Introduction to Quantum Mechanics
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Lecture – 05
Heisenberg's formulation of quantum mechanics: expressing kinematic variables as matrices

What we done so far yes did the Wilson Sommafeld quantum conditions, and got some result more importantly.

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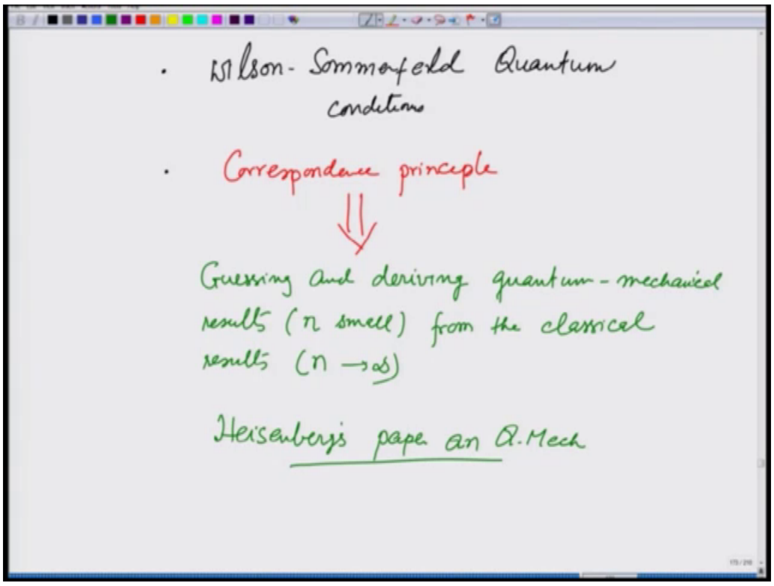

We have also discussed quite in detail the correspondence principle that lead to guessing and deriving quantum mechanical results, analyze a quantum mechanical results I would mean the quantum number n small, from the classical results n tending to infinity and we could get some answers. The questions that arises that quantum mechanics is not complete.

We do not know how to do quantum calculations themselves and that is where Heisenberg's comes and gives his first paper which has also being called the magical paper of Heisenberg. I will give that reference I will upload it on the forum and you can read it, he says that if you want to describe a quantum system do not refer to classical physics, do not try to this extra pollution all that, but calculate quantum mechanical things directly.

So, that is going to be the topic of discussion today Heisenberg's paper on quantum mechanics, and how he use correspondence principle to come up with the quantum version of the whole theory.

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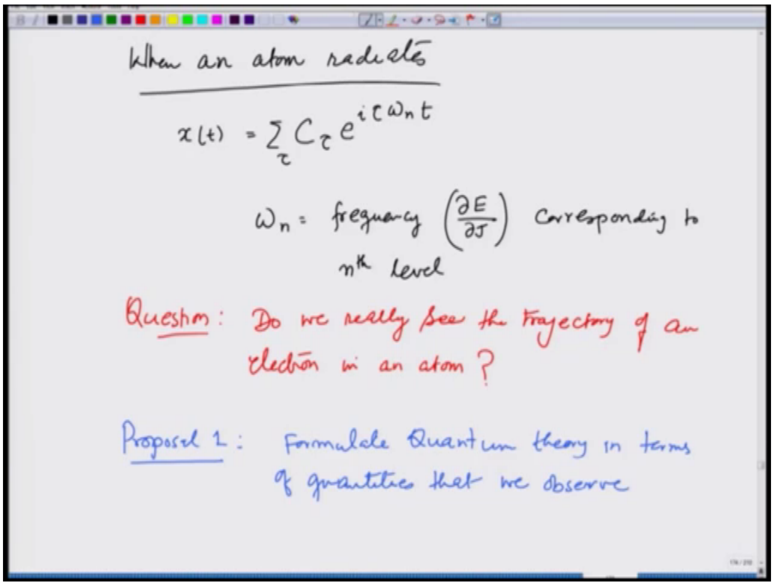

So, let us see what happens. So, far what we have seen when an atom radiates n by atom I mean general quantum mechanical system, it radiates according to suppose is performing simple harmonic motion then I would say that it is has some coefficient or amplitude C tau e raised to i tau omega n t.

This is how is motion looks and the general motion is equal to summation over tau of this. Where omega n is the frequency partial E partial J corresponding to n th level. Heisenberg asked the question do we really see the trajectory of an electron or a quantum particle which is radiating in an atom, and the answer is no I do not theory no which ways the electronic moving.

So, it is not proper to talk about things that we do not observe, and his first proposal one was formulate quantum theory in terms of quantities that we observe. Historically is this thinking has been historically was inspired by remark by Einstein in seminar. So, he says formulate quantum theory in terms of quantities that we observe in an atomic system, and remain as close to the classical as possible make minimum changes.

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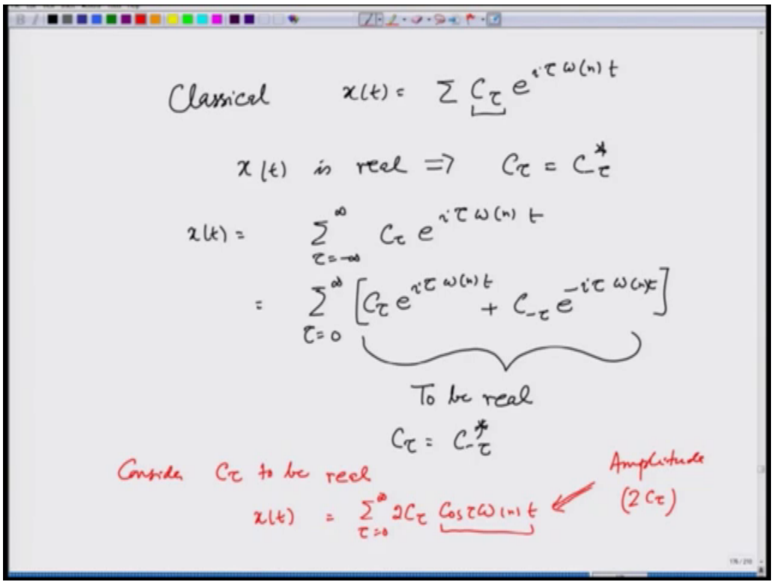

So, what he proposes is I am going to go back and forth between classical in quantum classically, we had x t is equal to summation C tau, e raised to i tau omega for the n th level t. Where these are the amplitudes this is xt and which xt is real and this implies that C tau should be equal to C minus tau star how do we see that? To see that write x t is equal to summation tau equals minus infinity to infinity C tau e raised to i tau omega n t, I can write this as C tau e raised to i tau omega n t plus c minus tau e raised to minus i tau omega n t sum t tau equals 0 to infinity.

Because now it covers both plus and minus and for this to be real, I should have C tau equals C minus tau star. Not only that if I do that and write now let us just consider C tau to be real then I have x t equals 2 C tau cosine of omega n tau t some over tau equals 0 to infinity.

So, the amplitude if the real cosine tau omega t is taken the amplitude of motion is 2 C tau. Write thus the classical result let us see the corresponding quantum mechanical result what Heisenberg proposes.

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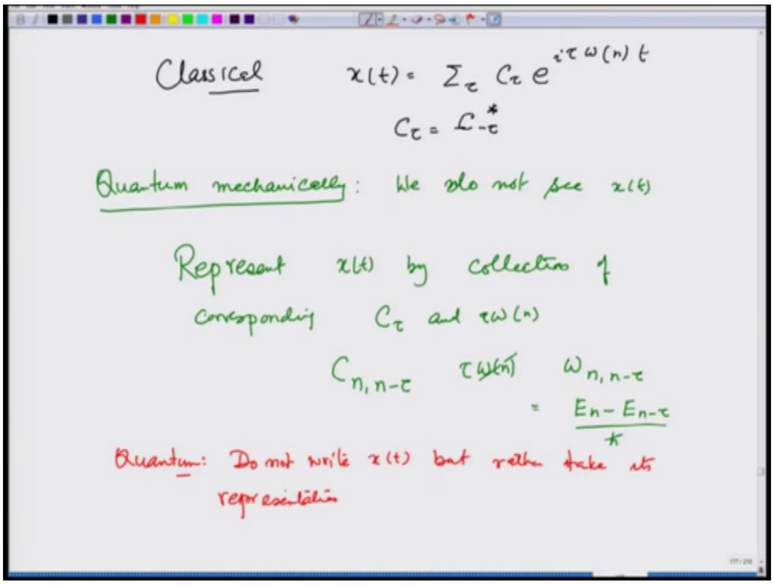

So, classical let me write and again classical x xt equals summation tau C tau e raised to i tau omega n t, C tau equals c minus tau star for x tend to be real quantum mechanically, we do not see xt we do not see xt.

That means I do not observe the trajectory therefore, what Heisenberg proposes represent xt by collection of corresponding C tau and frequency tau omega n, which quantum mechanically are nothing but C tau is actually C n, n minus tau that is the amplitude corresponding to transition from n to n minus tau level, and tau omega n is nothing but omega corresponding to transition from n to n minus tau, which is equal to En minus E, n minus tau divided by h cross.

So, I am not going to write an xt quantum mechanically do not write x t, but rather take its representation and how are we taking the presentation.

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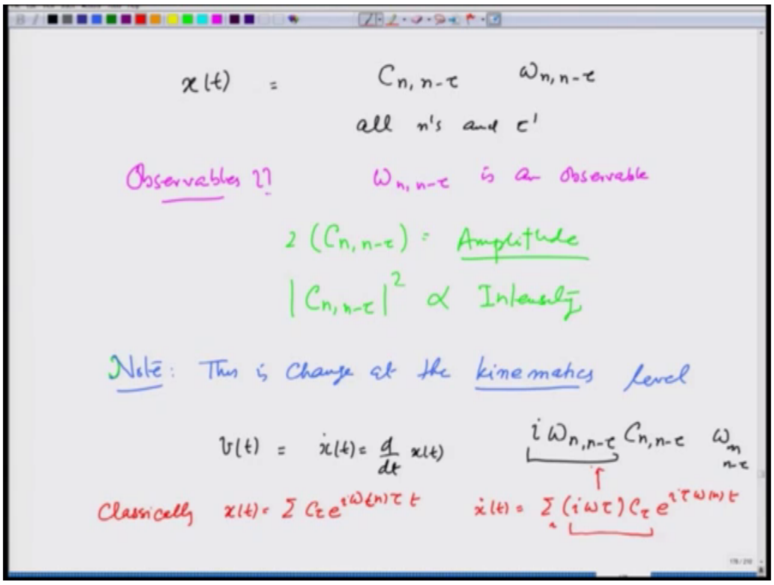

So, given and xt it is going to be represented by C n, n minus tau and the corresponding frequency omega n n minus tau which are given what they are, and all ns and taus this is how represent. So, it becomes a collection of numbers all right and that is how you represent how are the observable because I earlier I said that he had formulated theory in terms of observables. So, are they observables omega n, n minus tau is an observables because I see that is the frequency of radiation and C n, n minus tau times 2 is the amplitude and therefore, mod C n, n minus tau square is proportional to the intensity.

So, we are proposing 2 quantities omega n n minus tau and C n, n minus tau, which represent the quantum nature of the system. I just want to note this is changed at the kinematics level; that means, kinetics is not in decided we just set kinematically this is how we are going to represent the quantities how about the corresponding velocity? The corresponding velocity v t which is nothing but x dot t, which is d by dt of xt is going to represented by i omega n, n minus tau C n, n minus tau and the corresponding frequency n, n minus tau.

How would I get this factor? This factor is because let me explain again what happened classically we had xt equals summation C tau e raised to i omega tau, or tau omega n t its derivative, x dot t is going to be summation tau i omega tau, C tau e raised to i tau omega n t and this is being replaced by this whole thing.

So, we are making correspondence with classical, but making changes that now any Fourier component of the motion is going to be replaced by number corresponding to the transition from n th to n minus tau level.

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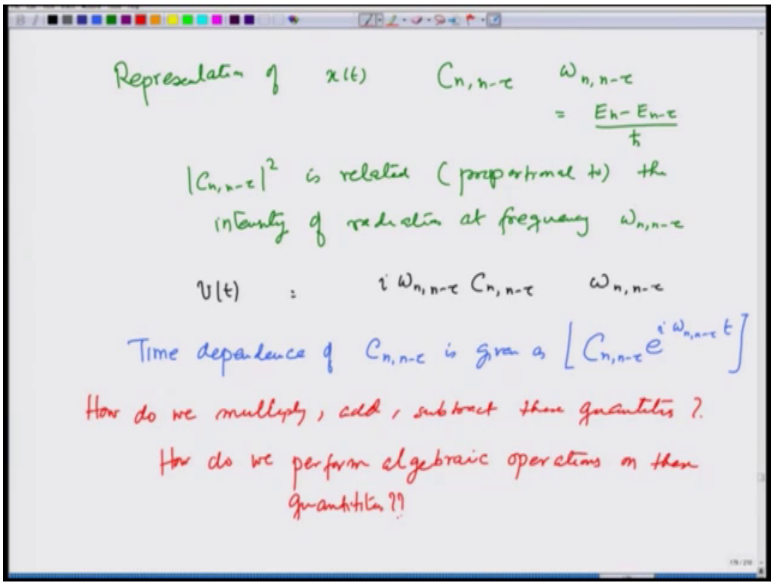

So, we have representation of xt which is the collection of numbers C n, n minus tau and the corresponding frequency omega n to n minus tau, which is equal to En minus E, n minus tau over h cross. Mode C n, n minus taus square is related infect proportional to the intensity of radiation at frequency omega n, n minus tau. So, this is the kinematic change is that suggested an all the corresponding other mechanical variables can be calculated.

For example we said that we tau is going to be represented by i omega n n minus tau C n, n minus tau and omega n, n minus tau right and this comes because we are going to write. So, let me write this time dependence of C n, n minus tau is given as C n, n minus tau e raised to i omega n n minus tau t. This is taking directly from the classical result and therefore, I could write vt by taking the derivative similarly I can write angular momentum or whatever. The next question that arises is how do we multiply add subtract these quantities.

In short how do we perform algebraic operations on these quantities and for that Heisenberg was guided by how frequencies combine.

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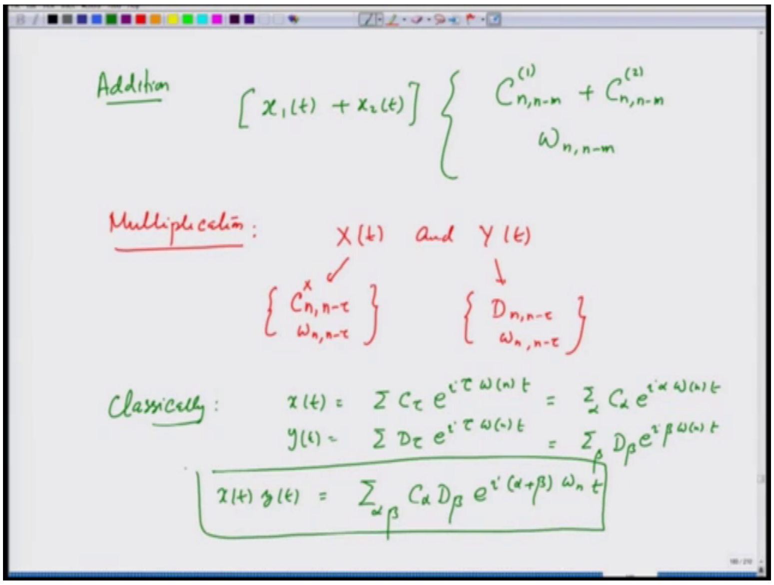

So, first question add the two quantities supposed I want to add x 1 t, plus x 2 t which also takes care of the subtraction this will be represented by C n, n minus m further first quantity plus C n, n minus m for the second quantity, and the corresponding frequency omega n, n minus m. The frequency is that I keep should be the same as those that I observe addition and subtraction are taken theorem more important it is multiplication. Suppose I have two quantities Xt and Y t, now these are represented by these numbers, Cn let us call it x n minus tau and omega n, n minus tau and this is represented by let us say D n, n minus tau and omega n, n minus tau how do I combine the two.

So, classically or in the classically limit using corresponding principle, what would be have xt is equal to summation C tau, e raised to i tau omega n t that is it. Y t will be equal to summation D tau e raised to i tau omega n t. I can also write this the x as summation alpha c alpha e raised to I alpha omega n t and I can write why a summation beta D beta e raised to I beta omega n t and therefore, classically x t times y t would be equal to summation alpha beta C alpha D beta e raised to i alpha plus beta, omega n t is this how I want to represent it quantum mechanically also.

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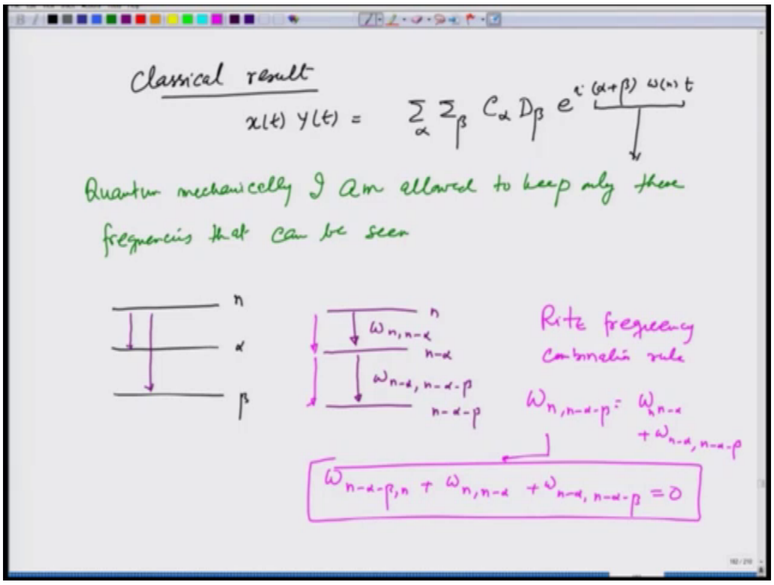

So, let us see that let us write the classical result for xt times y t is summation alpha summation beta. C alpha D beta e raised to I alpha plus beta omega n t, and this is not going to be in the case in quantum mechanically because quantum mechanically I am allowed to keep only those frequencies that can be seen. So, any arbitrary frequency cannot be seen for example.

Let us say if I have two levels, three levels, n alpha beta. If there is a transition taking place from n to alpha and n to beta this will be the some of these to frequencies, but I cannot see the frequency the frequency that I can however, see is frequency omega n, n minus alpha and one from this is n minus alpha this is n, n minus alpha minus beta and one which is omega n minus alpha two n minus alpha minus beta I can see that frequency.

So, what is known as rites frequency combination rule, according to which I can see frequency n, n minus alpha minus beta which is summation of omega n minus n, n minus alpha plus omega n minus alpha n minus alpha minus beta. Not all these frequencies can be combined right all the frequencies seen this is sometimes also written in a cyclic form which is omega n minus alpha minus beta and plus omega n, n minus alpha plus omega n minus alpha, n minus alpha minus beta is equal to 0 what is known as rally frequency some rule or combination rule.

So, all the frequency is data seen follow this rule, you can visualize this that only I see only those frequencies that come from combination of one level to the second level and from second level to the third level I cannot the combine frequency arbitrarily. So, while writing this classical result in a quantum mechanics sense, I have to be careful I can choose only certain combinations. So, let us see that.

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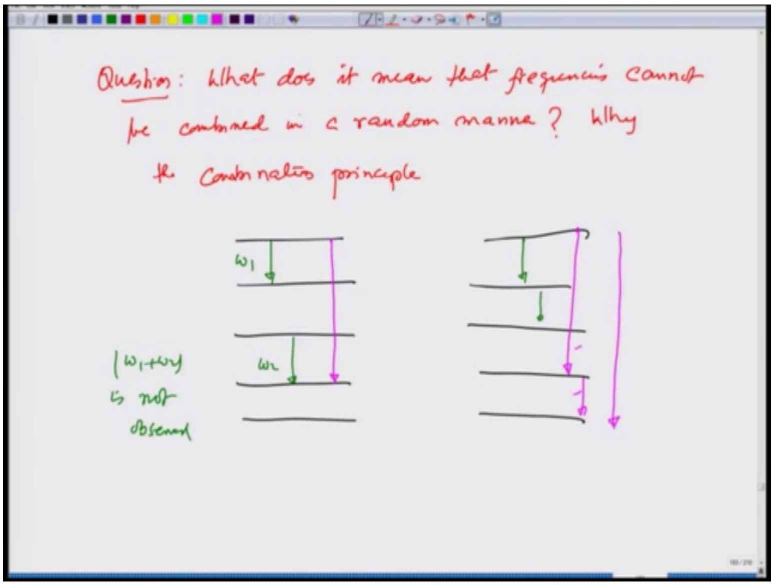

So, question so, let me there is a question here what does it mean that frequencies cannot be combined in a random manner, in other words why the combination principle. So, as I said earlier suppose I take many many levels, the frequency that I observed cannot be for example, I cannot observe this combination if I take suppose this is frequency omega 1 this is frequency omega 2 omega 1 plus omega 2 is not observed.

So, I cannot take all the frequency and combine them in any manner like, what is observed; however, is this plus this or second one this plus this. So, they have to be combined in a manner that follows Ritz combination principle or you can say I see one particular frequency where our total transition can be broken into two consecutive transitions that is a physical way of visualizing it. Here I cannot break for example, in the classical case I cannot break I am showing it on the left, the transition shown in pink into these two transitions and therefore, I cannot combine frequency in such a (Refer Time: 22:58).

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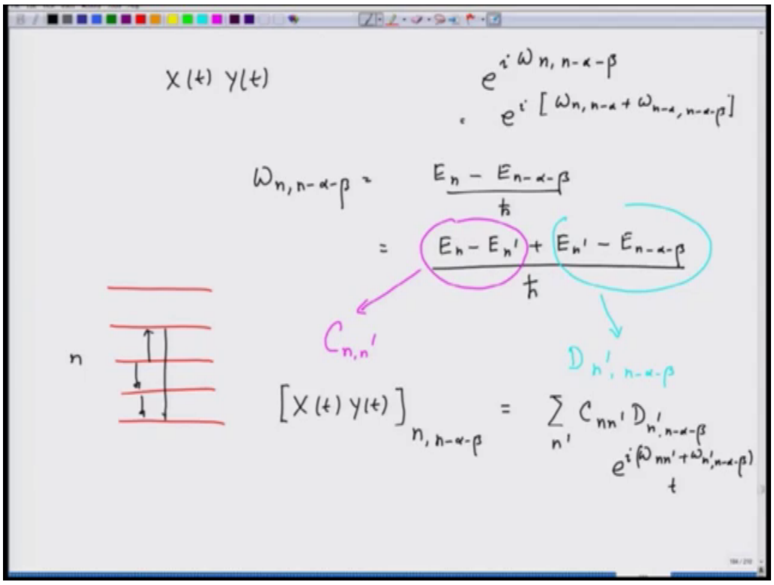

So, back to doing Xt Yt. So, I should have only those frequencies e raised to i omega n, n minus alpha that is a minus beta which is nothing but e raised to i, I think combine them only in this manor omega n, n minus alpha plus omega n minus alpha n minus alpha minus beta. This is the only way I should combine which makes sense because I can write omega n, n minus alpha minus beta which is E n minus E n minus alpha minus beta over h cross, as E n minus E n prime, plus E n prime minus E, n minus alpha minus beta over h cross where n prime now could be any intermediate states.

For example again going back to this example I will make these energy levels. Suppose this is my nth level what I am representing this, this frequency omega n n minus either would like this going up and coming down and all these things, but they have to be combined in this manner.

And therefore, if I am combining frequency like this the corresponding if I take see according to this at will C n, n prime, and this one for y would be d n prime n minus alpha minus beta and therefore, I should have X t Y t suppose I want to get this term n, n minus alpha minus beta, this should be equal to summation all n primes C n, n prime, D n prime n minus alpha minus beta and I can write this as e raised to i omega n, n prime plus omega n prime n minus alpha minus beta t.

So, you done it very very systematic manner combining an experimental result call they rites frequency combination rule, and then really seeing how I could combine this.

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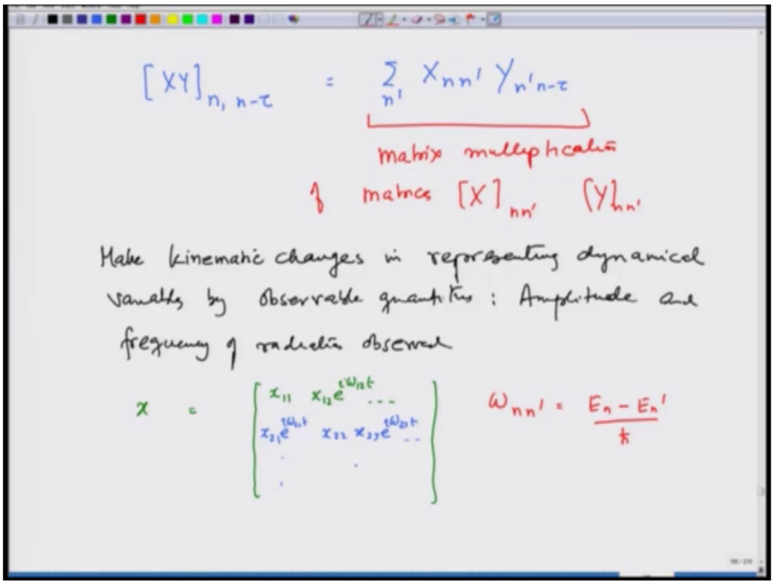

So, what we are getting is therefore, I can write in general is x y some n, n minus tau component is going to be given by summation X nn prime, Y n prime n minus tau sum over n prime and in module language I know this is nothing but matrix multiplication. M ultiplication of matrices X n, n prime and Y n, n prime the two matrices I am making their product.

So, Heisenberg did not know matrix mechanizes that time physics is did not use them he discovered this through this. So, which is another greatness. So, he has now shown that make kinematic changes and representing dynamical variables by observable quantities and that means, the amplitude and frequency of radiation observed.

So, a quantity like x would be represented by let me now write it. X 11, x 1 2 e raised to i omega 1 2 t and so on the vertical side x 2 1 e raised i omega 2 1 t, x 2 2, x 2 3 e raised to i omega 2 3 t and so on. Where omega n, n prime is given as E n minus E n prime over h cross and it could be negative or positive depending on what nn primes are. So, all quantities I going to be represented by matrices and the moment I say matrices I also know how they should be multiplied consequence of this is that.

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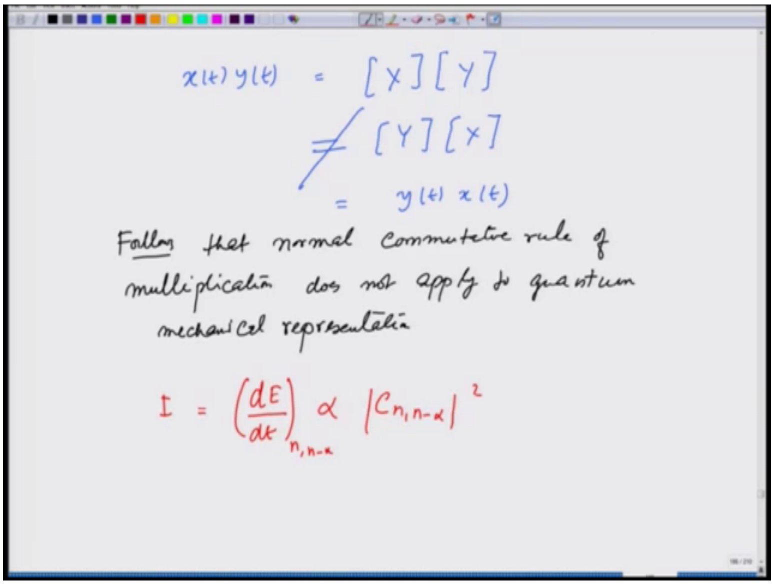

If I take xt yt, which is the matrices multiplication of X matrix and Y matrix it is not the same as Y X this we no formative algebra.

So, this is not the same as y t xt. So, by this representation it also follows that the normal commutative rule of multiplication does not apply to quantum mechanical representation this is a big kinematic change from the classical way of thinking that. Now x times v is not going to the same as v times x, and this was a departure and the rate of transition or rate of energy coming out which is the intensity for an atom is going to be proportional to C n, n minus alpha mode square.

Let us say this is transition taking place from n n minus alpha. If I could calculate C n omega directly I am answer I solve the problem quantum mechanically. How it is done, what are the quantum conditions, and how it is the dynamic followed will be subject of next two lectures.