Introduction to Quantum Mechanics

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Lecture – 06

Black Body Radiation VI- Wein's distribution law and Rayleigh- Jeans distribution law

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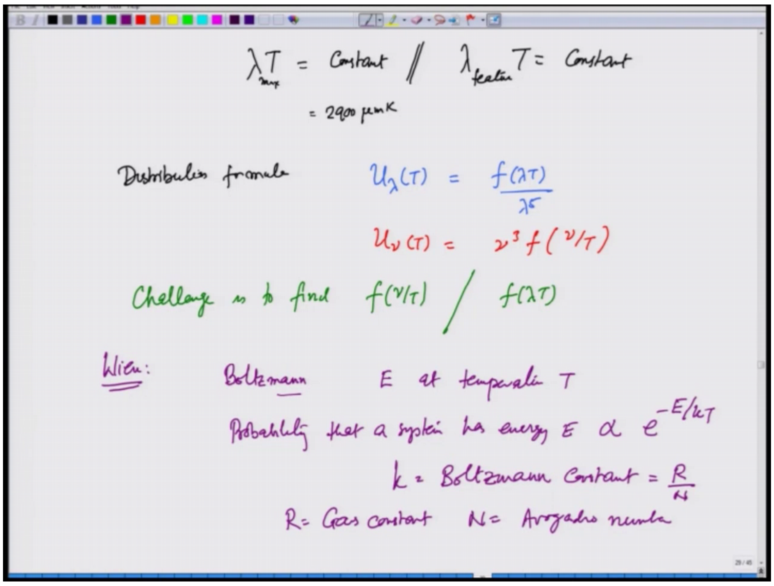

So, what we have learnt so far about black body radiation and its spectral density is that the spectral density is going to be the radiation is such that lambda max times T is a constant. In fact, what is true is at lambda any feature times T is a constant and that feature be happened to take to be the lambda max, and this comes out to be the 2900 micrometer Kelvin. The other is the distribution formula which says that u lambda T is going to be of the form some function lambda T over lambda raised to five or of the form u nu T is equal to nu cubed some other function I may call it function f this is not the same function as the one given above nu over T. Up to this point everything is exact thermodynamically derived, the challenge is to find f nu over T or equivalently at function f lambda T.

So, Wien did a simple analysis. And he got a distribution formula what he said is I will I will just state this without any proof he used some name call the Boltzmann's analysis. What Boltzmann says and I will come back to this again that if there are systems that have energy E at temperature T. Then the probability that a system that can take many, many energies, but has energy E has energy E is proportional to e raised to minus E over k T we will use it again temperature is T and k is known as Boltzmann constant all right. And this is equal to R over N where R is the gas constants gas constant and N is the Avogadro number that is the value. So, the main point is the probability that a system has energy E if it can take many, many values under certain temperature T is e raised to minus E by k T.

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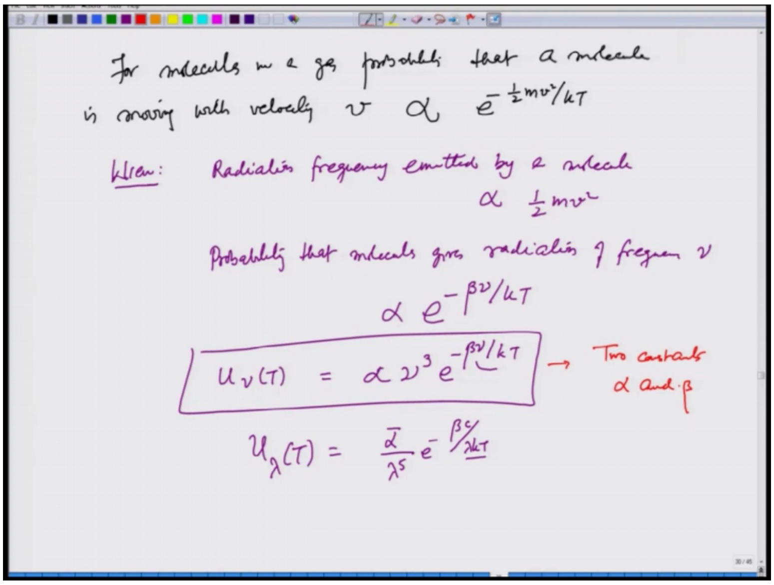

So, for molecules in a gas if they are freely moving the probability that that a molecule is moving with velocity v is going to be proportional to e raised to minuses energy one half m v square over k T. Now, Wien made an assumption. Wien says that the radiation frequency emitted by a molecule is proportional to its energy, this is an arbitrary assumption just to get the answer all right. So, the probability that molecule gives radiation of frequency nu is going to be e raised to proportional to e raised to minus some constant beta nu over k T. This is just an adhoc assumption. And therefore, he says that the u nu T is going to be some alpha, alpha is a constant times nu q because that is what this formula thermodynamically derived formula gave e raised to minus beta nu over k T.

Or in terms of lambda, this is going to be u lambda T is equal to some other constant right alpha bar over lambda raised to 5 e raised to minus beta over lambda k T. And there will be a c here all right these two formulas where alpha bar may involve some alpha time c; nonetheless this is the formula he proposed and you see this is of the same form it has a nu over T here and lambda T here. Exactly the form that he has proposed, but this is derivation is adhoc; however, there are two constants here right. So, let us just focus on this formula, two constants alpha and beta.

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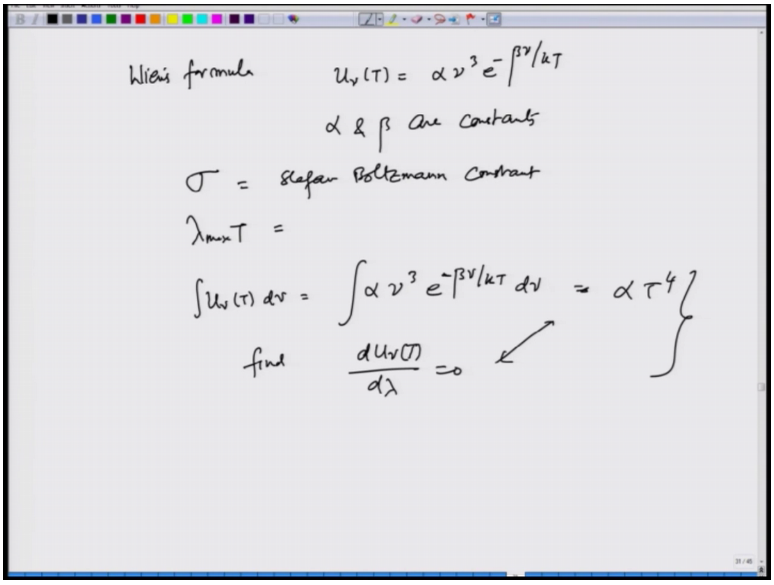

How do we determine these constants? So, Wien's formula is that u nu T is equal to alpha nu cubed e raised to minus beta nu over k T, alpha and beta are constants. How do we determine that? I already have two constants, I have sigma which is the Stefan-Boltzmann constant, and I have lambda max T which is displacement constant. So, I can integrate u nu T d nu which is equal to alpha nu cubed e raised to minus beta nu over k T integrated over d nu, and get an answer which will be proportional to T raised to 4. And I can also do find d u nu T over d lambda equals 0 and that gives me where is maximum at what lambda is maximum and through these two equations I can determine I will find beta. So, he gave a formula all right, and that formula turned out to be very good.

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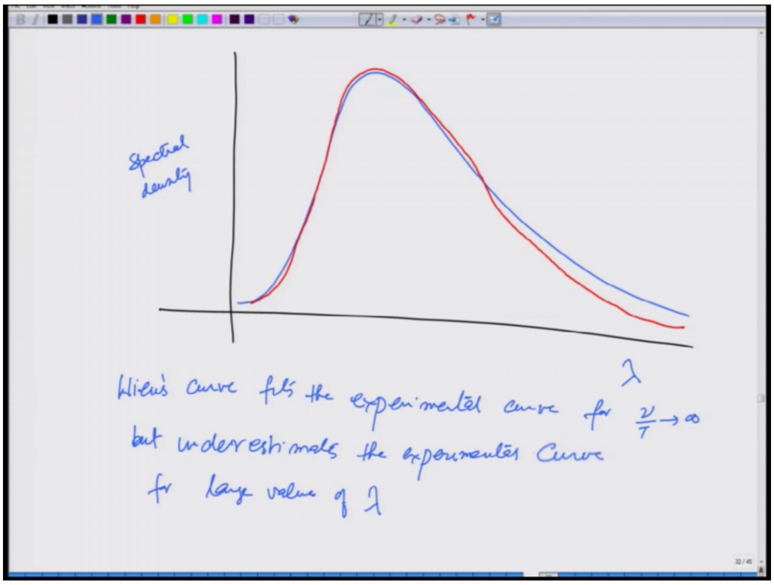

If you see it experimentally, the two curve is something like this, this is lambda, this is spectral density. And the Wien curve fitted right on top right on top except when you went to larger lambda it became slightly small. So, conclusion Wien's curve fits the experimental curve for nu over T going to very large value, but underestimates the curve for large values of lambda and this is when so the curve is very good the Wien was absolutely on the right track except that he was missing something.

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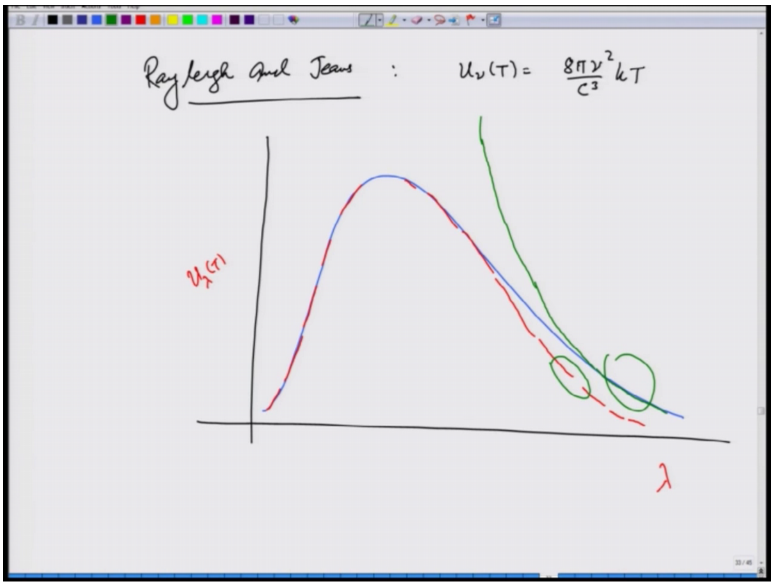

At the same time, there is another formula given by Rayleigh and Jeans, which said that the u nu T was 8 pi nu square over c cubed k T. How they got this formula, I will tell you in a few minutes, but let us see the two curves how do they look. So, this is the experimental curve the blue one. Rayleigh-Jeans is right on top right on top and when you reach large lambda, so this is lambda this is let us say u lambda T, where you got and then it is start deviating I am exaggerating, but starts deviating.

And the Rayleigh-Jeans formula matches beautifully out here and, but then it becomes large here. So, it matches only here and this is Wien's formula the true formula is somewhere in between that should match everywhere it should go to Wien's formula when you go to small lambda and it is toward to Rayleigh jeans formula. When you go to large lambda and that is where Planck came in and quantization hypothesis actually resolve this and that is what we want to do next.

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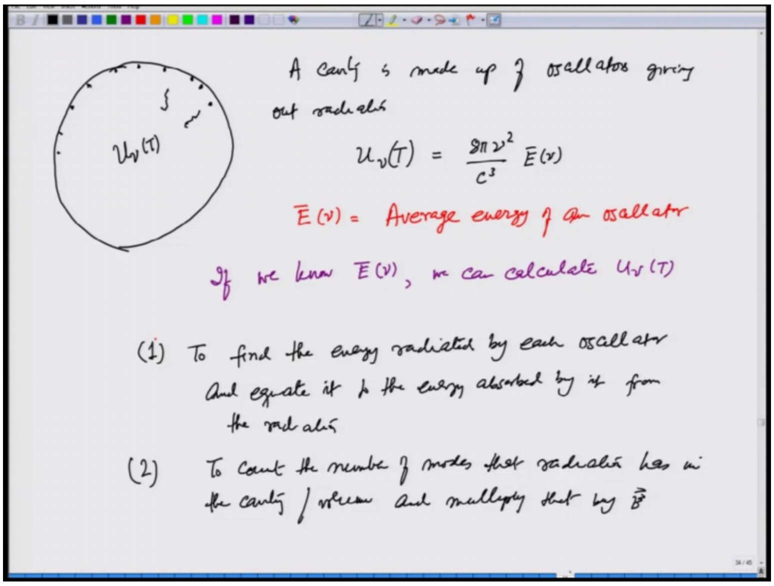

So, to understand how it comes about let us first write what is going to be our line of attack as planted. We are going to assume that there are lot of radiators, which give out lot of oscillators and radiators inside right, which give out radiation. So, a cavity is made up of oscillators giving out radiation, so that all this radiation fills up the cavity. And what we are going to show is that u nu T is going to be equal to 8 by nu square over c cubed times E bar nu, where E bar nu is the average energy of an oscillator.

So, what this does for me is if we know E bar nu we can calculate u nu T all right. Now, this can be derived in many different ways, I am not going to do it because this is going to take time and it slightly beyond the scope of this course, but if you are interested I can send you material and I can explain it to you later forum. But the weight is derived as a two ways one is to find the energy radiated by each oscillator and equate it to the energy absorbed by it from the radiation, you do that and you get precisely this formula. Second way is to count the number of modes that radiation has in the cavity per unit volume and multiply that by E bar both ways, this is the formula you get.

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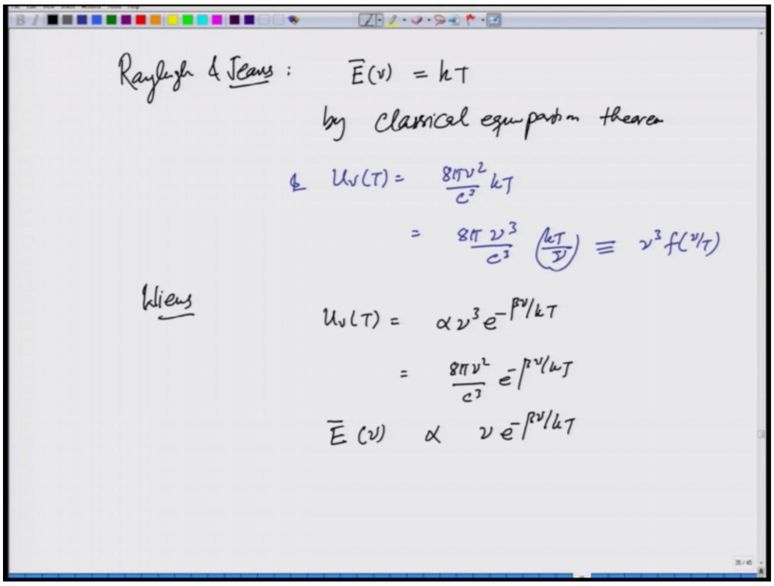

Now, what Rayleigh and Jeans did, was they said E bar nu is going to be go to be equal to k T by classical equipartition theorem and that gave them immediately that u nu T is going to be 8 pi nu square over c cubed k T, which can be written as 8 pi nu cubed over c cubed k T over nu. And you see this fits the formula nu cubed some function of nu over T. On the other hand, Wien said that n nu T is of the form some alpha nu cubed e raised to minus beta nu over k T. If I equate this 8 pi nu square over c cubed e raised to minus beta nu over k T it tells me that E bar nu is proportional to nu e raised to minus beta nu over k T.

And truth is somewhere in between and that is where quantum hypothesis is comes in and that is going to be the subject of the next lecture.