Introduction to Quantum Mechanics
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Lecture – 08
Brief introduction to matrix mechanics and the quantum condition in matrix form

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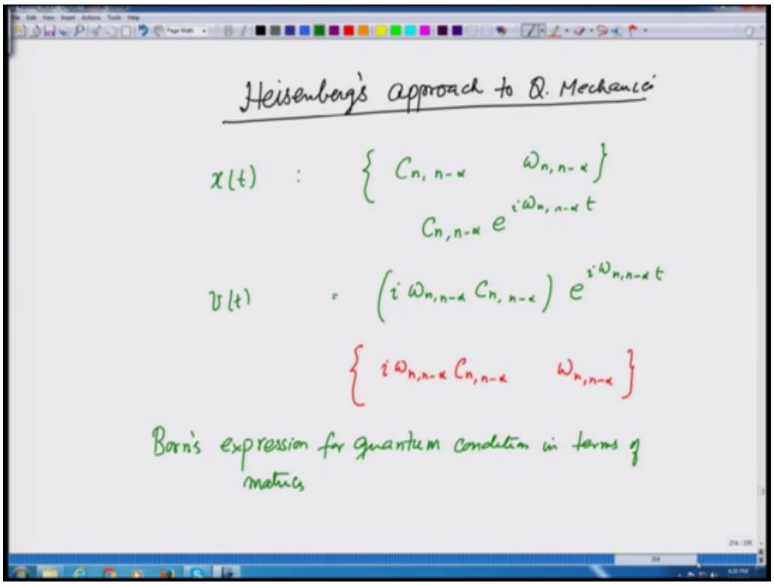

So, you have seen so far in the previous 3 lectures that we have discussed Heisenberg's approach to quantum mechanics although we have been saying that what he writes these quantities are in terms of matrices at the time when he did it, he did not know that he was dealing with matrices. In fact, he rediscovered in a way dictated by physics the matrix multiplication rules born on reading Heisenberg's paper realized that the quantities that Heisenberg was talking about and the algebra, he was talking about was actually that of matrix algebra; the language was that of matrix algebra and everything was dealt with matrices.

So, he tries to develop and in fact, became successful in developing the matrix version of Heisenberg's quantum mechanics and first paper was written on this by Jordan and Born and later developed fully by Jordan, Born and Heisenberg himself. What I am going to give you in this lecture is a very brief introduction to this method of doing quantum mechanics. It becomes quite complicated in applying to systems other than very simple system like a harmonic oscillator. And therefore quantum mechanics really gained popularity as a calculation tool after wave mechanics by Schrödinger came along which will be covering in the next week.

But to give you a brief introduction and what it sets the tone for the relations that are derived in wave mechanics also. So, it is necessary to do this. So, what Born and Heisenberg, Jordan; they together did was to consider that x t was given by a collection of numbers C n and minus alpha and omega n, n minus alpha and its time dependence the coefficients time dependence was given by e raised to i omega n, n minus alpha t and therefore, the corresponding velocity became i omega n, n minus alpha C and n minus alpha times e raised to i omega n, n minus alpha t. And therefore, the coefficient for velocity was i omega n, n minus alpha C and n minus alpha and omega n, n minus alpha omega of course, had to be the same as a system transition frequency.

Using these born wrote the quantum condition in a very neat way and that is what we are going to do first. So, Born's expression for quantum condition in terms of matrices in fact, the condition that he derived. So, fundamental that it is also engraved on a tombstone and over the forum maybe I will send you a picture of that tombstone.

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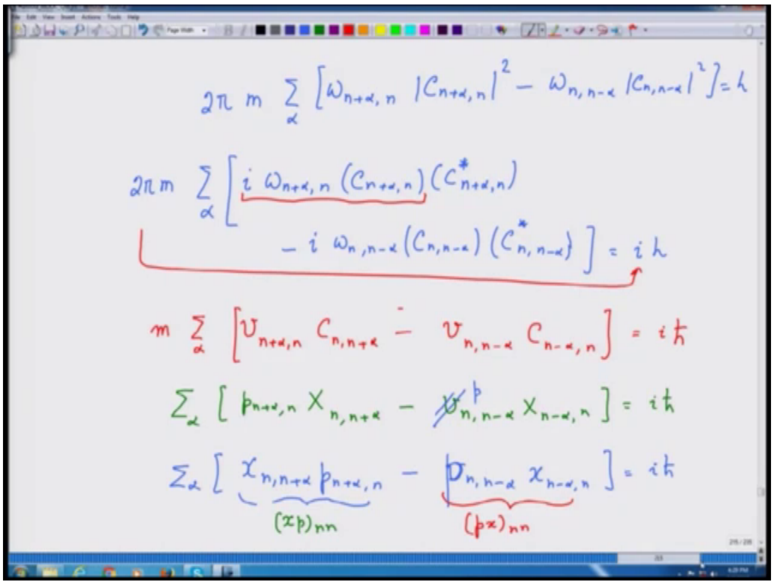

So, we call that the condition that Heisenberg wrote was 2 pi m summation over alpha omega n plus alpha n mod C n plus alpha n square minus omega n, n minus alpha mod C n, n minus alpha square equals h.

Let us rewrite this slightly. So, this becomes 2 pi m summation over alpha I want to write this in terms of p and x or p and q where q is the generalized coordinate. So, this becomes I am going to write this as multiplied by i omega n plus alpha n C n plus alpha n and the other one is going to be complex conjugate. So, let me any right first in terms of complex conjugate n plus alpha n minus i omega n, n minus alpha C n, n minus alpha times C n, n minus alpha complex conjugate is equal to i h.

Let me take 2 pi to the other side and write m summation over alpha recall that i omega n plus alpha n C n plus alpha n is nothing but the velocity n plus alpha n and the other one is C star plus alpha n. So, I am going to write a C n, n plus alpha minus same thing I get V n, n minus alpha C n minus alpha comma n is equal to i h cross m multiplied; the velocity is nothing but a momentum and therefore, I can write this as summation alpha p n plus alpha comma n x n, n plus alpha minus p n, n minus alpha x n minus alpha n equals i h cross.

Let me reshuffle the numbers and write this as summation over alpha x n, n plus alpha p n plus alpha n minus V n p this is V p even in the top this should be P n, n minus alpha x n minus alpha n is equal to i h cross recall that I am summing over alpha. So, this the present this whole quantity represents nothing but x P n, n by matrix multiplication root and similarly the other quantity represents P x n n.

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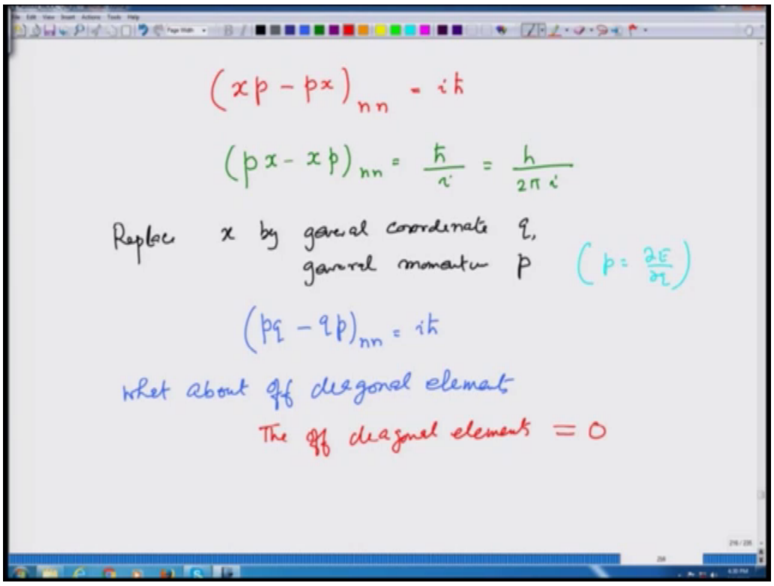

Therefore when I write this in terms of matrices i get x p minus p x n, n equals i h cross or return the way boar rotate p x minus x p n n equals h cross over i or h over 2 pi i.

Generalizing this if I replace x by general coordinate q and corresponding general momentum p recall from previous lectures, I have told you that p is p E d q we get the condition to be p q minus q p n, n equals i h cross. So, the diagonal element of p q minus q p is i h cross is a constant what about off diagonal elements let me give you the answer the off diagonal elements then we will justify it r equal to 0; that means, if the off diagonal elements are 0; that means, this matrix is a constant.

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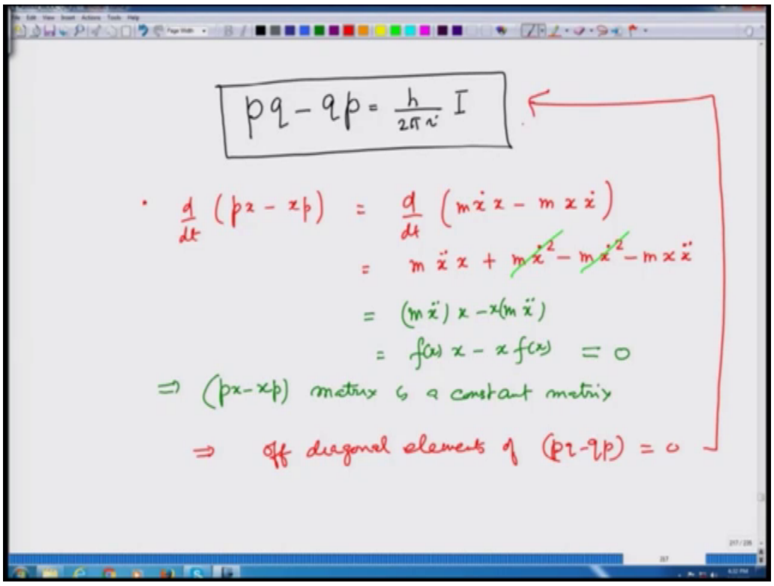

And what; that means, is that I can write the quantum condition as p q minus q p equals h over 2 pi I times the identity matrix because all the off diagonal limits of identity matrix are 0.

I can think of this as an assumption or if I look at p x minus x p and take it d by d t, it will be d by d t of m x dot x minus m x x dot which is equal to m x double dot x plus m x dot square minus m x dot square minus m x x double dot. These 2 terms cancel and I get this equal to m x double dot x minus m x double dot x, on the left hand side notice that i am being very careful in writing the order and this is nothing but the force x times x minus x force x if force depends only on x x time x is same as x times x. So therefore, this must be 0 and this implies p x minus x p matrix is a constant matrix and that immediately gives me that off diagonal elements of p q minus q p; I am taking the liberty of writing q are 0 and this leads to this condition this is the equation that is written on (Refer Time: 11:43) tombstone.

So, that was the first equation; the quantum condition expressed in terms of matrices like this.

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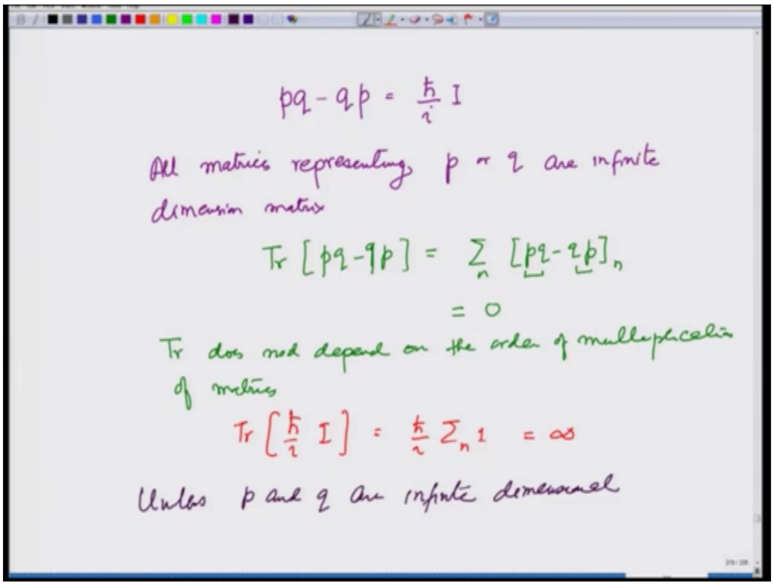

One consequence of the quantum condition that p q minus q p is equal to h cross over i times the identity matrix is that all matrices representing p or q or any other variable are infinite dimension matrix and that is seen by realizing that if I take the trace of 2 sides of the quantum condition which means i sum over all the diagonal elements, it is supposed to be 0 for such kind of product because the trace does not depend on the order of multiplication of matrices.

On the other hand, if I look at the right hand side h cross over i and take it trace which will be h cross over i summation n 1 will come out to be infinity. So, left hand side is 0, right hand side is infinity and that is true for any finite matrix. So, left hand side is 0 and right hand side is infinity and that cannot be true unless p and q are infinite dimensional because the result that the trace does not depend on the order is true for finite matrices and not for infinite matrices.

So, necessarily all the quantities that are represented by matrix in quantum mechanics the representation is an infinite dimensional matrix.

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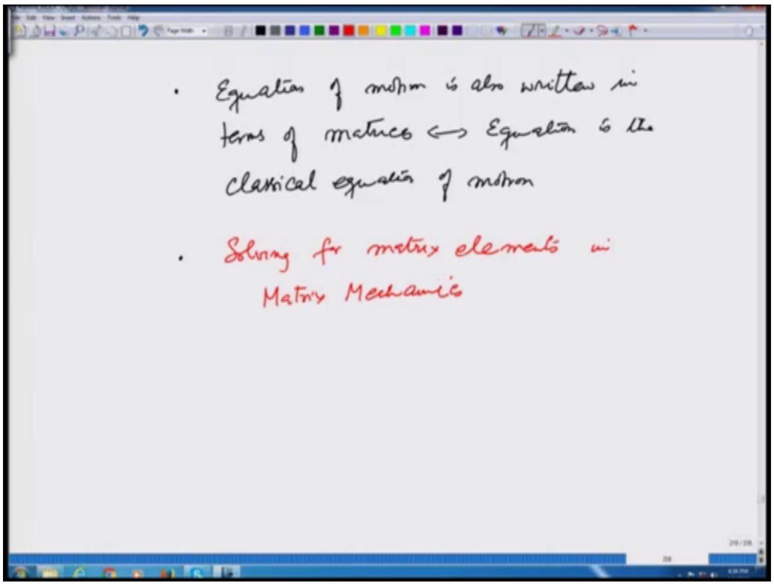

Number 2 equation of motion is also written in terms of matrices and the equation is the same as classical equation of motion. In fact, the classical equations are written in the form which is known as Hamiltonian dynamics and if I write those that will take me out of the domain of this course and therefore, I am not going to delve on that any further and third when you write the equation of motion and then through that you start doing your equations as was done earlier.

But what I should point out that solving for matrix elements and what is now we will call matrix mechanics at that time was a very tough job. In fact, the hydrogen atom problem was solved by probably using some conservation principles, but it did not go very far it became very complicated. And therefore, the easy application of quantum mechanics had to wait the development of wave mechanics that will be covered in the next lecture onwards.

So, this is a brief introduction to matrix mechanics. And this week has been devoted to the formal development of quantum mechanics in terms of matrix mechanics. And we stop here on this, and next week onwards we start on wave mechanics.