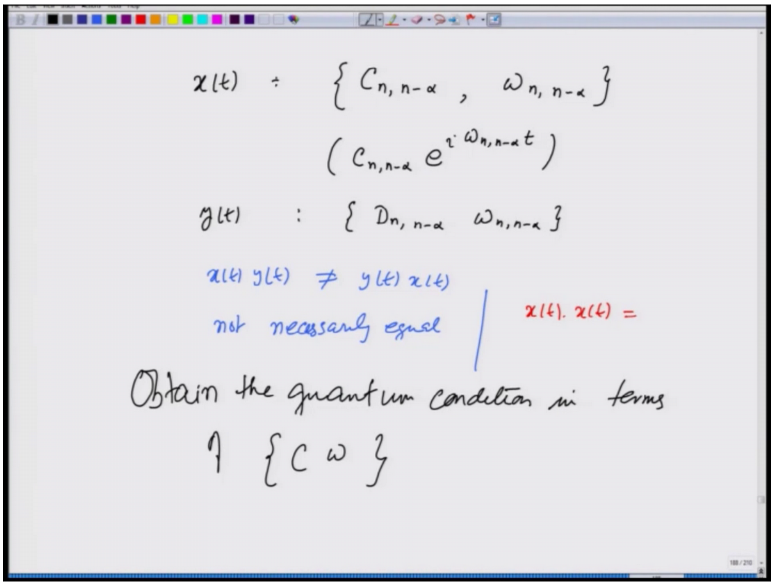
Introduction to Quantum Mechanics
Prof. Manoj Kumar Harbola
Department of Physics
Indian Institute of Technology, Kanpur

Lecture – 06
Heisenberg's formulation of quantum mechanics: the quantum condition

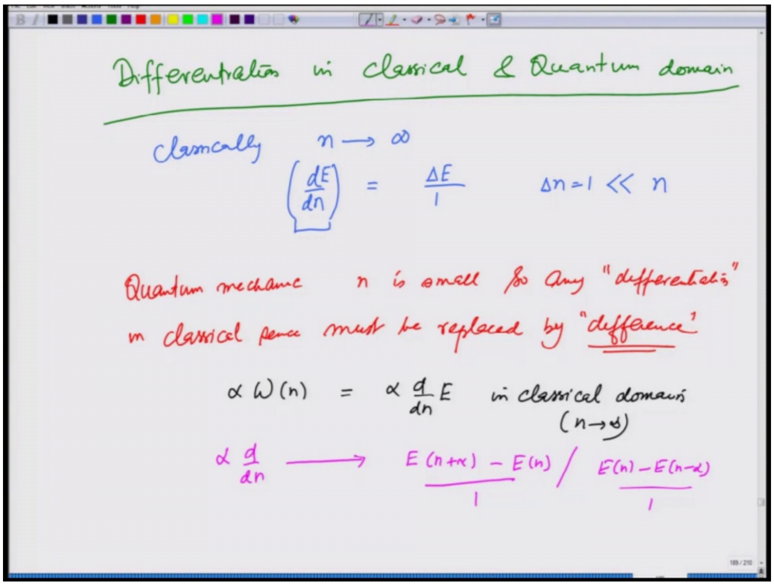
(Refer Slide Time: 00:23)



In the previous lecture, we introduced the quantum mechanical formulation of kinematic quantities by Heisenberg, we said if there is a quantity x t it is represented by a collections number C n, let is say n minus alpha and the corresponding frequency omega n, n minus alpha and the time dependence of C n, n minus alpha is given as e raise to i omega n, n minus alpha t, then this collection of numbers gives this, there could be another quantity y t which can be represented by collection of D n, n minus alpha omega n, n minus alpha and there I said that x t y t is not equal to y t x t. So, what I should say is not necessarily equal to.

For example if I have x t and multiplied by x t that would; obviously, be the same if we change it here, but the way we calculate the representation has become, now matrix multiplication and what we want to do in this lecture is obtain the quantum condition in terms of these numbers C and omega that is our goal.

(Refer Slide Time: 01:59)

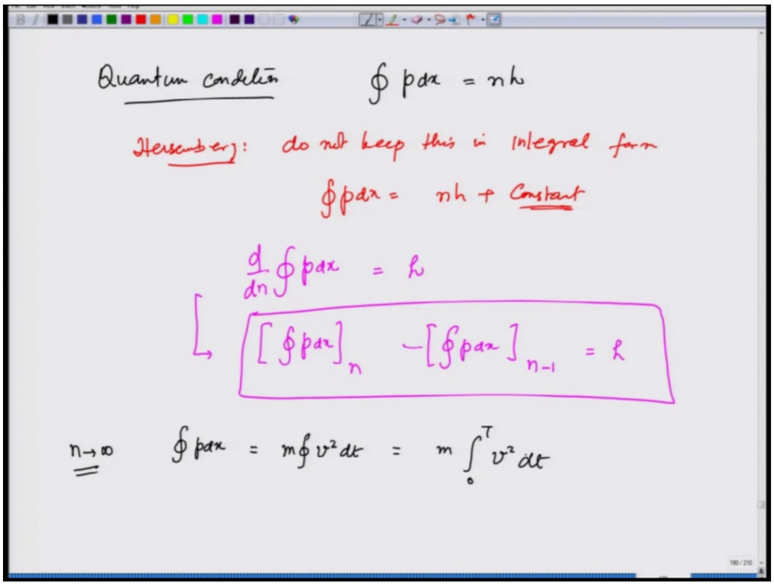


So, again we will be going back to the correspondence principle, but before that let us now look for the time being differentiation in classical and quantum domain.

We have been doing it, but now let us explicitly say it classically, the quantum number n is very large and therefore, we can afford to do differentiation and which is the same as delta e suppose i, this d E by d n by one because delta n which is one is much much much much smaller than n in the classicology in quantum mechanics this d E by d n would not be possible because numbers are small. So, in quantum mechanics n is small. So, any differentiation in classical sense must be replaced by difference that is how we distinguish between the 2.

So, for example, recall that omega alpha omega n was equal to alpha d by d n e in classical domain; that means, n tending to infinity in quantum domain alpha d by d n will go over to E n plus alpha minus E n that is the difference or E n minus E n minus alpha divided by 1 because the delta n is 1, this is the difference when we make a transition from classical to quantum. It has to be differences and alpha d by d n is going to be this difference where the 2 quantum numbers differ by alpha and we are going to make use of it that is why I wrote it.

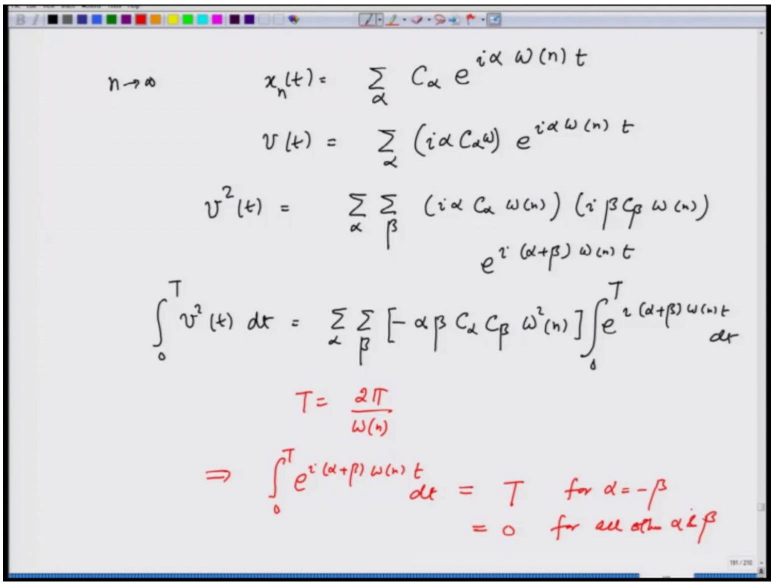
(Refer Slide Time: 04:39)



Now, recall that the quantum condition that gave us results before Heisenberg paper was this condition pdx equals n h first thing Heisenberg said; do not keep this in integral form why because this pdx could be n h plus some constant and we do not know this constant. So, it should be replaced by a differential form and therefore, what he said is replaces by taking derivative with respect to n on both sides pdx equals h and once you integrate it that constant may come out which would basically in quantum mechanical sense this would mean that integral pdx for n minus integral pdx for n minus 1 is equal to h, this is what he first replaces the quantum condition by and now we are going to derive this in terms of C and omega. So, let us do that now.

So, when we write pdx in the n tending to infinity regime you see again, we are going to make use of correspondence principle, I will derive some results from n equals infinity and then immediately translate them to quantum mechanical language and that is the use of correspondence principle that Heisenberg made, it is going to be equal to mass times v square d t over a period which I can also write as mass integer 0 to t v square d t that is the action.

(Refer Slide Time: 06:48)

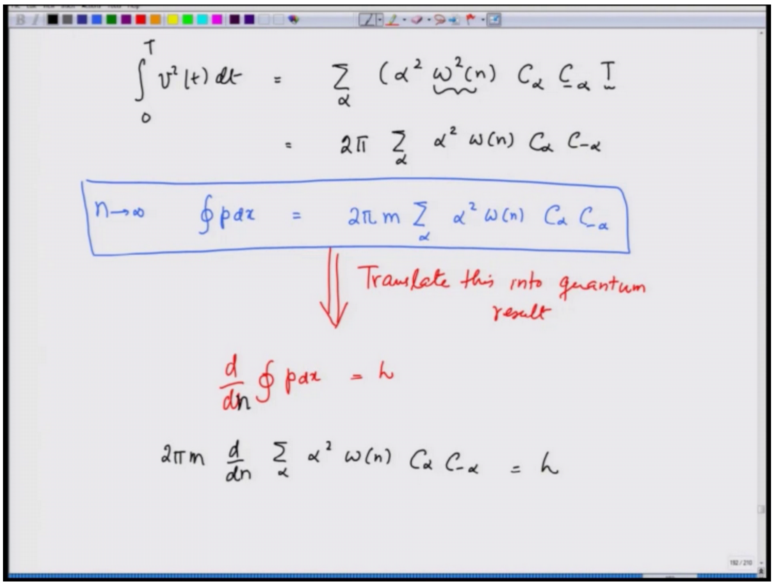


So, let us evaluate this is still in the n tending to infinity regime x t is summation, let us try it alpha C alpha e raise to i alpha omega which depends on this n t or the nth level. So, v this nth level t is going to be equal to summation alpha i alpha C alpha e raise to r and there is a omega also alpha omega which depends on n t that is v alpha or v t.

So, v square t is going to be equal to be equal to summation alpha summation beta i alpha C alpha omega n times i beta C beta omega n e raise to i alpha plus beta omega n t and when I do integration 0 to t v square t, d t is going to be summation alpha summation beta, I can write this as minus alpha beta C alpha C beta omega square for the nth level e raise to i alpha plus beta omega n t d t 0 to t.

And let me remind you this t is nothing but 2 pi over omega n, the period for the nth orbit the for the lowest frequency all the other are the harmonics. And therefore, this implies that integral 0 to t e is to i alpha plus beta omega n t d t is going to be equal to t for alpha equals minus beta and 0 for all other alpha and beta because in that case i raise to i alpha plus beta omega add capital T would be same at 0 and t.

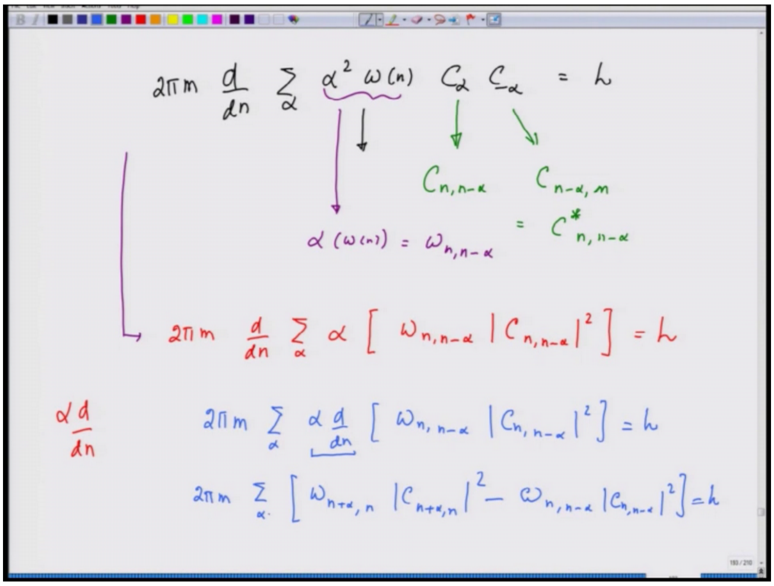
(Refer Slide Time: 09:21)



So, that is the period and only when alpha equals minus beta or beta equals minus alpha then this quantity becomes one and therefore, the answer becomes capital T and therefore, I can write integral P square t d t 0 to t is equal to summation alpha beta is now replaced by minus alpha. So, this is going to be alpha square omega square n C alpha C minus alpha t that is all and 1 omega and t multiplication gives me 2 pi. And therefore, I can write this as 2 pi summation alpha square omega n C alpha C minus alpha that is the classical result.

So, in the limit of n tending to infinity integration pdx comes out to be there was an m also there. So, I get 2 pi m summation alpha; alpha square omega n C alpha C minus alpha, right. Now I need to that is the classical result or intend to infinite result. Now I need to translate this into quantum result and not only that I need to write d by d n of integral pdx is equal to h. So, let us write that if I do that d by D n d by d n of summation alpha 2 pi m, I can write outside alpha square omega n C alpha C minus alpha is equal to h. Now I am going to make use of the correspondence principle.

(Refer Slide Time: 11:18)

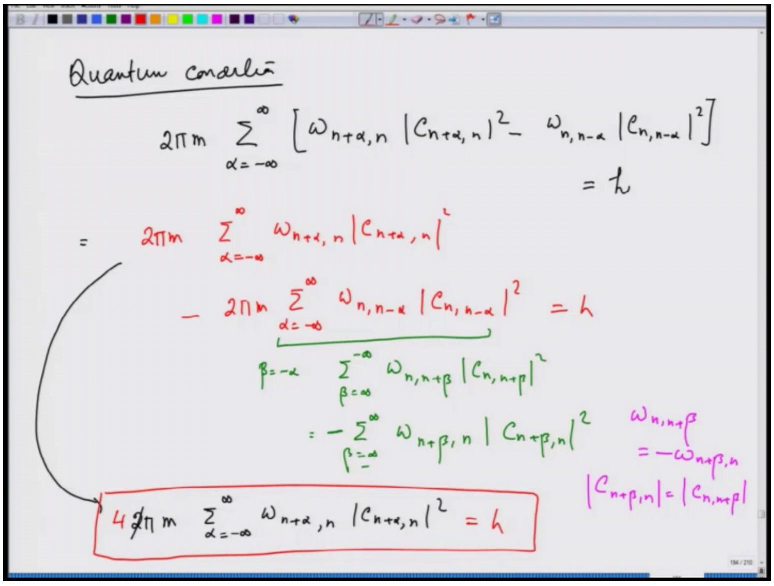


So, I have 2 pi m d by d n of summation alpha; alpha square omega n C alpha C minus alpha is equal to h when I do quantum mechanics, what does this mean? Remember, C alpha and all these things are corresponding to the nth level.

So, C alpha; let me write each number separately, C alpha would be C n, n minus alpha C minus alpha would be nothing but C n minus alpha m which is same as C star and n minus alpha, we have done this in the previous lecture and from here, I am going to take alpha omega n as omega n, n minus alpha. So, when I translate, I will be very careful in this i will write 2 pi m d by d n of summation alpha 1; alpha remains in the brackets, I am going to have omega n, n minus alpha modulus of C n n minus alpha square is equal to h.

Now, recall that I had done something with what alpha d by d n means in quantum mechanics and I am going to use that now. So, I am going to write this as 2 pi m summation alpha, I can take d by d n inside because that is operating only on the brackets square brackets. So, I am going to write this alpha d by d n omega n, n minus alpha modulus of C n, n minus alpha square is equal to h and this is nothing but difference over alpha. So, I am going to write this as 2 pi m summation over alpha omega n plus alpha n C n plus alpha n modulus square minus omega n, n minus alpha modulus C n, n minus alpha square is equal to h alpha varies from minus infinity to plus infinity.

(Refer Slide Time: 13:53)



So, let me rewrite this to the quantum condition comes out to be 2 pi m summation alpha equals minus infinity to infinity omega n plus alpha n modulus of C n plus alpha n square minus omega n to n minus alpha modulus C n, n minus alpha mod square is equal to h, I can do even better and write this as 2 pi m summation alpha equals minus infinity to infinity omega n plus alpha n C n plus alpha n mod square minus 2 pi m summation alpha equals minus infinity to infinity omega n, n minus alpha modulus C n, n minus alpha mod square equals h, I am going to change the second term this term.

I am going to replace beta by minus alpha and write this as summation beta equals infinity to minus infinity is the same thing, whichever order you run it, n omega n, n plus beta modulus C n, n plus beta whole square which I can write as summation beta equals minus infinity to infinity omega with a minus sign, I will change the omega n plus beta n modulus C n plus beta n mod square I could do this because omega n, n plus beta is minus omega n plus beta n, n C n plus beta n modulus is same as modulus C n, n plus beta.

If I switch the 2, right, it does not really change whether transition is from n to n plus beta or n plus beta 2 beta. And therefore, I can write this quantum condition also as 2 pi m summation alpha equals minus infinity to infinity omega n plus alpha n modulus C n plus alpha n square and the second term comes from the other term. So, it becomes instead of 2 it becomes 4; 4 pi m equals h.

So, we have all the C ns and omegas, we have the quantum condition transfer them and now we need to apply these conditions to calculate C and omega which we will do in the next lecture.