Introduction to Quantum Mechanics
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Lecture – 01
Introduction to waves and wave equation

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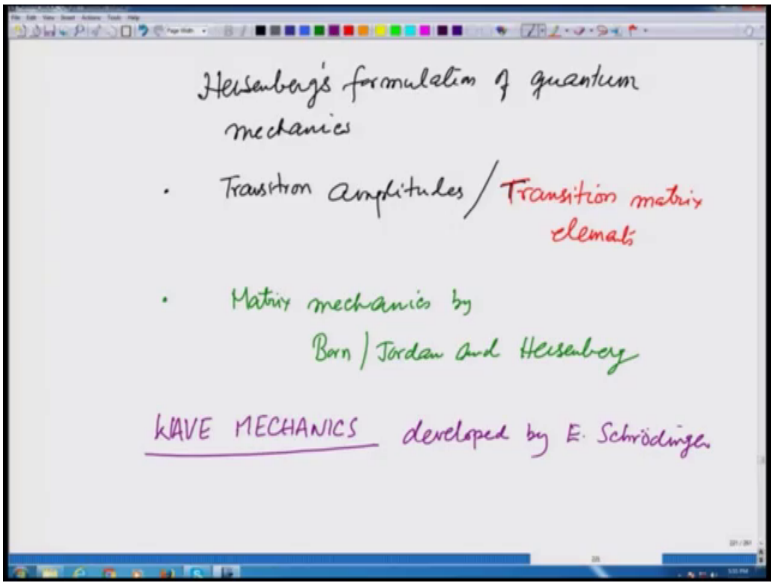

What we have done so far in past 3 weeks is seen how the quantum mechanics idea developed and culminated with Heisenberg's formulation of quantum mechanics which initially was written in terms of transition, amplitudes or, what I will also call transition matrix elements and this was reformulated then in terms of matrix mechanics by Born, Jordan and Heisenberg.

However as I commented towards the end of the last lecture, previous week, solution of quantum mechanical systems through this method becomes complicated and it was made easy with the birth of wave mechanics which was developed by Ervin Schrodinger and it actually treated particles like waves and the full machinery of wave mechanics is what we are going to develop over the next 2 weeks and show that this indeed is equivalent to Heisenberg's formulation and that kind of complete still picture of development of quantum mechanics and along the way we keep applying it.

But what I want to do in this lecture is give you an idea as to how waves are described in physics.

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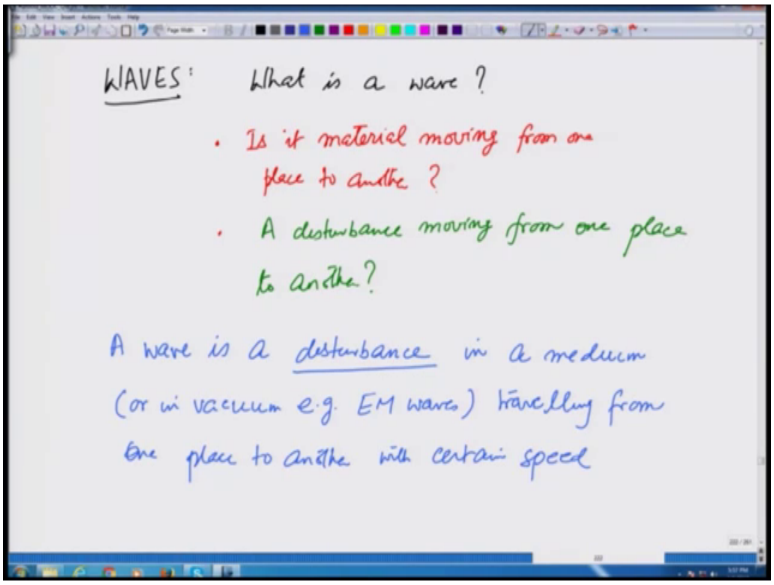

So, we start with description of waves and the question is; what is a wave? Is it a particle moving from one place together? So, let us raise this question; is it material moving from one place to another or is it a disturbance moving from one place to another and the answer is second one. For example, if you take a dish of water and dip your finger in it, you create a disturbance that travels from one place to another, but material does not go from one place to the other. Similar, when am talking I am creating a disturbance operation; disturbance in the air which travels; that disturbance travel from one place to other, but the material; the air does not go from one place to another.

So, what a wave is; a wave is a disturbance in a media for the time being, but electromagnetic waves; for example, do not require any medium. So, your say medium or in vacuum; for example, electromagnetic waves which is the disturbance of electrical magnetic field travelling from one place to another and; obviously, when it travels, it travels with certain speed; that is the wave and how do we describe it mathematically let us see that.

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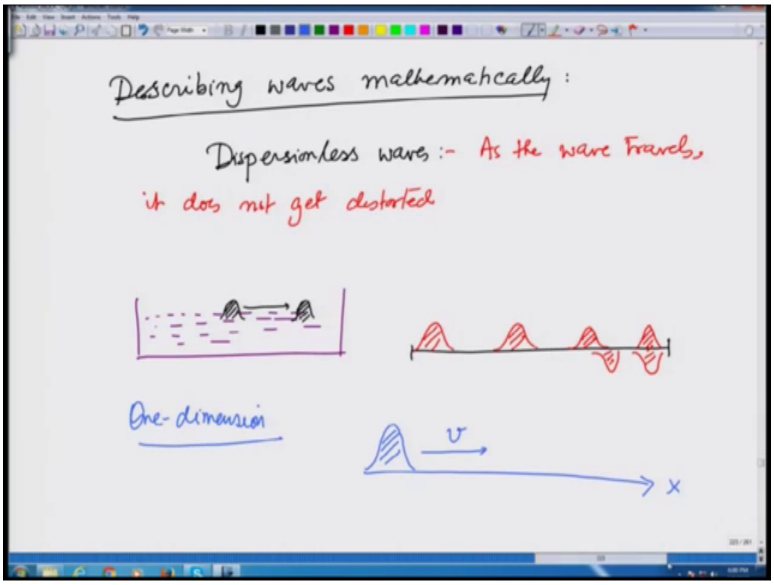

So, describing waves mathematically and we are going to restrict ourselves to dispersion less waves that is a (Refer Time: 05:33) us and what; that means, as the wave travels and by that now you understand as the disturbance travels it does not get distorted and what; that means, is suppose I have this dish of water and I create a disturbance in at. So, I kind of let say create a disturbance in it and as a travels it retains the same sheet or when I am talking to you, when I am talking to somebody outside, even if the person is standing 20-30 meters away, if I have attired a word he hears or she hears the same word; that means, the word or the disturbance is not gotten distorted these are called dispersion less waves.

Similarly on a string, if I create a pulse for examples a pulse like this it travels down the same shape and this, what I mean it is dispersion less and that it may get reflected and come back. So, this is a kind of waves we are talking about and this is what we want to describe mathematically. So, what I just now said is suppose I have and I am going to restrict myself to one dimension to convey the ideas and that all immediately write the equation for 3 dimensions also as we go along.

So, in one dimension suppose I am travelling towards the x axis and I create, I am not travelling the disturbance is traveling towards the x axis and travels with speed v.

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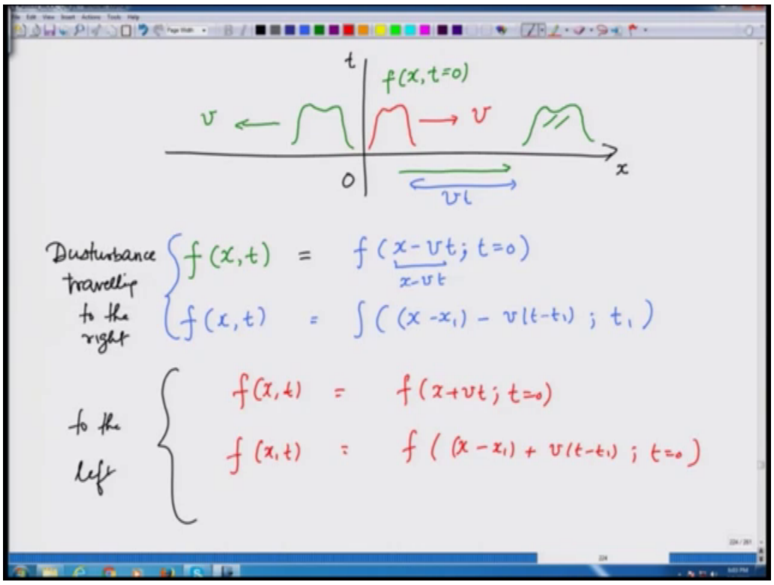

Let me go to the next slide and show this. So, here is my x axis here is 0, I will take 2 situations in one in which this disturbance which could be any kind is traveling to the right with speed v, I can take another situation in which the same disturbance travels to the left with speed v without getting distorted and I want to know, suppose this is function f of x at time t equals 0, I want to know what would the disturbance be as a function of time at a distance x this very simple this, if I am observing something at x. So, let us say I am observing it somewhere here at a distance x this would be the same disturbance has was at a position x minus v t at time t equal to 0.

So, this is a function of x minus v t in general, I can write if there was a disturbance at x 1 and t 1 and I am observing it as a function of x and t this would be same as x minus x 1 minus v t minus t 1 at t 1. So, because it has travel that much all that happened is a center of this has shifted by this much distance v t, let me write this; this is the disturbance or the wave travelling to the right, how about that traveling to the left, this would be given by f at x t would be what; it was at a positive distance x plus v t at time t equal to 0 and if I want to write in general f x t is going to be equal to f x minus x 1 plus v t minus t 1 at t equal to 0. So, travels to the left the sin changes, I can look at it another way.

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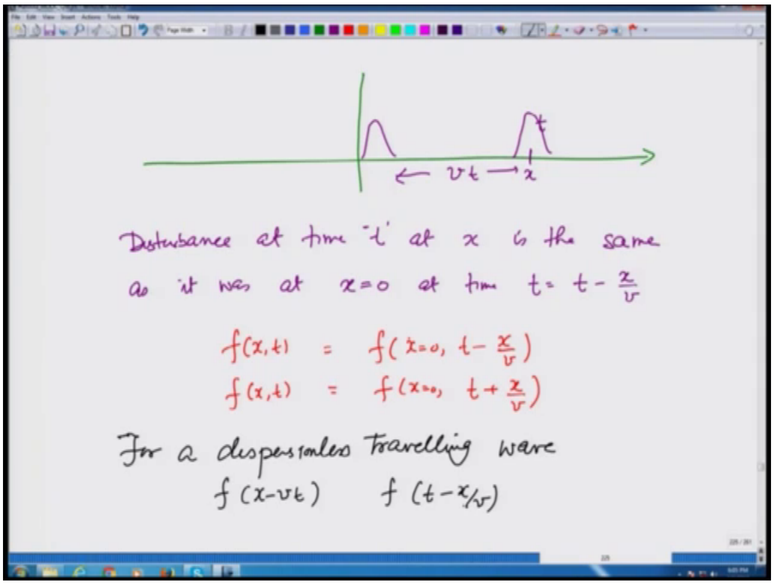

Other way, I can looking at it is if I am looking at point x at time t, the disturbance at time t at x is the same as it was at x equal to 0 at time t equals t minus x over v because it has traveled the pulse has traveled that much distance v t. So, it has traveled that much.

So, it took time x by v to travel that much. So, I can also write that f of x at t is same as f at x equal to 0 at time t minus x over v, if it is travelling to the right and if it is travelling to the left f x t is equal to f at x is equal to 0 t plus x over v, what we have understood what I did in the last slide and here that for a dispersion less travelling wave what we have is f as a function of x minus v t, I am dropping that 0 or a function t minus x over v which is a same as f x minus v t just 2 ways of looking at it that is how a travelling wave would look.

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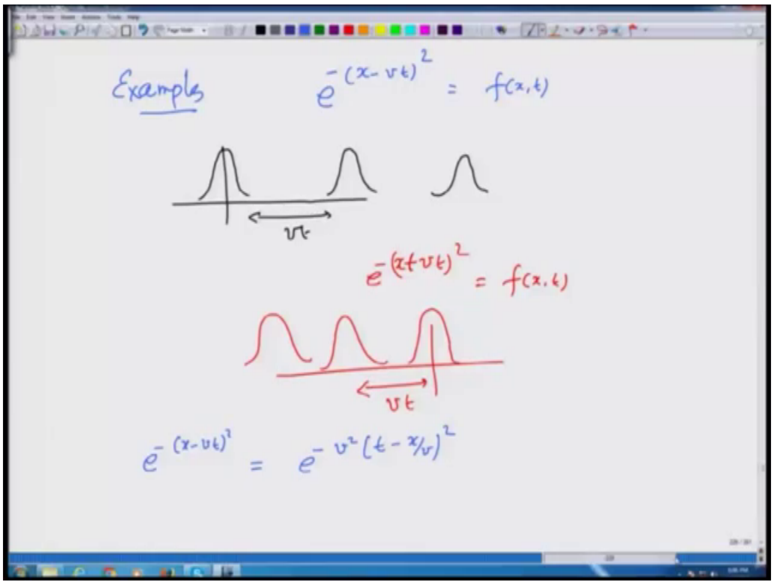

So, for example, if I have e raise to minus x minus v t square as a function this would be a travelling wave to the right and how does it look at time t equal to 0, it look like e raise to minus x square and with time progressing will keep travelling to the right by distance v t. If on the other hand, if I had e raise to minus x plus v t square is equal to f x t it will be a wave which will be traveling to the left with speed v or this is how describes it, I could also write this as thus e raise to minus x minus v t square as equal to e raise to minus v square which becomes a constant t minus x over v whole square which is another way of looking at it that this is a disturbance that I am seeing at time t, this is what it was t minus x over v time at x equals to 0.

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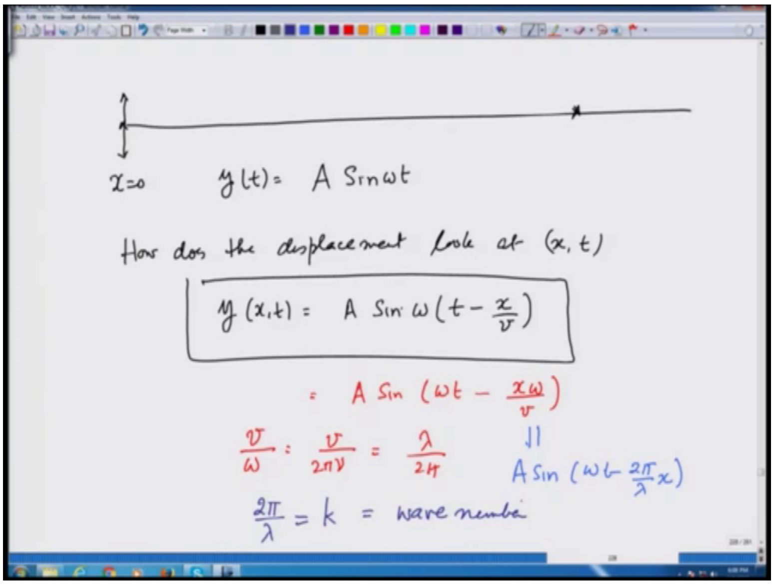

So, this is 2 ways of looking at the wave; suppose I take the long strings tide somewhere and start shaking this and let see this is A x equal to 0 and start shaking this end and I start moving it with y t equals some amplitude let us say sin omega t some shaking it up and down how does displacement look at x as a function of x and t. So, again going by what we did just now y at x and t would be equal to A sin of omega t minus x over v. So, if I want to see displacement after the wave has started travelling of course, t is such wave has passed through that point this point it look at this points some distance away at time t is the displacement would be what it was at x equal to 0 time t minus x over v.

So, this is A sin sinusoidal wave travelling down, I can write it slightly differently which is A sin, I can take omega inside minus x over v times omega v over omega is v over 2 pi nu which is the wavelength over 2 pi and therefore, I can also write this expression as A sin of omega minus 2 pi over lambda x omega t 2 pi over lambda is denote is usually as small k and called a wave number. So, I can in general write that is sinusoidal wave y t and x is A sin omega t minus k x where k is 2 pi over lambda which is omega over v.

So, this I can also write as some other amplitude which is minus A sin of k x minus omega t.

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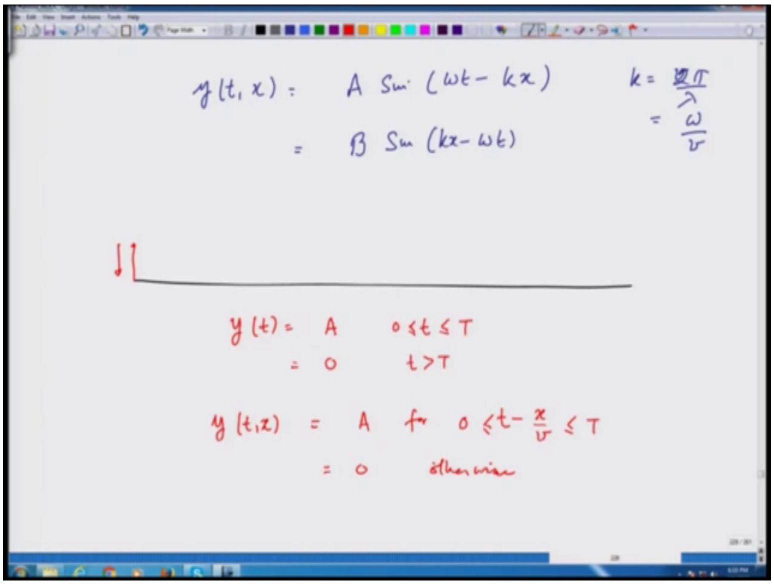

No matter how I write it, but there is a combination x minus v t or t minus x over v and that only represents a travelling wave is travelling wave restricted to being A sin wave the answer is no. For example, I can take the same string and move it up for some time hold it there and then come back; that means, I create a disturbance y t as equal to some disturbance a for time between 0 and sometime t and then 0 again or t greater than t how would it look as a function of x and t. So, why t and x would be equal to a for t minus x over v between 0 and t and 0 otherwise.

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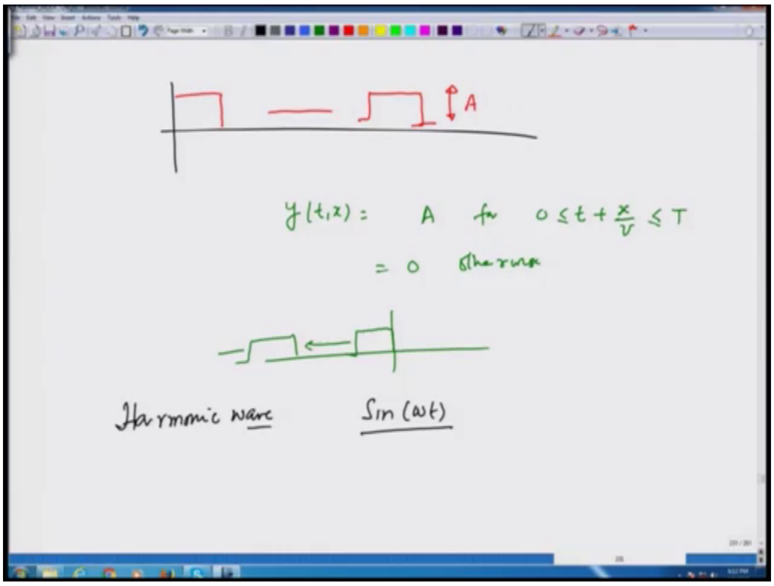

So you can do a little calculation. So, and show that what it looks like is I am actually creating a square wave which travels down with this being a on the other hand it could also be that y t x is equal to a for t greater than equal to t plus x over v greater than equal to 0 less then equal to t and 0 otherwise then this wave would be traveling to the left.

So, what I am try to get at is that the disturbance could be traveling to the left to the right a pure frequency or harmonic wave, I am just dropping lot of term harmonic wave would have a dependence which is sin omega t at a given place right and where as in general it could be any shape this is harmonic wave or sinusoidal wave and others could be any shape. So, is the disturbance which goes distortion less it does not get distorted that is the traveling waves?

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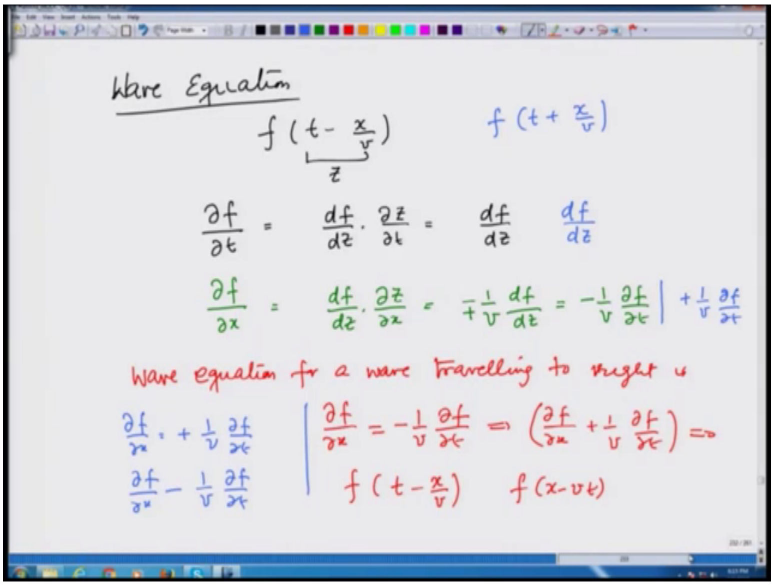

So now let us write the equation wave equation that governs the motion let us look at. So, this is a function of t minus x over v. So, if I take call this quantity z and take partial derivative of f with respect to t, I am going to get d f d z times partial z over partiality t which is same as d f d z .

And if I take partial derivative of f is respect to x this is going to be d f d z times partial z over partial x which is going to give me minus one over v d f d z and d f d z is same as partial and is by partial v. So, this is going to be minus one over v t f d t and therefore, the wave equation for a wave travelling to write is partial f by partial x is equal to minus one over v partial f by partial t implies partial f by partial x plus one over v partial f by partial t is equal to 0 what about the solution is since built up from the solution is going to be of t minus x v x by v.

So, solution is the function of t minus x over v shape could be anything or a function of x minus v t is the same thing for their wave traveling to the left is going to be a plus sin because this is going to be replaced by f t plus x over v for wave travelling to the left this remains d f by d z this; however, becomes plus one over v partial f partial t and therefore, for the wave travelling 2 the left is going to be parietal f by partial x is equal to plus 1 over v partial f by partial t or partial f partial x is minus one over v partial f partial t.

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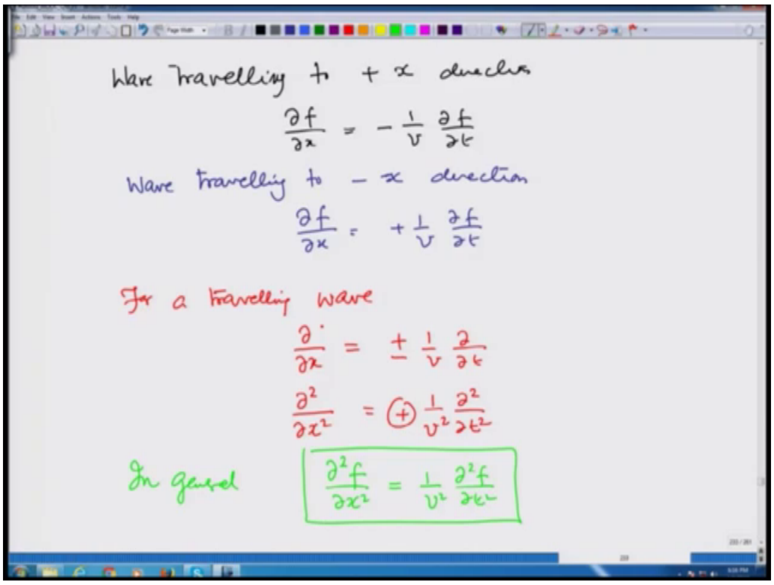

Wave travelling to plus x direction equation is partial f partial x is equal to minus 1 over v partial f partial t and wave travelling to minus x direction, we have partial f over partial x is equal to plus 1 over v partial f partial t and becomes rather cumbersome to remember which ways the wave travelling. And therefore, whether I should plus sign and minus sign we combined 2 equations, how do we combined them you realize that for a travelling wave partial derivative with respect to x is the same thing as you can see from the questions above either plus or minus one over v partial derivatives is respect to t.

So, if I take the second derivative with respect to x this is going to be 1 over v square second derivative with the respect to time and this becomes plus now because minus square is plus plus square x plus. And therefore, in general I can write for a wave d 2 f by d 2 d x square is equal to 1 over v square d 2 f by d t square and that describes a wave travelling either to the left or to the right.

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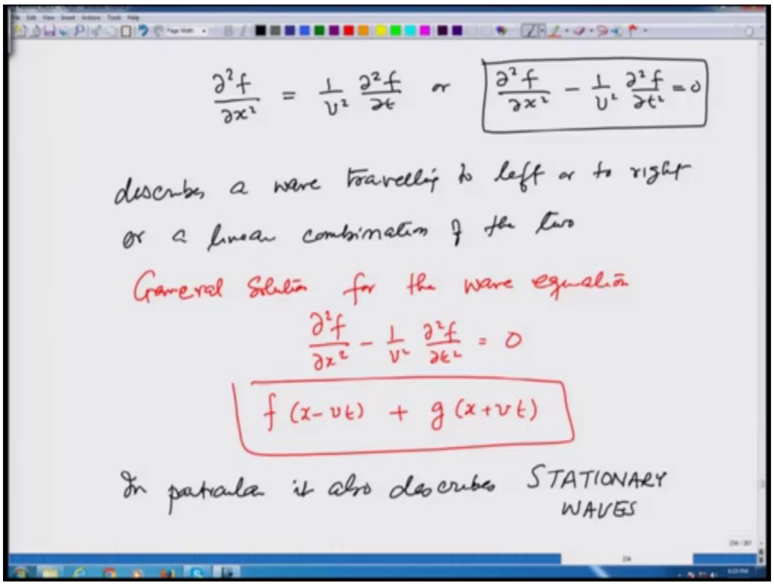

So, let me write this d 2 f by d x square is equal to 1 over v square d 2 f by b t square or d 2 f by d x square minus 1 over v square d 2 f by d t square is equal to 0 describes a wave travelling to left or to right or it could be linear combination or linear combination of the 2.

So, a general solution for the wave equation d 2 f by t x square minus 1 over v square d 2 f by d t square is equals to 0 is a function of x minus v t and could be some other function g of x plus v t we are both satisfy the equation given above notice that f describes the function in disturbance travelling to the right, g describes a disturbance travelling to the left and there combination could also be there which is the solution, because both satisfy the equation given in the wave equation which combines travelling to the left or to travelling to the right in particular. It also describes what would I will call stationary waves; these are waves which are linear combination of equal amplitude waves travelling to the left and traveling to the right.

So, they stationary they not move they are not traveling which will discuss in the next lecture.