Introduction to Quantum Mechanics
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Lecture – 03
Matter waves and their experimental detection

In the previous 2 lectures are introduced you to the idea of how to describe waves how to write the mathematically, and also how stationary waves arise and a general displacement of a vibrating string can be express in terms of the Eigen functions, and Eigen frequencies of that string. We want to develop a wave theory for particles and therefore, those lectures will given. So, if you want to develop wave theory of particles why should it be done, and it is done because a moving particle has waves associated with it, and we want to describe this wave. What does it mean?

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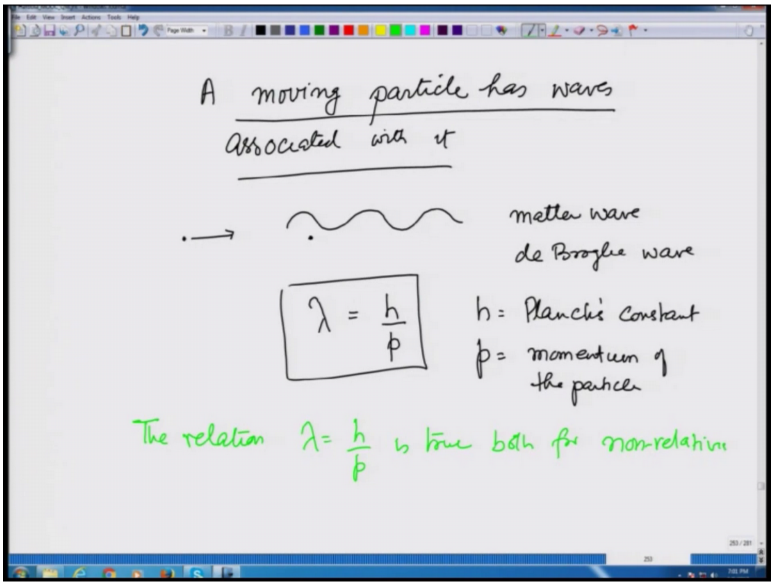

Let us understand that, what it means is that if there is a particle which is moving there is a associated wave.

So, particle is moving independently it has a wave associated which is known as matter wave or de Broglie wave, and it has a wave length lambda which has given by h by p where h is the Planck's constant, and p is the momentum of the particle. And this relationship which I am putting in a box is true relationship, lambda equals h over p is true both for non relativistic and relativistic particles. Let me explain what is that mean in there is a particles of mass m.

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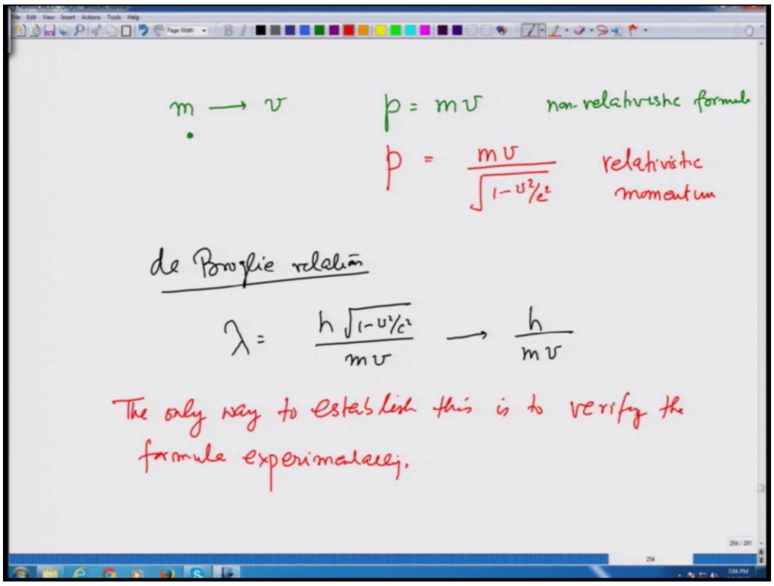

Moving with speed v is momentum you know is given as m v if the v is very large the correct formula for p is m v over square root of 1 minus v square over c square this is known as relativistic momentum, which for very small v goes over to the non relativistic formula. This is non relativistic formula and what de Broglie relationship says is that lambda will be given by h square root of 1 minus v square over c square over m v.

And goes to h over m v for a small speeds this is the proposal, this is cannot be derived cannot be obtained from any other equation, but it is a fundamental equation, it is that particles have this wave associated which is has a wave length h by m v. So, anybody who says that we are deriving this is I will cannot be derived, the only way to establish there. So, let me write this the only way to establish this is to verify the formula experimentally and if it comes out true experimentally then you know it is a relationship which is true.

So, I will describe in this lecture what experiments can be performed to check this relationship, but before that I will just want to give some sort of historical background has to how de Broglie arrived at this relationship, and I am just giving it is not a derivation it just that how people work how they think and propose something on the basis of they when they applying around with equations, but the ultimate check is experiments. If this formula that de Broglie proposed was not true experimentally, it would have no meaning. So, let us just see for the interest right. So, let me just write this historical.

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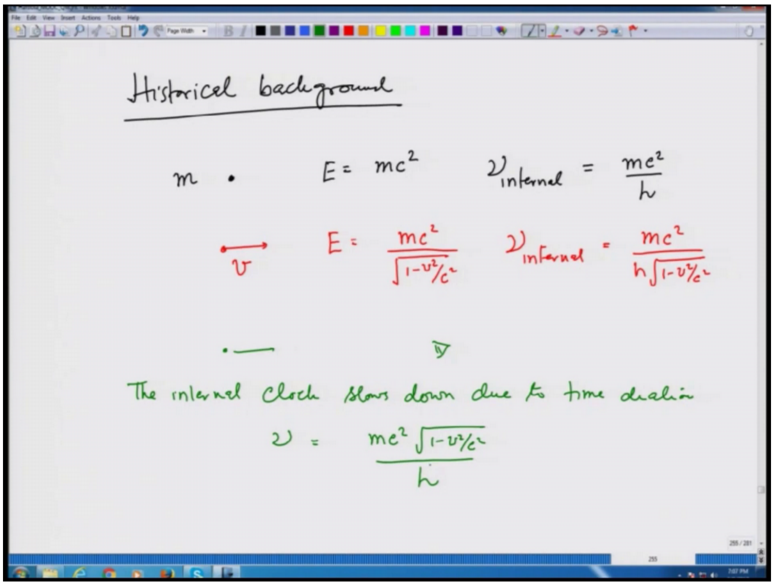

Background what de Broglie said is if there is a particle of mass m by Einstein relationship it has energy mc square and by Planck's relationship it has a some internal let us write internal frequency nu which is given by mc square over h. I am not deriving it I am just describing to you some historical background historical the way historically de Broglie was playing with his formulas, and when the particle moves with speed v is energy is given as mc square over square root of 1 minus v square over c square that is the correct relativistic formula.

And therefore, is nu internal is going to be equal to mc square over h square root of 1 minus v square over c square. You see nu internal has gone up a bit, on the other hand when this is moving and a person is watching it from here the internal clock that had this frequency nu internal slows down due to time dilation, and there is a different frequency nu which is observed from outside which is going to be equal to nu equals mc square root of 1 minus v square over c square over h, this frequency has gone down because the time dilation outside person sees this clock running slow.

So, for the same particle which is moving with speed v, we have 2 different frequencies, how do I reconcile the true, that was the dilemma that this person was facing.

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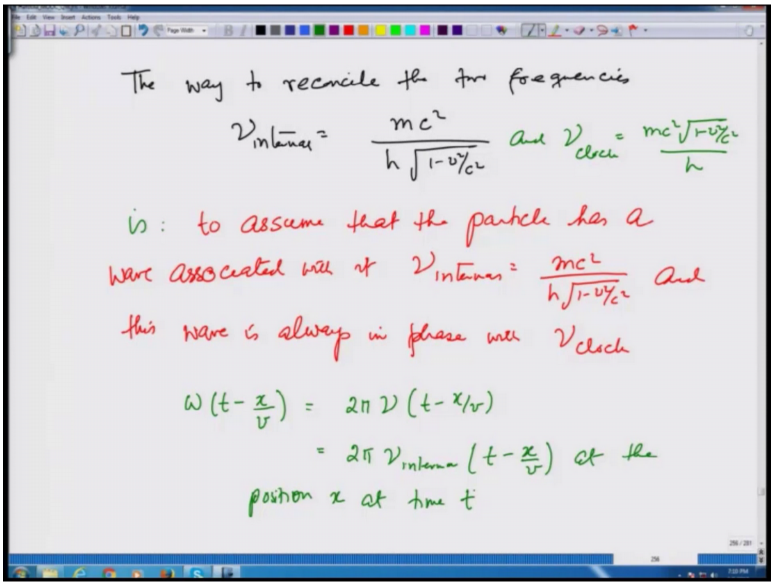

The way to reconcile the 2 frequencies nu internal which is mc square over h square root of one minus v square over c square.

And nu let us call it clock, which is mc square, square root of one minus v square over c square over h is to assume that the particle has a wave associated with it, and then the frequency nu internal equals mc square over h square root of 1 minus v square over c square, and this wave is always in phase with nu clock, that I am assuming remains with the moving particle recall from previous 2 lectures that this phase is given by omega t minus x by v which is nothing, but 2 pi nu t minus x over v. So, this wave is a frequency nu internal and therefore, the phase for the wave is going to be 2 pi nu internal times, t minus x over v at the position x at time t.

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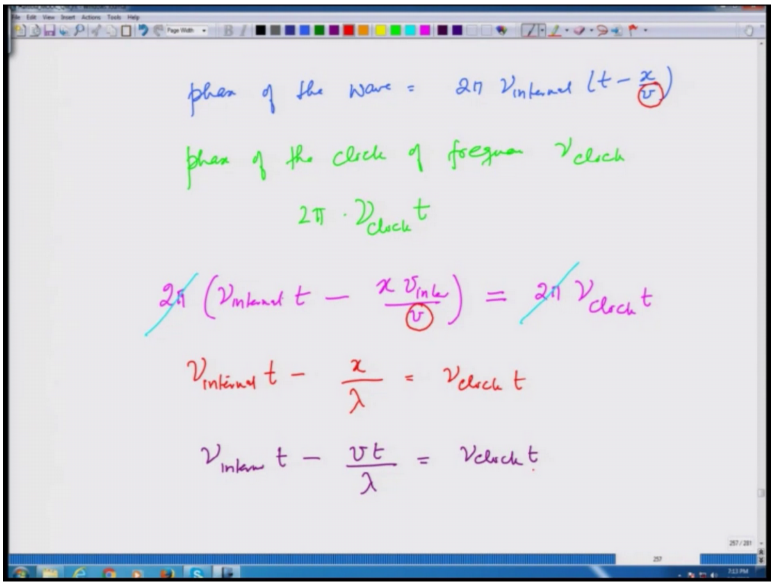

So, I have this phase of the wave is equal to 2 pi nu internal t minus x over v. How about the phase of the clock of frequency nu clock? This clock is with the particle and there phase in time t is going to be nu clock times t times 2 pi, and what de Broglie demanded that these to be equal these 2 phases are going to be equal. So, let us write the left hand side has 2 pi nu internal t, minus x nu internal divided by v and this we demand should be equal to 2 pi nu clock times t. 2 pi 2 pi cancels from the 2 sides and I get nu internal times t minus x v over nu internal can be written as lambda of these waves associated with the particle, is equal to nu clock times t and I am going to replace this x by v t now, for that is the v to the particle.

So, I am going to write nu internal t minus v t divided by lambda is equal to nu clock times t. You may wonder and you may feel at this point that may be we cheated, because this v out here which I am in circling by red in circling by red out here if I had put x equals v t probably out of canceled things, but that is not true this v out here, this v out here this v is actually v wave this v out here is v wave. So, even if I wrote this whole thing writing x equals v t write from the beginning, there will be no problem because I will have 2 pi nu internal t minus v t divide by v wave is equal to 2 pi nu clock times t.

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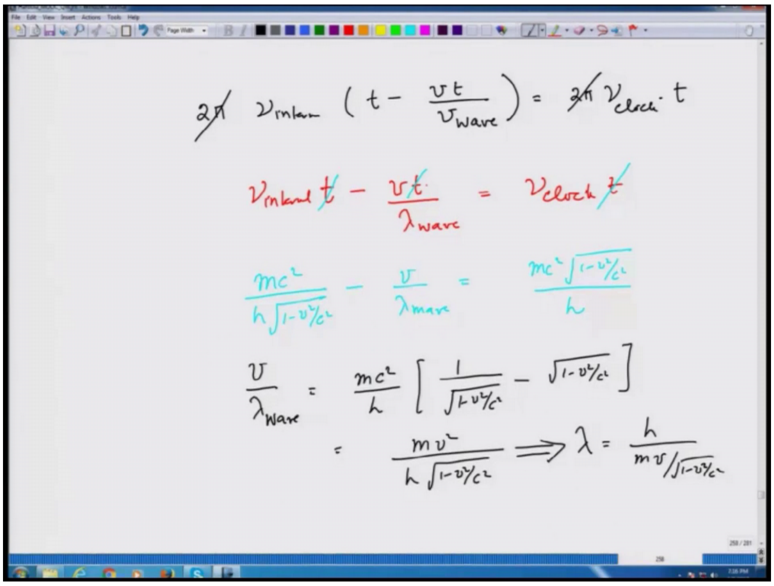

So, that is not an issue, and this t cancels and therefore, and 2 pi cancels. So, let us take nu internal n. So, you get nu internal times t minus v t and nu internal divide by v wave is nothing, but lambda wave is equal to nu clock times t, t again cancels and you substitute for nu internal which is nothing, but mc square over h square root of 1 minus v square over c square, minus v over lambda wave is equal to nu clock which is mc square, square root of 1 minus v square over c square over h.

We shuffle things around and you get v over lambda wave is equal to mc square over h one over square root of 1 minus v square over c square, minus square root of 1 minus v square over c square, and this gives you m v square over h square root of 1 minus v square over c square, leading to lambda equals h over m v over square root of 1 minus v square or c square, which is the de Broglie formula I again emphasize that this is not a derivation.

This is how de Broglie arrived at it this could have been wrong completely unless verified experimentally. So, in fact, you was ask in thesis defense this is was is thesis how would you check this crazy proposal and he said you do diffraction experiments, and in that experiment if you get the lambda to be the same as obtained by him then it is true.

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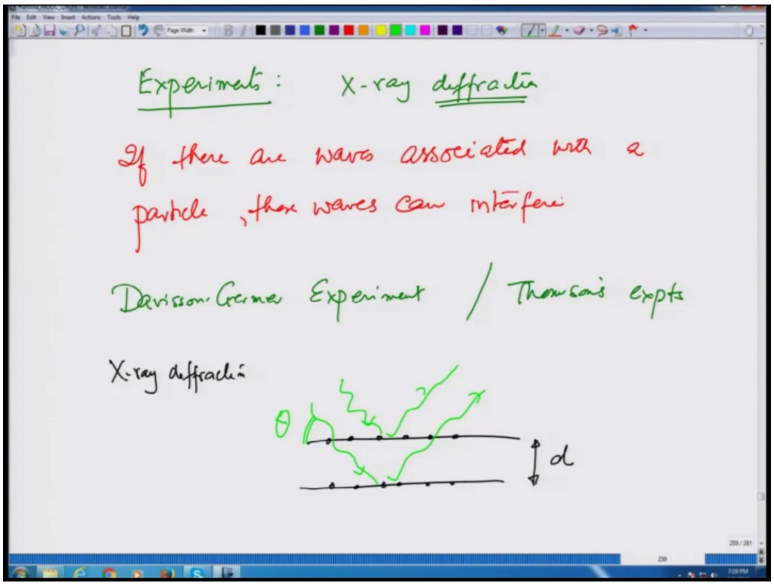

So, the experiments that are available that time what x ray diffraction, and this word diffraction is important because let me now say, if there are waves associated with a particle, these waves can interfere, that is a very specific wave property that waves interfere. So, if you can show the interference another form of which is diffraction of these associated waves then we prove it.

So, this is what is known as Davisson-Germer experiment now also Thompsons experiments. So, let me just tell you what happens in x ray diffraction, this give me the Bragg formula for it, suppose that 2 layers of a crystal, crystal have these atoms which are regularly placed and suppose I have these 2 planes formed by this at a distance d, and if x rays come in and go out what you see at that certain angles theta from the plane of the surface of the crystal, you see lot of interference you see lot of peace at other angles nothing really happens. So, the weight was explained we are on x ray diffraction.

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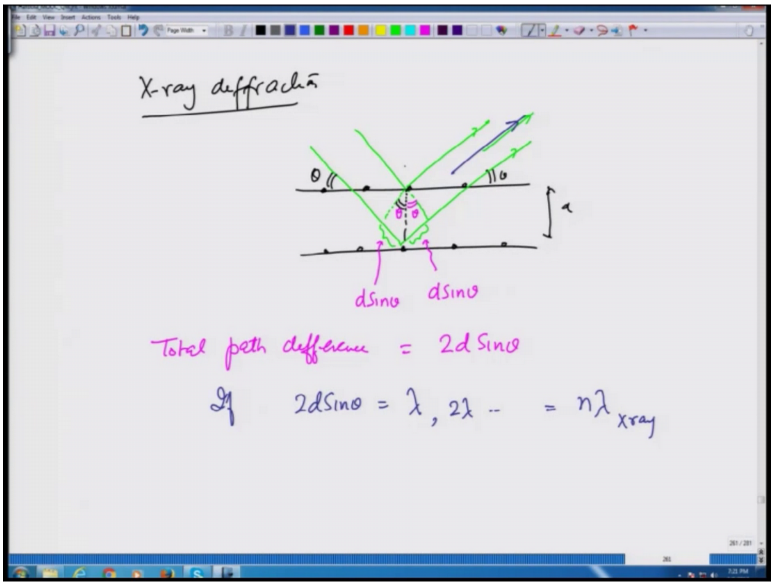

The weight was explained was that these planes, planes form by these atoms actually act like mirrors for x rays.

So, in x ray coming in gets reflected from the top surface gets reflected from the bottom surface and if the phase difference between the 2 reflected rays, which is given by this path difference shown by curly bracket is integer multiple of lambda of these x rays, then we would see lot of x rays reflected in that direction. So, let us see what is of corresponding formula if this distance is d and this angel is theta.

So, the angle would be if I draw perpendicular here this angle inside shown by pink is also going to be theta, this is going to be theta. So, this curly bracket distance is going to be d sin theta and on this side also is going to be d sin theta. So, total path difference between the x rays reflected from the upper surface and the lower surface is 2 d sin theta, and what Bragg suggested that if 2 d sin theta is equal to lambda or 2 lambda and so on in general n lambda integer multiple of the wavelength of the x ray.

I am going to see a constructive interference and I am going to see x ray is reflected at that angle, at any other angel you will not see that much of reflection. So, this is known as Bragg formula. So, one could do the same thing with electrons what one would do in this case is take these crystals.

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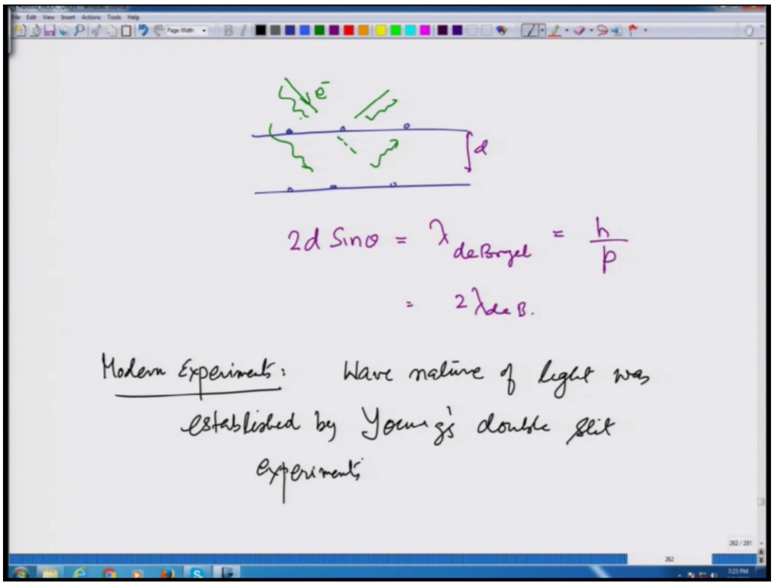

And instead of x rays you let electrons come in and the associated waves will also be reflected from the top surface in the bottom surface, and you should see peaks only at certain angles and those peaks will correspond to the constructive interference of the associated waves. So, if those waves really exist if I know d of this crystal, I should see a strong reflection of 2 d sin theta equals lambda de Broglie, which has h over p or 2 lambda d Broglie and so on and this was confirmed experimentally which established that lambda indeed is there, there is a wave associated and you see exactly x ray light diffraction pattern even when you do electron diffraction from crystal. This was the long ago in 1930s modern experiments are also there for example, if you recall form it was great.

The wave nature of light was established by Young's double slit experiment. So, can I do a Young's double slit experiment for electrons also can I prepare.

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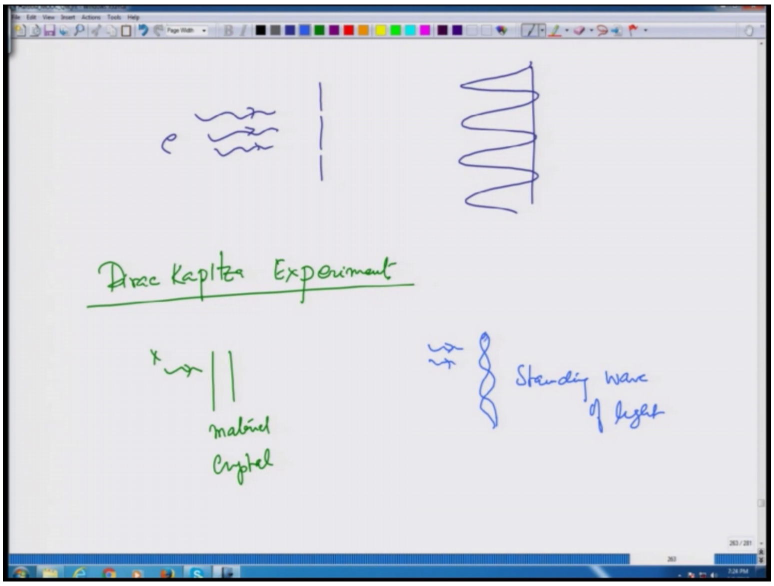

Double slit let electrons come in, and see if I obtain a fringe pattern of the (Refer Time: 21:26) is like an Young's experiment it was done, and now a days it can be done because we can make slits which are small enough we can create distance between them which are a small enough using other modern technology, and this experiment has also been done and that conforms that the wavelength indeed is lambda de Broglie. Third experiment which was very interesting into propose way back is Dirac Kapitsa experiment and in 1990s and after that these experiments have also been performed its required very high intensity light. So, the experiment is what Dirac and Kapitsa said is that if x rays can be diffracted by material that is a crystal, and if I electrons really have wave nature then electrons can also be diffracted by standing wave of light.

Because standing wave of light have this periodicity, and this is like a light crystal from which I can diffract electrons and these experiments have also been performed. They had to wait till very high intensity lasers who were invented they had give you intention of light to diffract electrons. So, let me just tell you that also in a minute.

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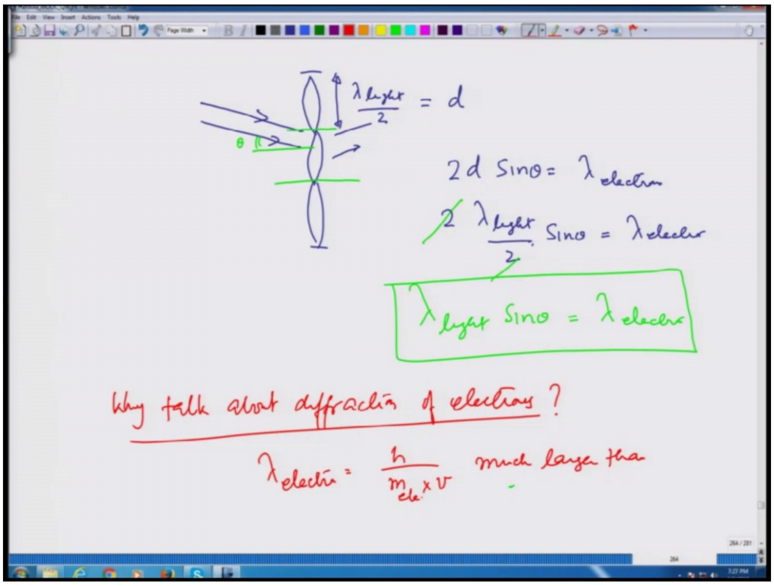

So, if I have standing wave of light between say 2 mirrors, then the periodicity is lambda light divided by 2. So, this becomes like the d of the crystal, and if I bring the electrons in and they diffract then I should have 2 d sin theta equals lambda electrons. 2 lambda light over 2 sin theta shall be equal to lambda electron this 2 cancels, and I should be able to satisfy this equation when I see a lot of electrons being diffracted in a particular angle from this light crystals, how to speak now you see if you notice here the crystal planes are like this. So, angle is going to be this angel this formula can also be derived classically; however, it is interesting to look at it from the wave perspective. So, these are the experiments that actually confirm the wave nature I have given you the classic Davisson-Germer experiment, I have talked about double slit experiment that is actually a wave interference phenomena.

And I have also talked about Kapitsa Dirac proposal which has been performed with high intensity lasers. So, these are the waves the experimental confirmation of lambda de Broglie for electron can be checked. Now you must be wondering so far how come I am talking about electrons why talking about diffraction of electrons. This is because lambda electron which is h over m electron times v is much larger than heavier particles, and larger the wave length easier it is to see the diffraction or interference in therefore, one works with electrons, other thing which I wish to point out is that when we looked at.

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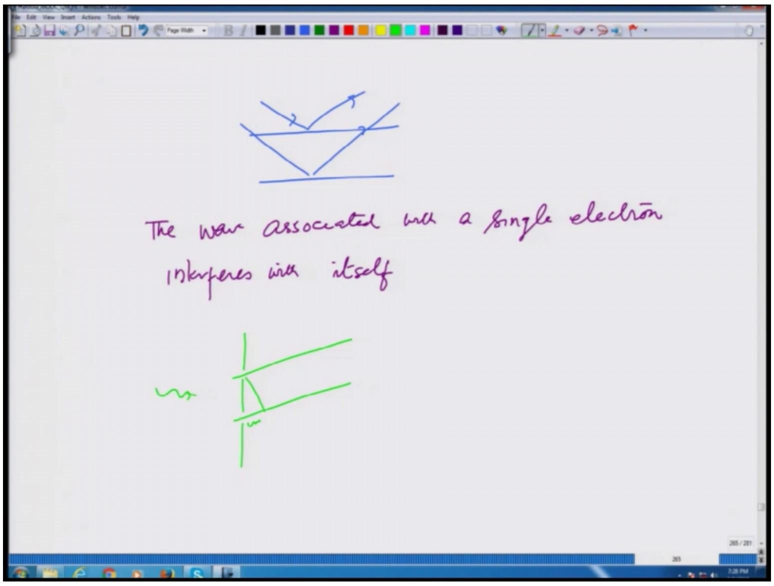

Electron diffraction from a crystal it is the wave associated with each electron that interferes with itself. So, the wave associated with a single electron interferes with itself. So, it is not that an electron interface over on electron, it is wave associate with each single electron that interfere with itself. Similarly when I talk about double diffraction double slit experiment with electrons each electrons wave interferes with itself, it is as if electron interfering with itself that makes you little uneasy, only thing I can say is welcome to the world of quantum mechanics this is how things work out. I have deliberately not derive the formula for Young's double slit experiment, because that you know well from your twelfth grade physics.

Accept that now, when we work with electron interference wavelength is that of electron waves. So, this is this concludes introduction to matter waves to you it tells you how the wavelength associated with the waves with the particle is given in terms of is momentum, and how it is experimentally verified.

On next lecture onwards we will develop a wave equation known as the Schrödinger equation for these particles in terms of de Broglie waves, and start working. With it that will be the Schrödinger explosion of quantum mechanics.