going to be e raised to minus n h nu over k T divided by summation n equals 0 to infinity e raised to minus n h nu over k T. Now we have normalized the whole thing, right. Now summation n equal 0 to infinity e raised to minus n h nu over k T is nothing but 1 over 1 minus e raised to minus h nu over k T which is equal to e raised to h nu over k T over e raised to h nu over k T minus 1.

So, this we have calculated. Therefore, average energy e nu is going to be equal to energy n h nu times probability that energy is n h nu summed over n equal 0 to infinity. So, I calculate multiply by the probability of it having energy n h nu and then sum over. So, this comes out to be equal to summation n equal 0 to infinity h n h nu e raised to minus n h nu over k T divided by that whole thing summation n equal 0 to infinity e raised to minus n h nu over k T.

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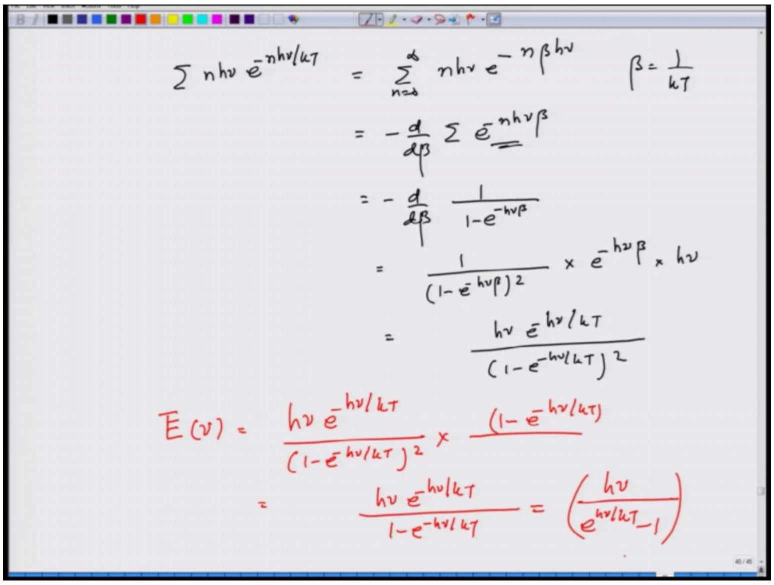

So, let us now calculate this, to do this I do a trick summation n h nu e raised to minus n h nu over k T, I write as summation n equal 0 to infinity n h nu e raised to minus n beta h nu where beta is 1 over k T and this can be written as minus d by d beta of summation e raised to minus n h nu beta, but this we have calculated. This comes out to be minus d by d beta of 1 over 1 minus e raised to minus h nu beta which is then equal to 1 over 1 minus e raised to minus h nu beta square times e raised to minus h nu beta times h nu.

So, this is equal to h nu e raised to minus h nu over k T divided by 1 minus e raised to minus h nu over k T whole square. Therefore, e average nu is equal to h nu e raised to minus h nu over k T over 1 minus e raised to minus h nu over k T square times whatever we had calculated earlier as summation over e raised to minus n h nu over k T and that is 1 over 1 minus e raised to minus h nu over k T which comes out to be h nu e raised to minus h nu over k T divided by 1 minus e raised to minus h nu over k T which is equal to h nu over e raised to h nu over k T minus 1.

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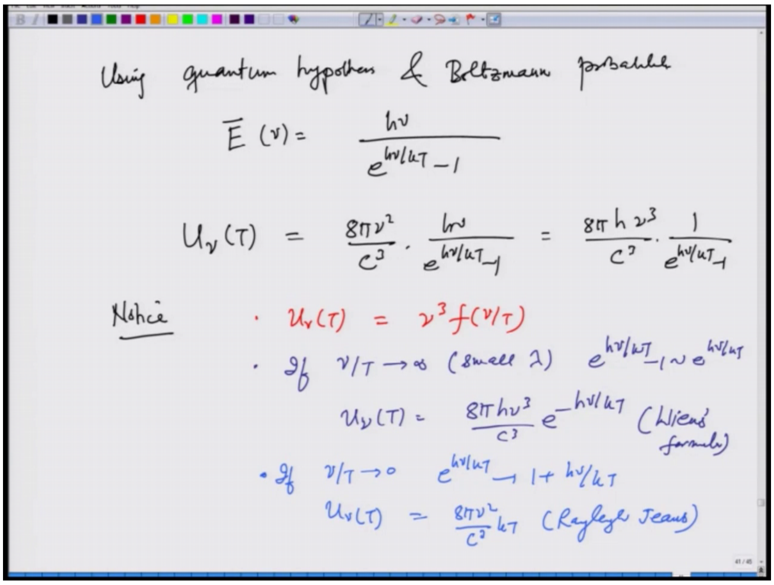

So, what we have found is that using quantum hypothesis and Boltzmann probability that average energy of the oscillator is h nu over e raised to h nu over k T minus 1 and therefore, u nu T is going to be 8 pi nu square over C cubed h nu over e raised to h nu over k T minus 1, notice which is 8 pi h nu cubed over C cubed 1 over e raised to h nu over k T minus 1, notice u nu T is of the form nu cubed f nu over T number 1. Number 2 if nu over T goes to infinity that is very large or small lambda then e raised to h nu over k T minus 1 will go to e raised to h nu over k T and u nu T becomes 8 pi h nu cubed over C cubed e raised to minus h nu over k T which is Wien's formula.

And if nu over T goes to 0 which is very small, then e raised to h nu over k T goes to 1 plus h nu over k T and u nu T goes to 8 pi nu square over C cubed k T which is Rayleigh Jeans.

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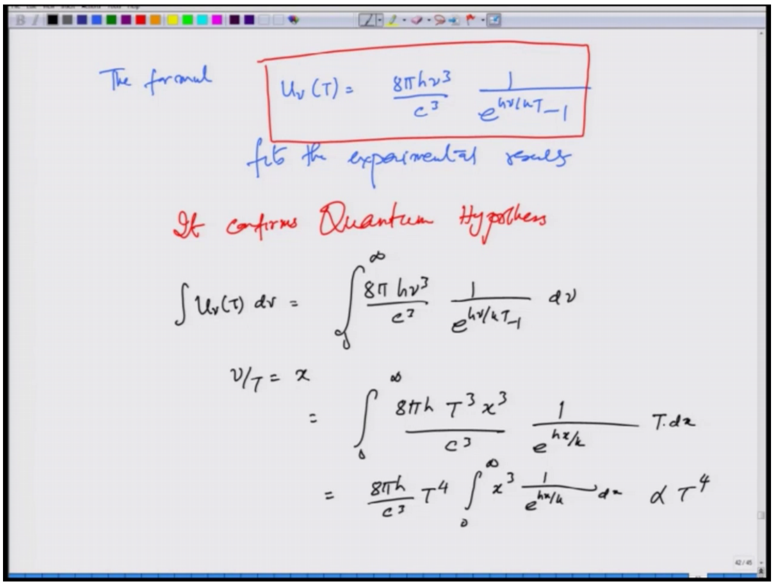

So, this formula derived by quantum hypothesis goes correctly to Wien's formula and Rayleigh Jean's formula in the right limits much more important, the formula u nu T equals 8 pi h nu cube over C cubed 1 over e raised to h nu over k T minus 1 fits the experiments. So, this is the formula for the spectral density and it confirms quantum hypothesis that is that the oscillator with frequency nu can take energies only in units of h nu.

How about Stefan Boltzmann's law? So, let us integrate u nu T d nu, it comes out to be 8 pi h nu cubed over c cubed 1 over e raised to h nu over k T minus 1 d nu going from 0 to infinity, substitute nu over T to be equal to some x then this formula becomes 0 to infinity 8 pi h T cubed x cubed over c cubed 1 over e raised to h x over k T d x which is nothing, but 8 pi h over C cubed T raised to 4 integral x cubed 1 over e raised to h x over k d x 0 to infinity which is proportional T raised to 4 and you can determine the coefficient for the Stefan Boltzmann coefficient for this.

So, to conclude this lecture what we have shown you is that the Rayleigh Jean's formula and the Wien's formula are 2 limits of the correct formula which is derived from quantum hypothesis. So, from now on we assume that the energy levels of an oscillator are quantized.

What other evidences for quantization could be there that we will see in the next lecture that you can extend this hypothesis further to include radiation that radiation should also be quantized and also a mechanical oscillators should be quantized. So, far we have said only electromagnetic oscillators, those oscillators that are giving out radiation are quantized, we will see that mechanical oscillators will also be quantized and they explain the behavior of specific heat of solids at low temperatures. So, that the evidence for quantization becomes stronger and stronger and then will take off from there.