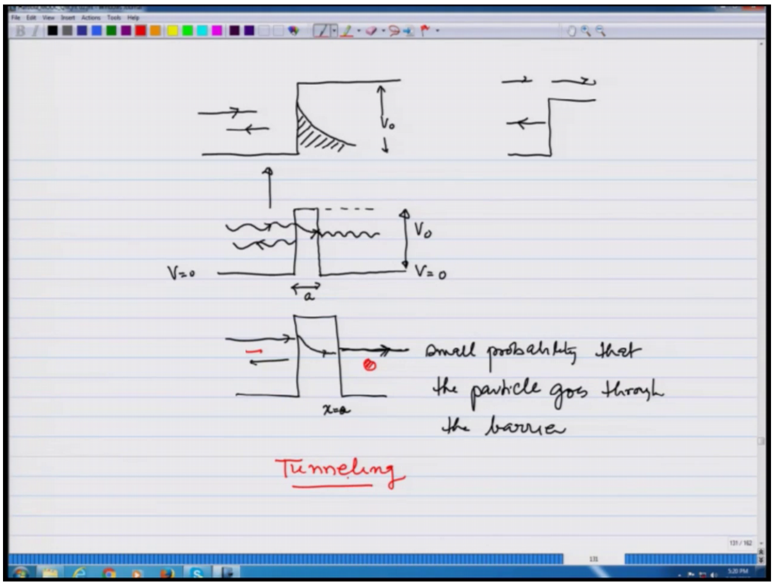
Introduction to Quantum Mechanics
Prof. Manoj Kumar Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture – 06
Quantum- tunneling and its examples

In the previous lecture we considered the reflection and transmission of particles, coming from the left to a barrier of height V 0 and showed that there will be particles that will be reflected there is a possibility that particles may be found in the region, where they cannot enter classically.

(Refer Slide Time: 00:17)

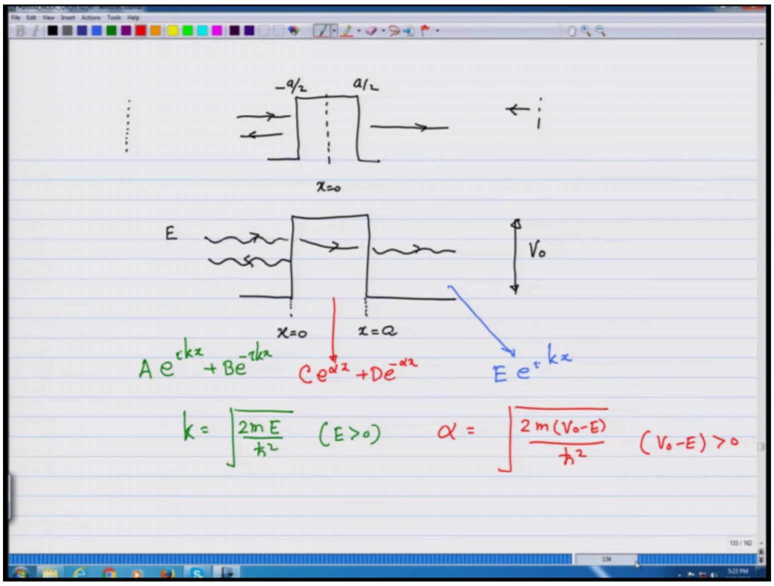


And we also showed that for energy is greater than V 0 energy is greater than V 0 particles can go through and also still reflect. The first case I also motivated you to think that if I make this barrier thin; that means, it does not go all the way to infinity, but I make it of finite width and potentiality become 0 again. So, this is V 0, and in between there is a height V 0 and the width of this is a then the particles which are coming in have the wave function coming in wave function going back. In the region in the middle the wave function may die down, but however, now since it is finite at the other boundary and the wave function has to be continuous to the other side, it will not be 0 to the other side.

So, what would happen is particle would come in let me make this picture again would come in get reflected, but there is a finite probability that will go through. Now since the wave function decay is as you go to the right, this wave function when it becomes continuous across boundary at x equals a would be small to start width, but non zero and therefore, there is a small probability that the particle goes through the barrier. Which is a purely non classical effect classically the particle would just go back, it cannot penetrate through, but here even if it is energy is low. So, as the particle has coming in suddenly you will find the sometimes is found on the right hand side. And this is known as the phenomena of tunneling. It is as if the particle has tunneled through the barrier.

So, we will study this phenomena in this lecture. Way before that I just want to answer a question sometimes a student's ask that earlier I had told you that the wave function is symmetric if the potential is symmetry.

(Refer Slide Time: 03:11)



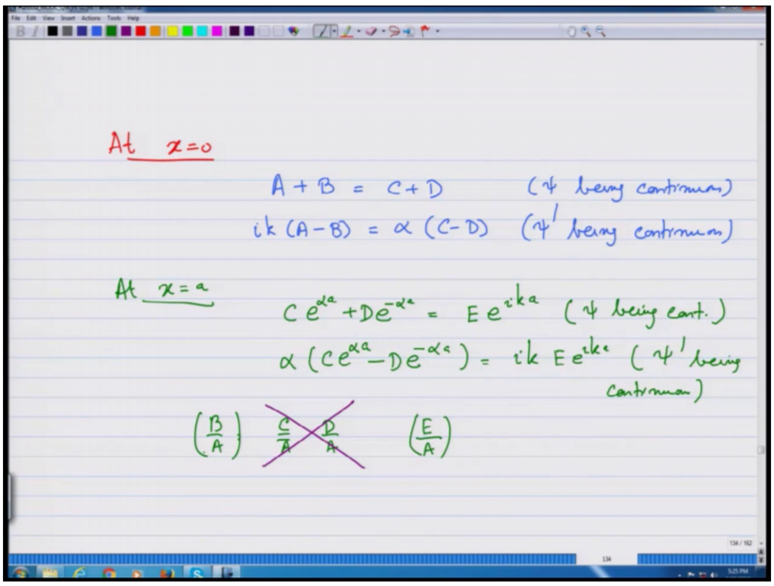
So, if I consider the barrier to be symmetric about x equals 0, this is minus a by 2 this is a by 2. How come I am only saying that particle has coming in going back and then going only to the right? And the wave function is not symmetric. This is because we have not made the boundary conditions symmetric. We are considering particles coming from the left, but on the right hand side there is no symmetry in that initially we did not consider any particle coming to the left.

If we made the whole thing stationery then the wave function will be symmetric there will be equal number of particles coming to from the right, equal number particles coming from the left and then they will be tunneling through, but we are taking a symmetric problem a symmetric boundary condition right from the beginning. And therefore, the solution is not symmetry now to the problem. So, what we have is this barrier of height V 0 and again to facilitate solutions I will shift back to one end of the barrier being at x equals 0, and the other hand being at x equals a.

There are particles coming from the left with energy less than V 0 with some energy e and therefore, by boundary condition there will be some reflection there will be some solution here and then there will be some transmission. So, let us write the wave functions on the left I am going to have a e raise to i k x plus B e raise to minus i k x i will define in a minute what k x are in the middle here I am going to have c e raise to alpha x plus D e raise to minus alpha x.

And on the right side, again I am going to have some solution E e raise to i k x k and alpha come from the solution the Schrödinger equation and we have done it again and again. So, I am not going to go over it and write directly that k is square root of 2 m E over h cross square, E of course, is greater than 0. And alpha is square root of 2 m V 0 minus E over h cross square, V 0 minus E is greater than 0. So, you found solution in 3 regions the only thing I have to now do is do the boundary condition matching.

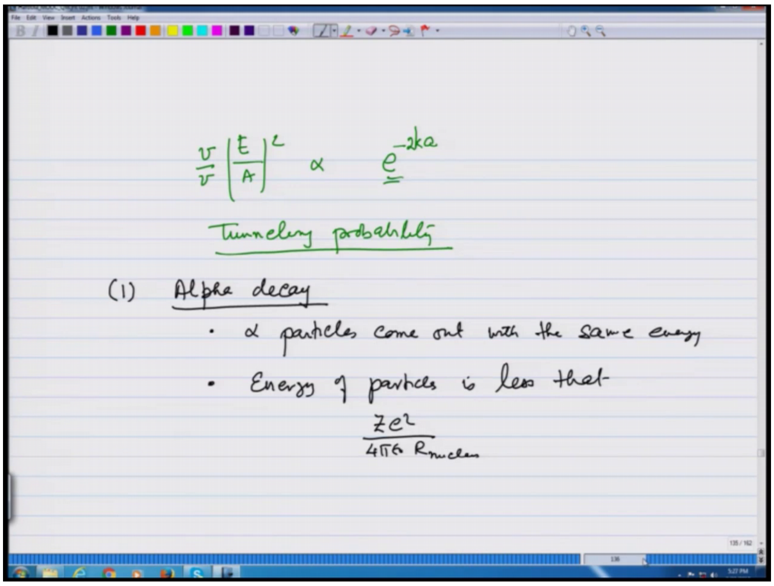
(Refer Slide Time: 05:58)



So, at x equals 0 I am going to have A plus B equals C plus D this come from psi being continuous. And I am going to have i k A minus B equals alpha C minus D and this comes from psi prime being continuous. And then at x equals a, I am going to have C e raise to alpha A plus D e raise to minus alpha a equals E e raise to i k a this again comes from psi being continuous.

And I am going to have alpha C e raise to alpha a minus D e raise to minus alpha a equals i k E e raise to i k a. And this comes from psi prime being continuous. So, I have got 4 equations and 5 unknowns all I am interested in is B over A, C over A, D over A. And In fact, if I am looking only for transmission probability I am not even interested in these and finally, I am interested in E over A, which gives me the transmission probability. So, we can match all these boundary conditions and get those answers, and what you find finally, is that E over A mod square V over V.

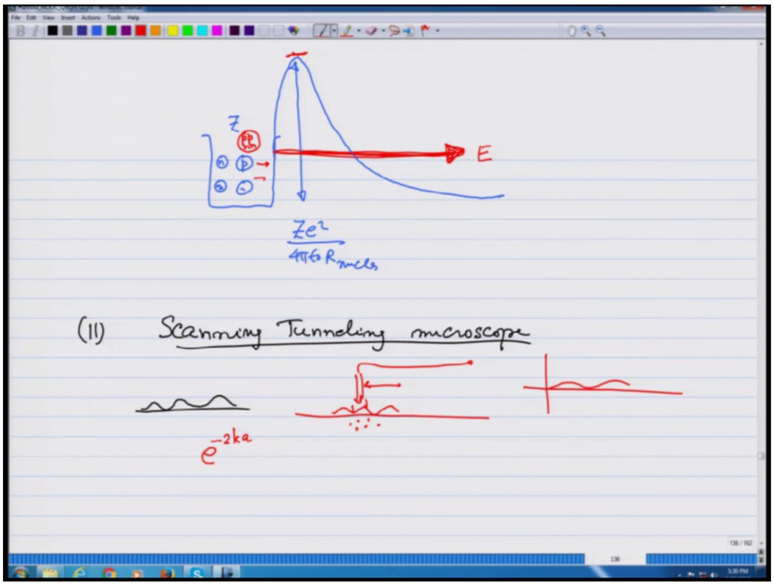
(Refer Slide Time: 07:55)



Because the velocity is the same on both sides comes out to be proportional to e raise to minus k 2 k a. So, as a increases the probability goes down nonetheless there is a non 0 probability that the particle tunnels through. So, this is tunneling probability. I will give you in the assignment the problem to calculate this exactly. What I want to focus on in this lecture is how this phenomena is used in explaining different subatomic events.

So, one it has been uses is to d explain alpha decay. I am not going to go quantitative on this I will describe this only qualitatively. What is seen in alpha decay is alpha particles come out with the same energy, number 1. Number 2 energy of particles is less than Z e square over 4 pi epsilon 0 r nucleus. So, how do they come through?

(Refer Slide Time: 09:34)

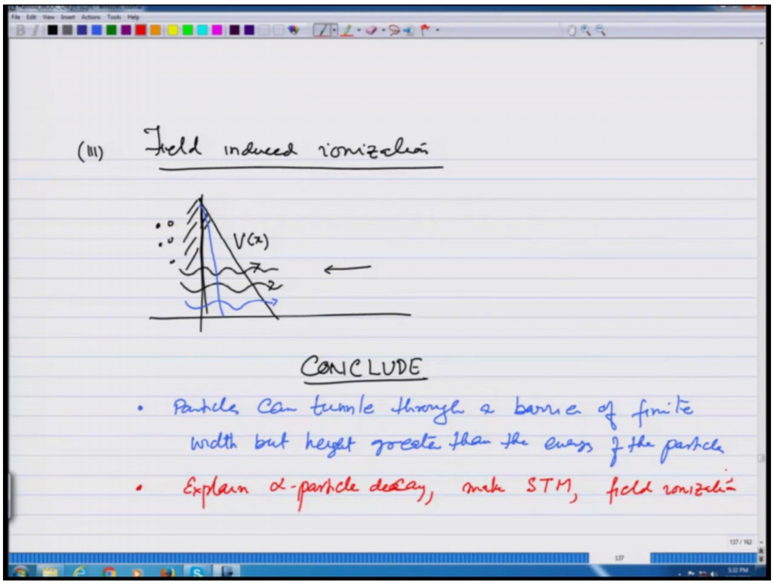


Let me explain this little more in details. So, suppose there is a nucleus in which these neutrons and protons neutrons and protons are there. And is charge is z, if I bring a positive charge particle from far away it is a coulombic potential which at the nucleus radius becomes of the height Z e square over 4 pi epsilon 0 r nucleus. And what is seen is that these alpha particles come at energy much lower than that. So, how did they climb up width? So, this was a mystery and the way it was explain was through tunneling. What people proposed is that inside the nucleus the alpha particle maybe already in the form of 2 protons 2 neutrons, and when they hit the wall of the nucleus then they can tunnel through. And therefore, they come out with the same energy and they cannot go up to the barrier, and then come down if they did.

So, the energy would be larger than the barrier height, but the energy is lower; that means, they are not actually climbing the barrier and coming down there all tunneling through. And then one can calculate this tunneling probability and relate that to the decay of nuclear through alpha particle decay and that fitted the curves. So, tunneling phenomena explained the alpha particle coming out from a nucleus with energy much less than the barrier height. Number 2 more recently with in past 30, 40 years tunneling has also been used to make something call the scanning, tunneling microscope that actually gives you how a surface where is on our surface has roughness. So, ideas is again that you take a tip, a small tip and make it go over a surface that you want to map or scan.

Now as I showed you earlier that the tunneling probability is proportional to minus 2 k a. So, if there this electrons inside depending on the roughness of the surface as the needle is passing over this surface, it will have more current if the surface is lifted up and less current if the surface is down. And therefore, the current in this if that is measured would be going up and down and then that can be used to map the roughness of the surface. And this has been used to see the roughness in the surface of the atomic level.

(Refer Slide Time: 12:43)



Third of phenomena which is known as field induced ionization. So, consider a metallic surface, and there these electrons inside. If I apply a strong field e it creates a potential V x wearing in front of this surface. And now electrons can tunnel through this barrier. Stronger the field large is the slope of v x and therefore, thinner the barrier.

So, stronger the feel more electrons can tunnel through and this is known as field induced ionization. So, these are 3 phenomena with that show you that tunneling does take place and whatever calculations are done through tunneling ionization tunneling probability does match whatever we measure. And so, is a very important phenomena purely of quantum mechanical origin. So, I will just conclude this lecture which is a very short one that particles can tunnel through a barrier of finite width, but height greater than the energy of the particle. And this has been used to explain alpha particle decay and to make scan in tunneling microscope use it in field ionization and so on.