Introduction to Quantum Mechanics
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Lecture - 03
Born-interpretation of the wavefunction and expectation values of x and p operators

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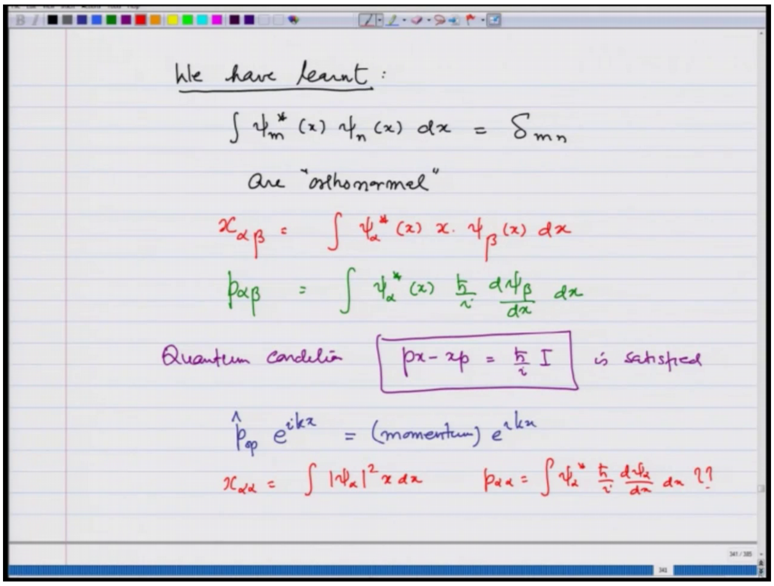

So, what we have learnt so far that wave function psi which are the solutions of the stationery state Schrodinger equation are orthonormal. So, are; I am repeating this. So, that you get familiar to these words and we have also found that I can define x alpha beta as integration psi alpha star x x psi beta x d x, I am using these indices alpha beta m n repeatedly. So, that you know that these are dummy indices and p alpha beta has integration psi alpha star x h cross over i d psi beta over d x d x and through these, we also learnt that quantum condition is satisfied p x minus x p equals h cross over i I is satisfied.

This made sense to define these operators like this because we also learnt that p operator acting on e raise to i k x gave me momentum times e raise to i k x again. So, this is how we extracted or got the moment out of the particle moving to the definite momentum. The question I left you with in the last lecture was what about x alpha alpha which is integration mod psi alpha square x d x and p alpha alpha which is integration psi alpha star h cross over i d psi alpha over d x d x; what do they imply and what does psi square mean and that is the question we are going to address in this lecture.

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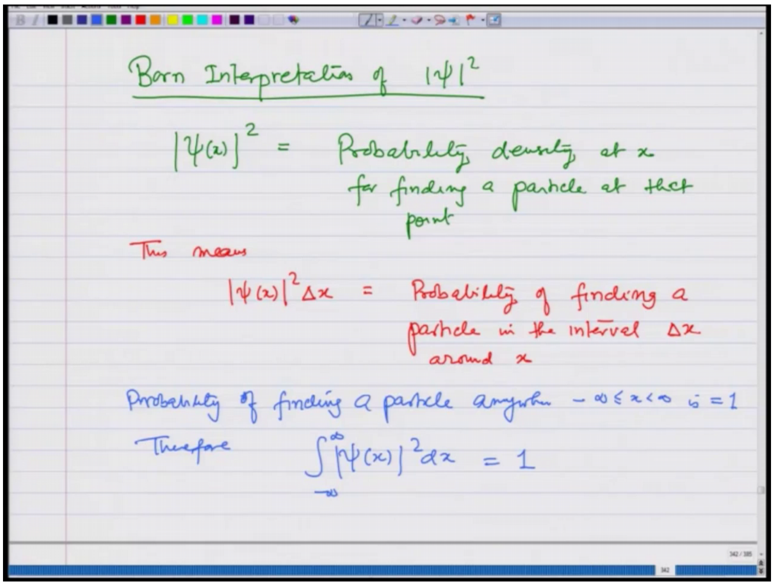

So, let me straight away give you born interpretation of mod psi square and then everything just sits together well. So, this becomes a very self consistent theory. So, psi x mod square is probability density at x for finding a particle at that point what does that mean this means that mod psi x square delta x is equal to probability of finding a particle whose wave function is psi x in the interval delta x around x. So, this is the probability density the psi x square is the probability density and this is the probability now probability of finding a particle all over spaces one. So, probability of finding a particle anywhere between minus infinity and infinity is one.

And therefore, we demand that psi x square d x p equal to 1 that is the total probability of finding a particle between minus infinity to infinity. So, remember from the first lecture this week there are saying that we are going to demand that the wave function being normalize. So, normalize wave function, now has an interpretation that this represents the probability of finding particle anywhere between minus infinity and infinity on the x axis and psi square x delta x gives me the probability of finding a particle in the interval delta x around x.

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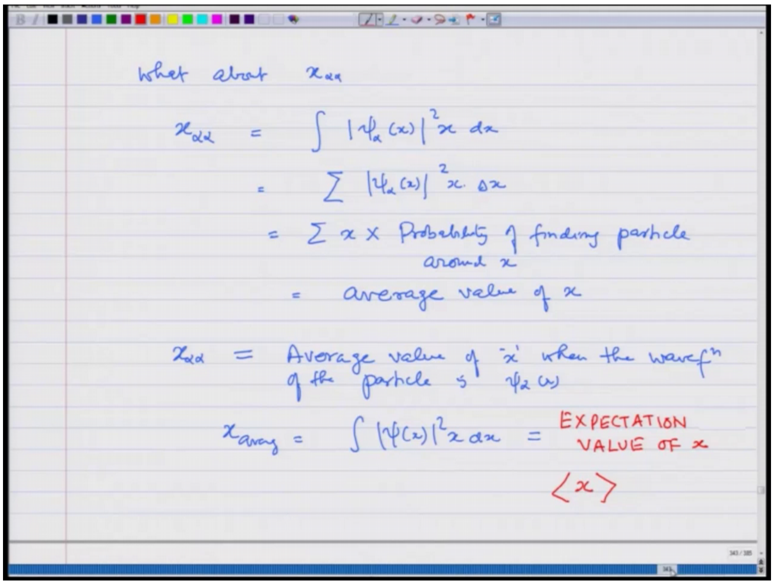

So, now let us see the value x alpha alpha. So, what about x alpha alpha? X alpha alpha is going to be equal to integration mod psi alpha x square times x d x which is nothing but if you break it into small intervals it is over each interval psi alpha x mod square except that point delta x which is equal to x times probability of finding particle around x and I am summing over all that small small intervals it. So, this is nothing but average value of x.

So, x alpha alpha gives me average value of x when the wave function of the particle is psi alpha x in general x average is going to be equal to mod psi square x d x and this is known as expectation value of x this is what you expect to see notice that once you represent the particle as a wave there is no way that you have a fix x for it. So, this is expectation or average value of x and this is denoted as x like this, this is the notation that you must follow what about p alpha alpha.

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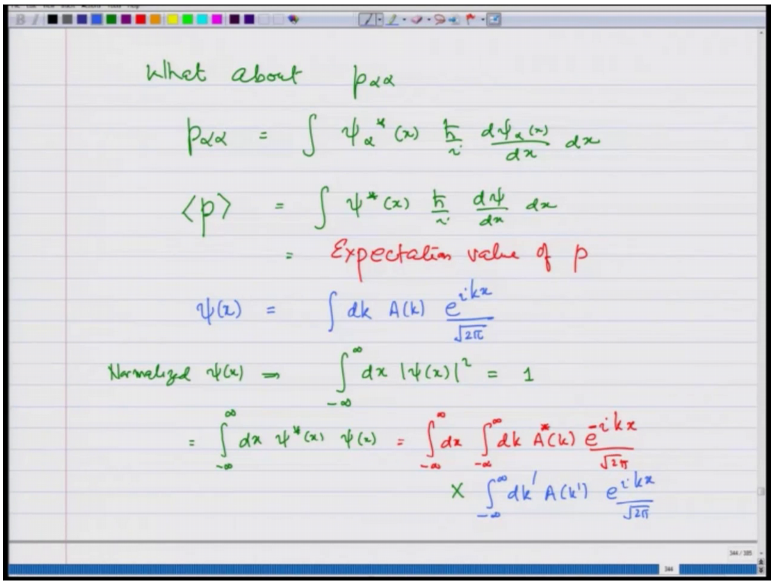

So, next question we raise is what about p alpha alpha, p alpha alpha is nothing but psi alpha star x h cross over i d psi alpha x over d x d x.

Or again going to call this the expectation value in general is going to be sum psi if the wave function is psi star x h cross over i d psi over d x d x integration. So, in general this is going to be called the expectation value of p or the average p and let me now tell you why. So, recall I can write any psi x in a Fourier transformation as integration d k a k e raise to i k x over square root of 2 pi this is the Fourier transformation and if psi x is normalized then a k should be such that they normalize the whole thing. So, let us see if psi x is normalize I have normalized psi x implies integration minus infinity to infinity d x mod psi x square is equal to 1.

Let us substitute that. So, I am going to write this as integration d x minus infinity to infinity psi star x psi x and write their Fourier transforms. So, that this becomes integration d x minus infinity to infinity integration d k a k e raise to i k x over root 2 pi, this is also minus infinity to infinity times or they should be a k star because I am writing psi star and there should be minus sign times integration minus infinity to infinity and I am going to write d k prime it is a dummy variable I should use different variables a k prime e raise to i k x over root 2 pi that is what psi star x.

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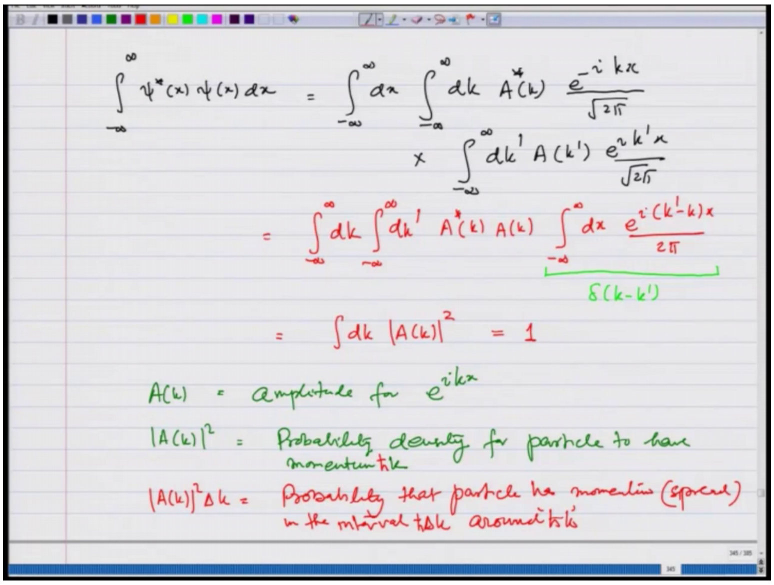

So, let us now write again. So, integration psi star x psi x d x minus infinity to infinity is equal to minus infinity to infinity d x integration minus infinity to infinity d k a k star e raise to minus i k x over square root of 2 pi times minus infinity to infinity d k prime a k prime e raise to i k prime x over root 2 pi which I am going to write as integration d k; I will pull this out minus infinity to infinity minus infinity to infinity d k prime a star k a k times integration minus infinity to infinity d x e raise to i k prime minus k x over 2 pi and this you can check through any mathematics book is nothing but delta k minus k prime and therefore, when I perform the whole integral this becomes equal to integration d k a k mod square and this is one because wave function is normalized.

So, this I can enterprise as is the amplitude for e raise to i k s which is the wave function for a particle having momentum h cross k and therefore, I can write that mod a k square is the probability density for particle to have momentum k and a k mod square delta k this is the probability that particle has momentum spread or momentum in the interval delta k h cross delta k around h cross k h prime h cross a psi and therefore, the net probability.

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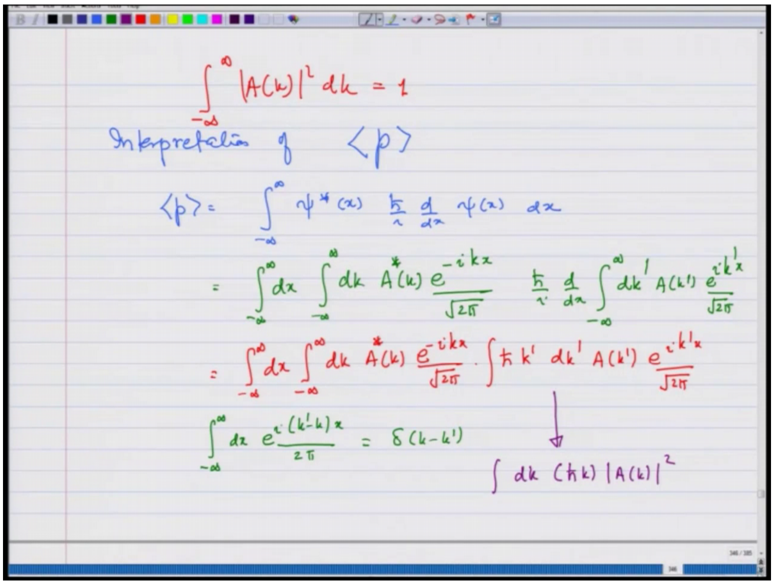

That has any momentum from minus infinity to infinity is one now for the interpretation of is p expectation value. So, I defined p as minus infinity to infinity psi star x h cross over i d by d x of psi x d x let me again write this in the Fourier expanded form Fourier transform this is minus infinity to infinity d x psi star becomes minus infinity to infinity d k a k star e raise to minus i k x over square root of 2 pi then I have h cross over i d by d x of integration minus infinity to infinity d k prime a k prime e raise to i k prime x over root 2 pi.

Which can be written as minus infinity to infinity d x minus infinity to infinity d k a k star e raise to minus i k x over root 2 pi times h cross k prime d k prime a k prime e raise to i k prime x over root 2 pi, I am going to again use the same trick as earlier and do the x integration first. So, minus infinity to infinity d x e raise to i k prime minus k x over 2 pi is nothing but delta k minus k prime and therefore, this integral becomes integration d k h cross k mod a k square which i live as an x psi for you. So, what we are getting?

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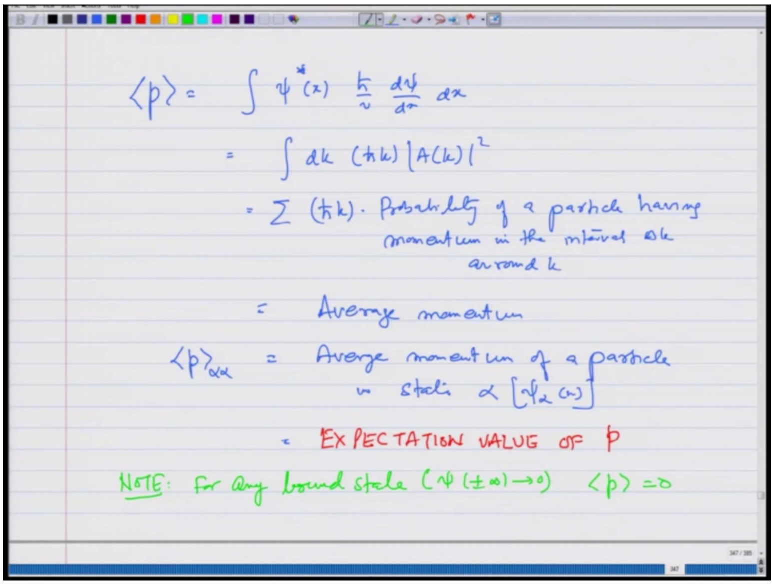

Now, is that this p expectation value which is psi x star h cross over i p psi over d x d x is nothing but integration d k h cross k a k mod square this is nothing but h cross k times probability of a particle having momentum in the interval delta k around k and this is sum to over all this intervals.

So, this is average momentum. So, p alpha alpha is average momentum of a particle n state alpha or psi alpha x i. So, this is also known as the expectation value of p. So, p alpha alpha gives you the expectation the; so, diagonal elements give you the expectation value or the average value for that quantity in a given state and of diagonal elements give you cross transition matrix elements. So, we have interpreted now the diagonal element through bonds interpretation of wave function just note that for any bound state and; that means, psi plus minus infinity going to 0 wave function does not go for away its bound expectation will u p will always come out to be 0 because wave function other the particle has equal probably of going to the left or going to the right this.

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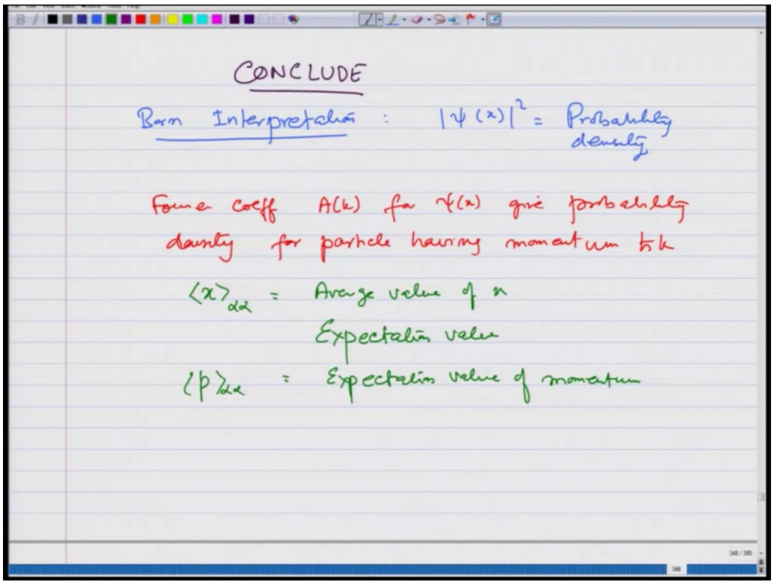

You can just right away you can also try to prove it. So, conclude this lecture, we have born interpretation at we says that mod psi x square is probability density along with it I also gave you the Fourier coefficient a k for psi x give probability density for particle having momentum x cross k and then we interpreted a x alpha alpha as the average value of x and gave it a name expectation value and p alpha alpha is also expectation value of momentum.

Now, again you will see that since the two formalism Heisenberg and Schrodinger are equivalent any conserved quantity that is time independent is going to have only diagonal terms quantities which are not conserved are going to have off diagonal terms also. So now, you shown in these three lectures that the 2 formalisms are equivalent and we have also interpreted the wave function through bonds interpretation; and we have also now know how to calculate the matrix elements through operators for the corresponding quantities.