Introduction to Quantum Mechanics
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Lecture – 04
Application of the correspondence principle: Einstein's A Coefficient of the harmonic oscillator and the selection rules for atomic transitions

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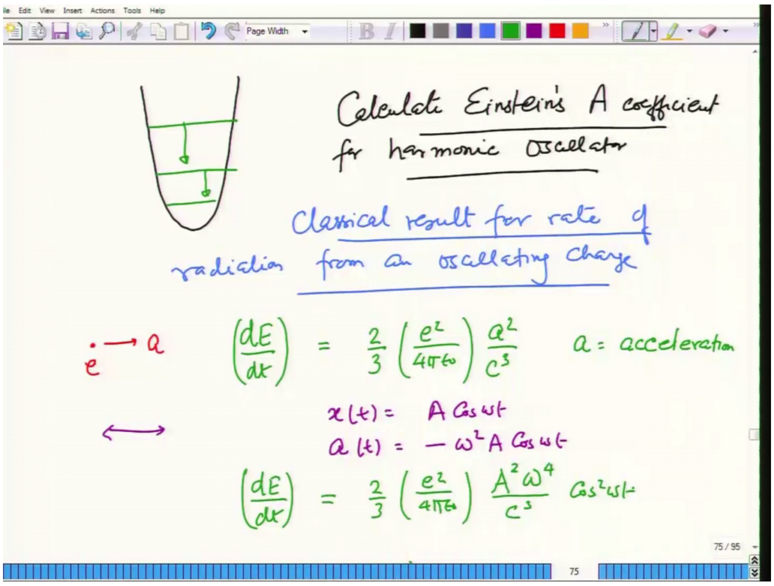

What you learned in this that in this harmonic oscillator when the jumps takes place, they take place at most by 1 delta n and not more than that. Let us now use correspondence principle to calculate the A coefficient for harmonic oscillator. So, I am going to calculate Einstein's A coefficient for harmonic oscillator, for this I will use a classical result and the classical result for rate of radiation from an oscillating charge and use that result and that says classically it is rate of radiation form a charge.

Let us say this charge is e and it is accelerating with the acceleration a, a is given by the formula 2 thirds which is known as (Refer Time: 01:37) formula e square over 4 pi epsilon 0 a square over C cubed where a is the acceleration. Now for a particle that is performing a simple harmonic motion. So, if x t is given as amplitude cosine of omega t, the acceleration we know is minus omega A square a cosine of omega t, And therefore d E d t, the rate of radiation would become 2 thirds e square over 4 pi epsilon 0, A square is the amplitude square omega raise to 4 divide by C cubed where C is the speed of light cosine square omega t.

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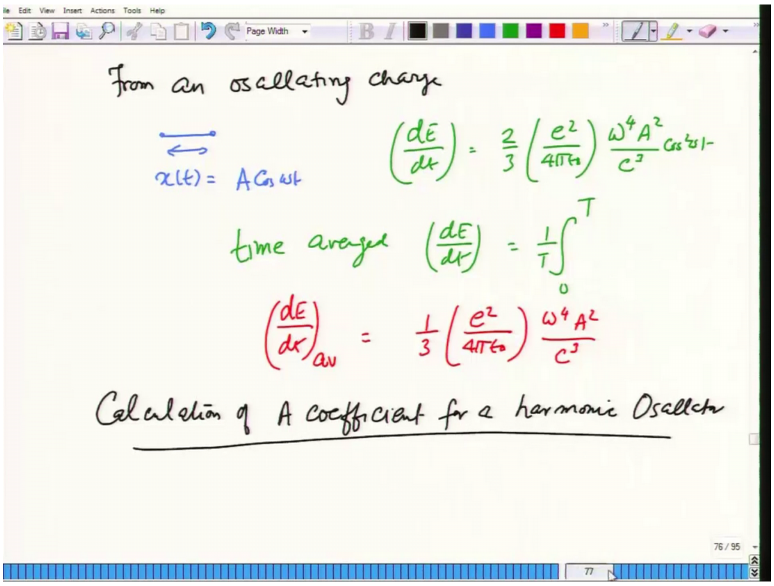

So, we have from an oscillating charge x t equals A cosine of omega t d E d t is equal to 2 thirds e square over 4 pi epsilon 0 omega raise to 4 amplitude square over C cubed cosine square omega t. So, time averaged d E d t which is nothing but integrate this from 0 to time period and divided by T gives you a half for cosine square omega T. So, time average d E by d t an average comes out to be 1 third e square over 4 pi epsilon 0 omega raise to 4 A square over C cubed. This is the average rate at which a charge particle radiates the charge particle is oscillating with frequency omega radiates.

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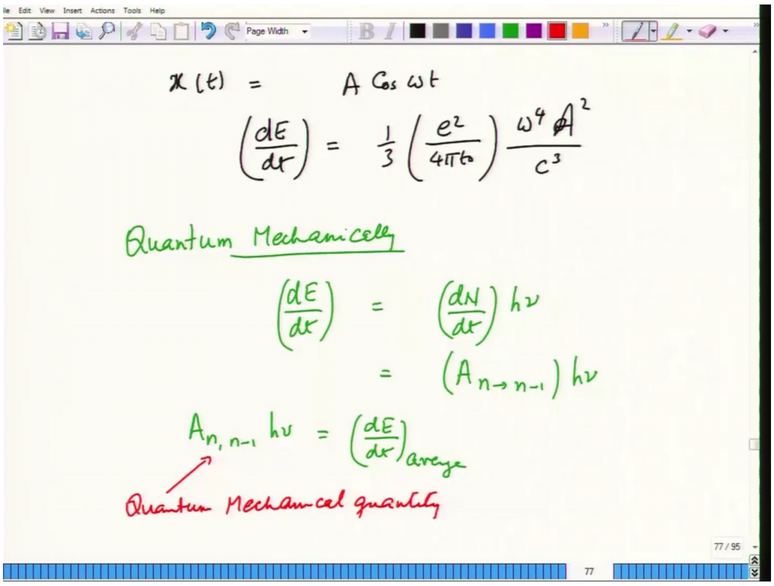

Now, let us see come to calculation of A coefficient for a harmonic oscillator. So, when a harmonic oscillator is performing motion x t equals a cosine of omega t, I have the rate of average rate of radiation is equal to 1 third e square over 4 pi epsilon 0 omega raise to 4 A square over C cubed.

Now, quantum mechanically, the rate of radiation would be the number of photons coming out from an atom. So, number of photons coming out would be nothing but d N by d t per unit time times h nu; that will be the rate and this I know is nothing but A n to n minus 1 that is 1, I am calculating the a coefficient h nu. So, A n n minus 1 h nu is equal to d E d t average. This I should remind you is a quantum mechanical quantity A coefficient because A n, n minus 1 gives you the probability through which the x r state decays and with that multiplied by h nu gives you the rate in the large n limit by correspondence principle the 2 result should match.

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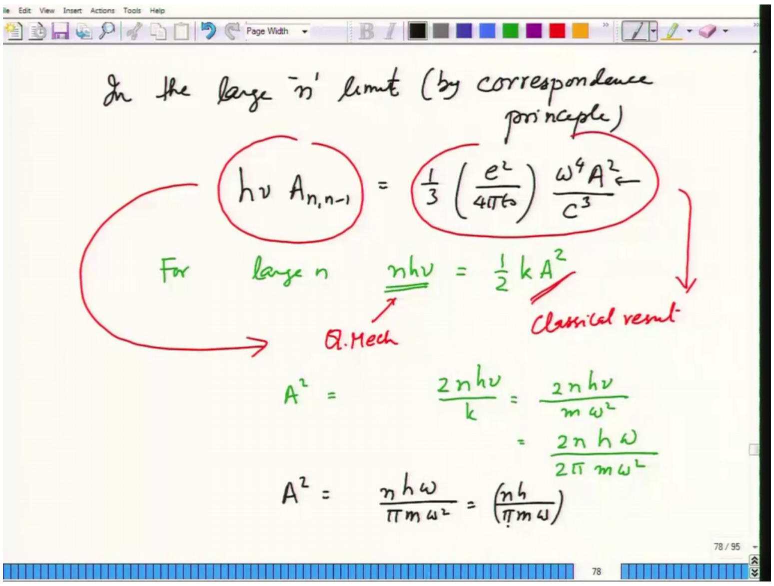

So, h nu A n to n minus 1 should be equal to 1 third e square over 4 pi epsilon 0 omega raise to 4 amplitude square over C cubed and then hopefully whatever a n, n minus 1, I calculate here that will to true in the lower limit also.

So, this is where I am using the correspondence principle. Now how do I calculate the amplitude? Now for large n for large n or small n the energy is n h nu and if it is large n, I can write this as 1 half k A square that is energy classically. So, on the left hand side, I am using a quantum mechanical result, on the right hand side, I am using the classical result. Similarly in the equation above, this is a quantum mechanical result and this is a classical result. So, I can write these arrows here also.

Because in the left hand side, the mechanism through probability on the right hands side; it is through and accelerating charge giving out radiation in any guess, I can now calculate the amplitude A square is equal to 2 n h nu over k which is nothing but 2 n h nu over m omega square. I changes to omega against 2 n h omega over 2 pi m omega square and therefore, I can write the amplitude square as n h omega over pi m omega square which as n h over pi m omega plus A square value and now I am going to substitute that A square value in the expression for h nu A n, n minus 1.

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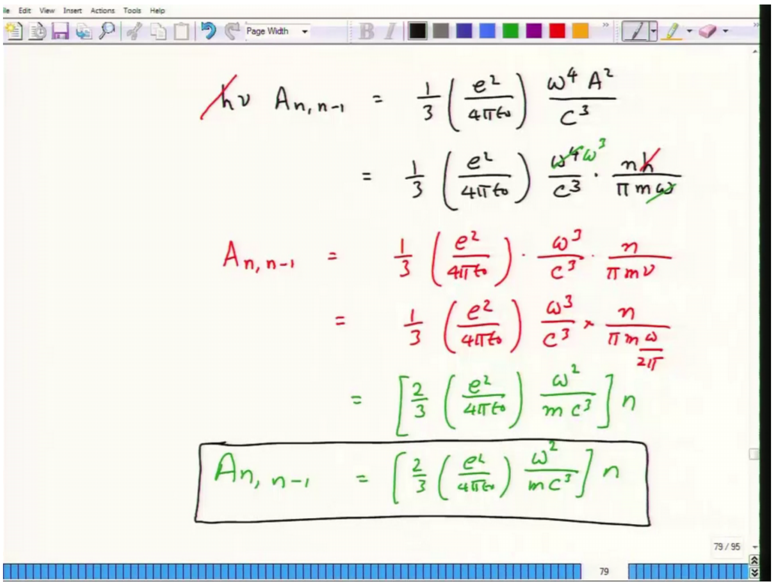

So, if you do that I have h nu A n, n minus 1 is equal to 1 third e square over 4 pi epsilon 0 omega raise to 4.

A square over C cubed and which is equal to 1 third C square over 4 pi epsilon 0 omega raise to 4 over C cubed and I just calculated the value of a square which is nothing but n h over pi m omega. Now I can cancel terms on both 2 sides is h cancels. So, notice how beautifully, I have actually converted a classical formula for amplitude to a quantum mechanical quantity n h cross energy.

So, I get A n, n minus 1 is equal to and here, I can cancel this omega to omega cubed is equal to 1 third e square over 4 pi epsilon 0 omega cubed over C cubed n over pi m nu again substitute nu for omega, I get 1 third e square over 4 pi epsilon 0 omega cubed over C cubed times n over pi m omega over 2 pi which gives me 2 thirds e square over 4 pi epsilon 0 omega over m C cubed n.

So, the Einstein's A coefficient A n, n minus 1 is this 2 thirds e square over 4 pi epsilon 0 omega over m C cubed times n. This is omega square omega square. So, A n, n minus 1 is 2 third e square by 4 pi epsilon 0 omega square by m C cubed n and this is a formula which is given purely in terms of the frequency of the oscillator the n index for the level form is the transition is taking place.

So, this is the formula which is quantum mechanical, we have used correspondence principle, in this we have mix rates of we know transition in classical, the regime to what would happen in the quantum regime, where it is given by the Einstein's A coefficient and using that wave found the formula. Similarly, we can apply to other places also and one of those, I will give you as an assignment problem finally; I also like to tell you about selection rules.

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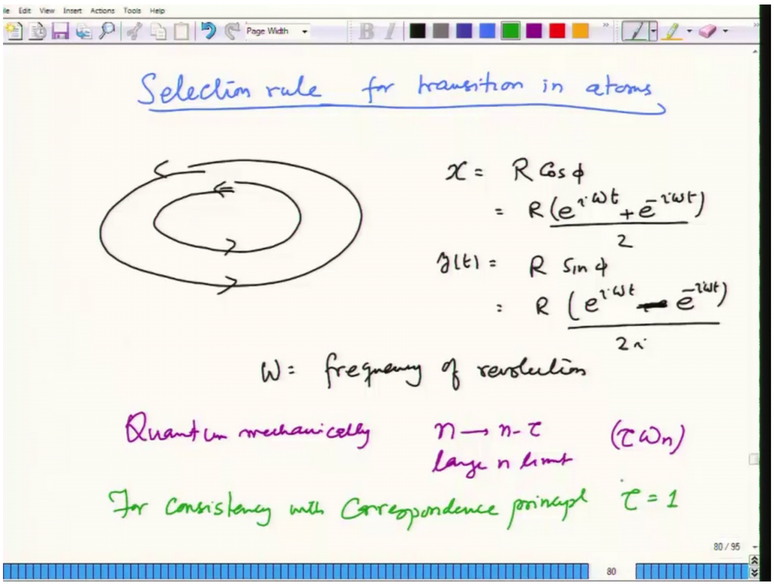

In fact, one selection rule for transition in atoms; in atoms I take the simplified version where the board orbits are there and particles are moving in circular orbits when the particles are moving in a circular orbits, x is the radius times cosine of phi which is R e raise to i omega t plus e raise to minus I omega t divided by 2 and y t is equal to R sin of phi which is equal to R e raise to i omega t minus e raise to minus i omega t divided by 2 i where omega is the frequency revolution.

Notice again that there is only one frequency that exists quantum mechanically, when the transition takes place from n to n minus tau in the large n limit, the frequency that can exist of tau times omega n, where as classically we see only one radiation frequency existing and that is the frequency of revolution and there for this implies for consistency which correspondence principle, I should have tau equals 1, it can be either from one up one down, but the change can be maximum one.

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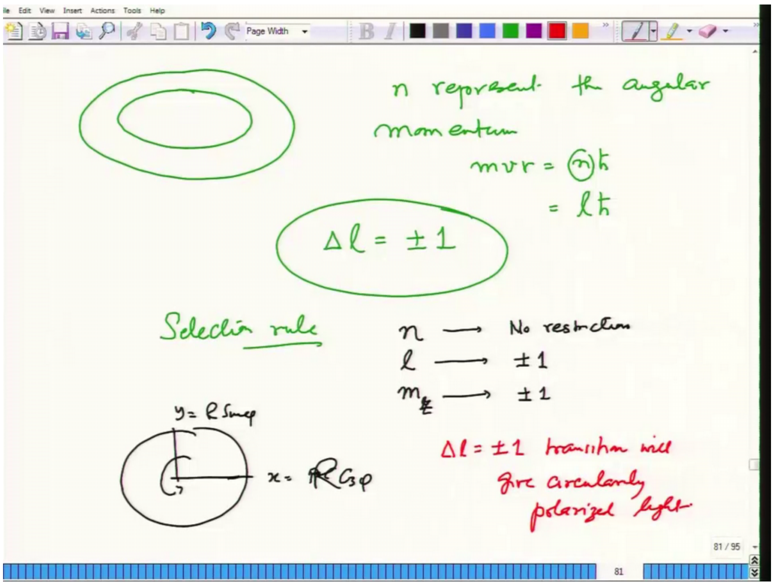

And therefore now remember when we have Bohr orbits, what does is n represents; n represents the angular momentum through the condition that mvr is equal to n h cross and this implies that this n that quantum number that gives you the angular momentum which is Sommerfeld case, we called l h cross the change of l can be maximum delta l can be plus or minus 1, if higher l changes were allowed in the classical regime, I would not see the fundamental frequency out, see harmonics which are not seen what I see classically is the only one frequency of radiation and that is the frequency at which particle is rotating. And therefore, maximum delta l can be 1. So, this is another selection rule. Now remember from the old quantum theory I had n l n m l or m z quantum numbers. So, what we found is l plus minus 1 is allowed. In fact, when you do the more recursive this is also comes out to be plus minus 1. However, there are more restrictions on n.

But the idea want to do give rather than just deriving these is that true correspondence principle, you can get some inside into what is allowed; what is not allowed; not only that you see one more thing when a particle is moving in a circle, in this manner that x is some radius cosine phi and y is some radius sin phi, then the radiation will be given out there will be given out will be circular polarized. So, we can say also by correspondence delta l equals plus minus 1 transition will give circularly polarized light.

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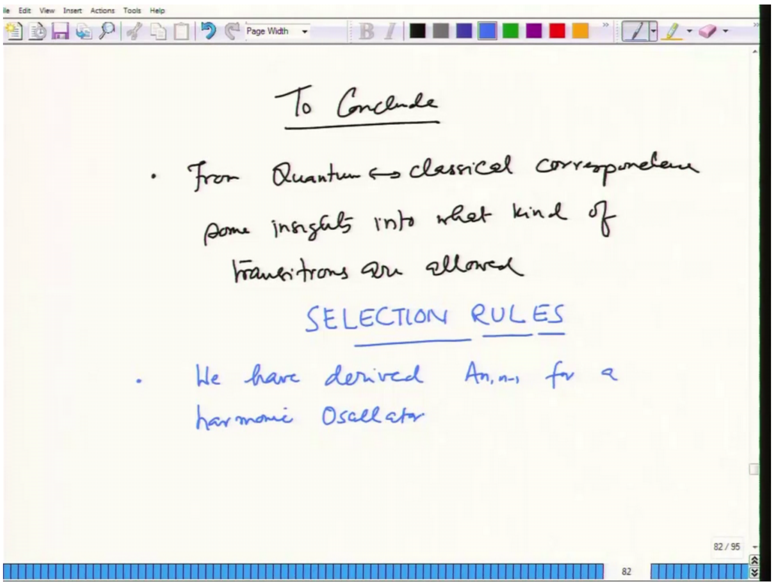

So, to conclude this lecture what we have done is we found from quantum classical correspondence some insights into what kind of transitions are allowed enough given a term to this selection rules and the important particle in spectroscopy and to we have derived A n, n minus 1 for a harmonic oscillator, you know ask why A n, n minus 1 why naught A n, n minus 2 that is because A n, n minus 2 represent transition from n 2 n minus 2 level and change of n by 2 is not allowed harmonic oscillators.

So, only nonzero points are A n, n minus 1 or A n, n plus 1 also recall from the lecture Einstein's A and B coefficients once you know the a coefficient, you can also calculate the B coefficient that is the coefficient for stimulated emission or absorption.

Thank you.