Introduction to Quantum Mechanics
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Lecture - 03
Black Body Radiation III- Spectral energy density and radiation pressure inside a black body

In the previous two lectures, we have given you some basic definitions of a Black Body and represented this by a spherical cavity and if I want to study it the radiation at temperature T.

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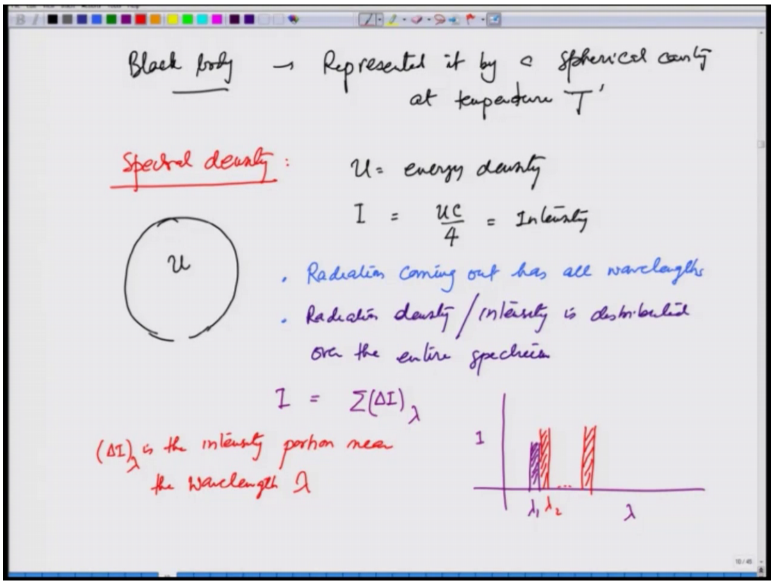

So, I would say spherical cavity at temperature T. What I want to define now is something called spectral density. You have already seen that in a cavity or in a black body, we define something called U; where U was the energy density and the result that we got in the previous lecture was that the intensity coming out of a black body is UC by 4, this is the intensity coming out.

Usually, if you have recalled your electromagnetic theory intensity UC and that is given for a plane wave. In this case, the intensity coming out becomes UC by 4 because the radiation is isotropic its coming from all direction as you saw in the derivation in the previous lecture.

Now, what spectral density does is that radiation which is coming out; so, let me write this slowly. Radiation coming out has all wavelengths, so it will have red color, yellow color, blue color, ultraviolet, infrared everything it has. And the radiation we can write the radiation density or intensity is distributed over the entire spectrum; that means, if I have the intensity I, it will be summation of some delta I for each lambda; let me explain.

So, if I look at lambda verses intensity curve, there would be some intensity delta I near say lambda 1; some other intensity I near lambda 2 and so on. So, in every interval there is some intensity which could be different for different lambda. So, let me write this, so where delta I lambda is the intensity portion near the wavelength lambda as I shown in picture and how does it look like.

I will now show through a picture downloaded from the internet.

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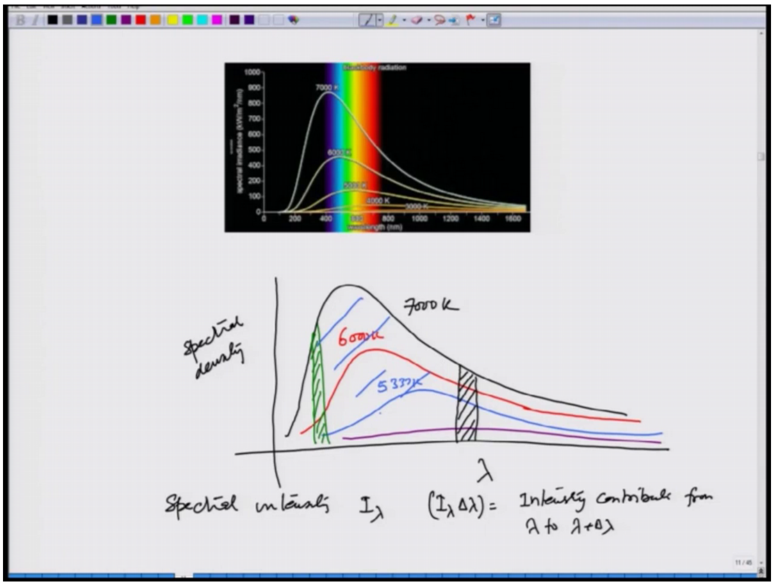

So, this is the image of intensity of radiation, so let me also show it to you; this is the image which you see and I will decide if it for in which intensity of radiation or what we will call spectral density of radiation is plotted and it is plotted at different temperatures. So, for example one curve looks like this, the second curve looks like this, third curve looks like this and the fourth curve looks like here. As the higher curve is the upper curve is at 7000 K; the middle curve red one is at roughly 6000 K; the blue one is at 5333 and so on; as the temperature goes down the curve becomes lower and lower and the P goes down and down.

What we mean here is if I take for spectral density; if I take a particular lambda and calculate the area under this curve; at this lambda or calculate area at say another lambda out here; this would give me the energy content in this d lambda. So, now I can define is spectral density for you is spectral intensity as such; I lambda as such that I lambda delta lambda is the intensity contribution from lambda to lambda plus delta lambda.

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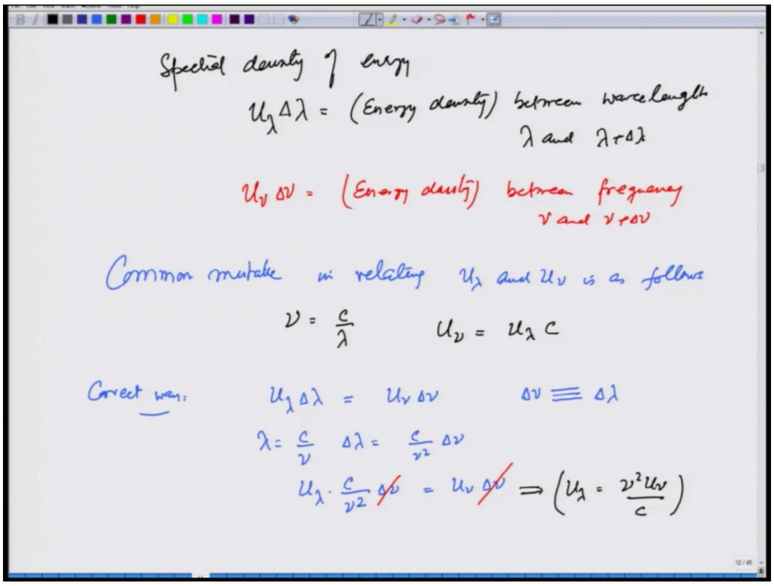

So, if I integrate over this entire curve it gives me the total intensity. In the similar manner I can define a spectral density of energy which U lambda such that U lambda delta lambda is the energy density between wavelength lambda and lambda plus delta lambda. Notice, equivalently I can also define U nu; delta nu which will be energy density between frequency nu and nu plus delta nu.

A very common mistake; let me point this out common mistake in relating U lambda and U nu is as follows. The common mistake is this see nu is equals C over lambda; one thinks that U nu should be equal to U lambda times C; which is not really correct. What you should look at is that the correct way is U lambda; delta lambda should equal U nu delta nu, where delta nu corresponds to delta lambda.

Since lambda is equal to C over nu delta lambda would equal C over nu square; delta nu and therefore, we have U lambda times C over nu square delta v is equal to nu delta nu. Because the energy content is the same and now I can cancel delta nu and I get U lambda is equal to nu square U nu over C since these are all magnitudes.

I am not varying about the minus sign, so be careful about this; this is a very common mistake that people make. So, I have designed a spectral density for you and theoretically analysis, I will need one more thing and that is I will do that and then this lecture gets over. The next quantity, we will need in our analysis is going to be pressure due to radiation inside a cavity and we will do a calculation of pressure due to radiation falling on this small area here from all over the place and that would be the pressure formula.

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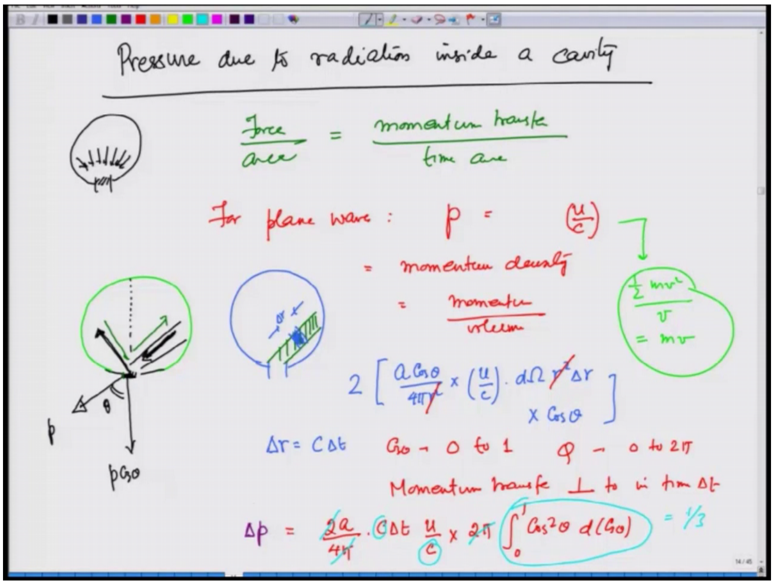

Now, for pressure what I need to calculate is force per unit area which is nothing, but momentum transfer per unit time; per unit area. So, what I will need now is the momentum that is falling on this area. Now for plane waves, if you recall from electromagnetic theory momentum P which is momentum content per unit volume is same as U over C.

Let us see that; so, this nothing, but momentum density that is momentum per unit volume per unit volume. You can see this is dimensionally correct because U has a dimension of one half m v square divided by v; which is same as m v. So, this is dimensionally correct this is an per unit volume; so, this is momentum per unit volume. Now what is happening is that in this cavity; at this area a, radiation is coming from all over and it is bringing in some momentum and that momentum applies force.

So, momentum per unit time whatever momentum transfer is there; the only component of momentum that matters is perpendicular to this which is at an angle theta. So, what I need to calculate is how much momentum is coming in take its component. So, suppose this is P; this has to be P cosine of theta and for every side that momentum comes in there is a momentum going out either by reflection or this part is also radiating so that there is a back by momentum conversation there is a back force.

So, net momentum is going to be I will calculate this and multiply it by 2. So, I can calculate the momentum coming from all sides. So, even if there is a momentum coming from this side; there will be momentum going out. So, I will calculate the total momentum coming in multiplied by 2 and that would give me the answer for the force. So, again we will do a calculation like what we did earlier.

I will consider this cavity and in this area; consider this volume from side from where the momentum is coming in. So, if I take again in d omega; this length delta r; then the momentum coming in is going to be a cosine theta over 4 pi r square, which is probability of the radiation coming this way. Then net amount of momentum would be U over C is the momentum content in this area.

In this volume; blue volume that I am showing and that volume is d omega; r square delta r. And for pressure calculation, I will further multiply this by cosine of theta and as I said earlier a factor of two because either the reflection or radiation emission from this area; so this would be the net momentum. Now, again as previously we have delta r; total length for time delta T is going to be C delta T.

So, I can write theta cosine theta is going to vary from 0 to 1; phi is going to vary from 0 to 2 pi. So, the momentum transfer normal to a in time delta T is going to be equal to 2 a over 4 pi; this r square cancels here delta r is C delta T. So, I have taken care of that then I have U over C then I have d phi integral which is 2 pi and then I have cosine square theta d cosine of theta 0 to 1 and that the momentum transfer; so, let us call this delta P.

Let us now cancel a few terms; this 2 pi cancels with this 2 and 4 pi; C cancels with this C. So, I get and this integral this integral is nothing, but one-third.

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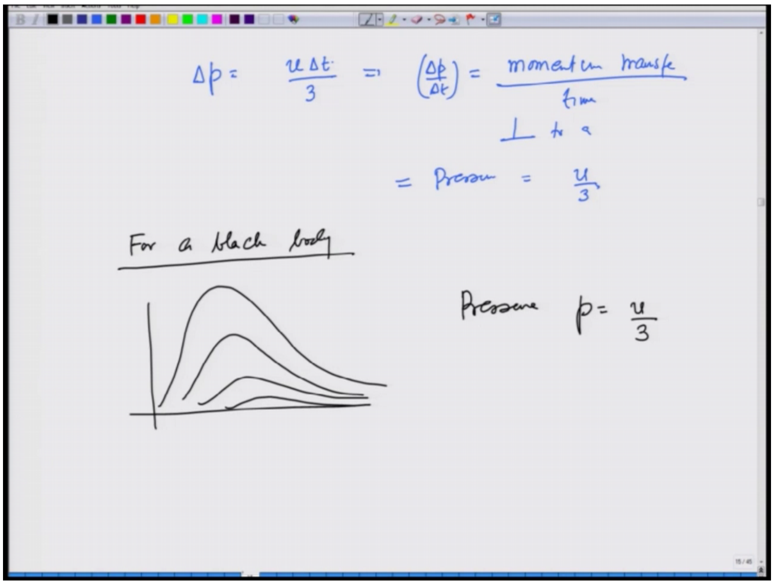

So, what I get is delta P is equal to U, delta T over 3 and this implies delta P by delta T is equal to momentum transfer per unit time normal to a. So, this is nothing, but pressure and this comes out to be U by 3. So, what we have seen now is that for a black body, this is spectral density which at high temperature is like this and as temperature level goes down; it becomes like this and pressure inside the cavity P is U by 3. We will use these for our analysis in coming lectures.