Introduction to Quantum Mechanics
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Lecture – 03
Selection rules (for transitions) through the correspondence principle

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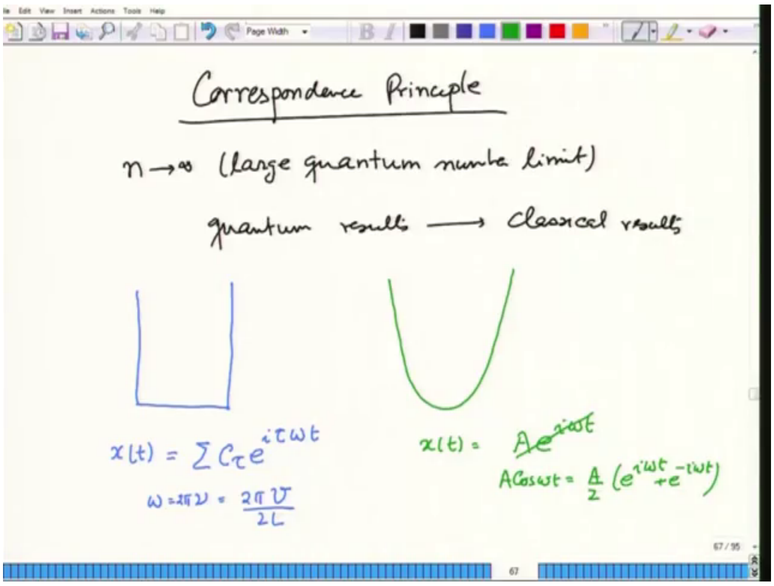

We have learnt so far about the correspondence principle that says that for n tending to infinity that is the large quantum number limit, the quantum results go to classical results. In this lecture, we are going to see how we can use this principle to derive something called the selection rules and also the A coefficient for the harmonic oscillator.

Recall, I took 3 examples in the previous lectures and I just consider 2 of these; the harmonic oscillator and the particle in a box and in the harmonic oscillator case, I said that the displacement can be written as summation C tau e raise to i tau omega t where omega is the basic frequency which is nu times 2 pi which was equal to 2 pi v over 2 L. On the other hand, in the case of harmonic oscillator x t has only one component and I can write this as the amplitude e raise to i omega t. In fact, I want to do better than that; this is usually written as A cosine of omega t which is amplitude divided by 2 e raise to i omega t plus e raise to minus i omega t.

Thus, you see that the particle moving in a box has the fundamental frequency V over 2 L and all its multiples and therefore, when it performs motion, it will radiate all kinds of frequencies. The fundamental and it is harmonics tau times that much where as a particle performing motion in a harmonic oscillator would give out only one frequency.

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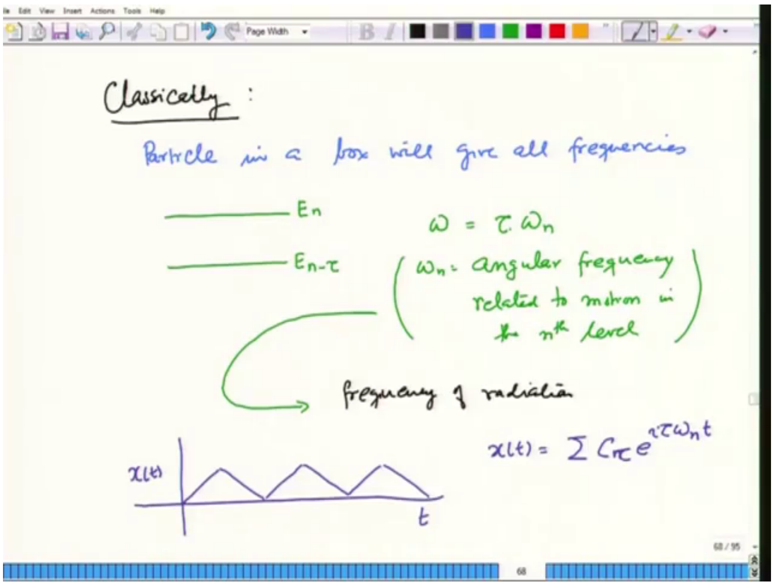

Let us see how we can use that. So, classically just learnt that particle in a box will give all frequencies. By all frequency, we do not mean that it will give out frequencies which are continuously varying, but if it makes a transition; if the particle makes a transition from nth level to say E n minus tau level, then the frequencies emitted would be tau times omega n; right where omega n is the angular frequency related to motion in the nth level.

So, classically this frequency omega n or the angular frequency connected with the motion in the nth level will be the frequency of radiation and what we saw last time is that if I write its displacement verses time, it is like this. It is not a pure harmonic. So, this is what tells you that x t actually has higher harmonic, also which I wrote earlier C; C tau e raise to i tau omega. Let us call it omega n t. So, it will give you frequency of radiation omega n and since the motion also contains other frequencies all the other frequencies will also come out.

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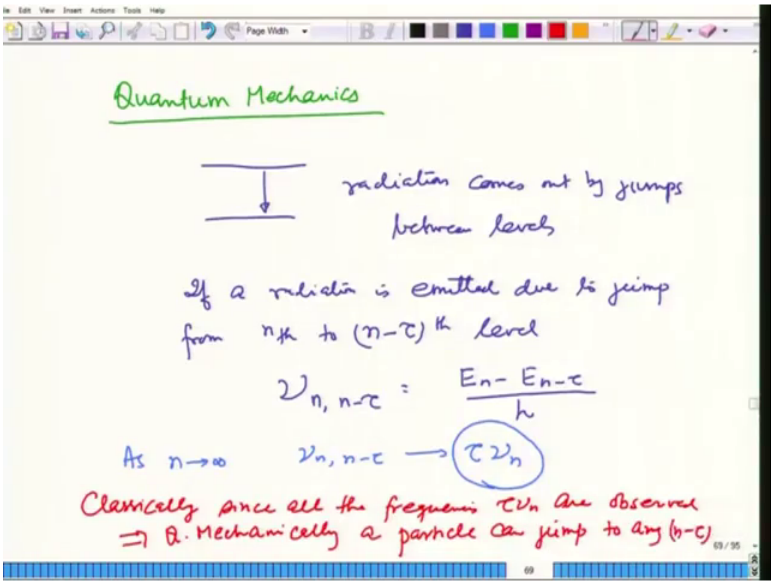

Let us see; what does it teach us about quantum transitions. So, let us now write quantum mechanically; radiation does not come out because a particle is oscillating with frequency nu rather it comes out by jumps. So, radiation comes out by jumps; radiation comes out by jumps between levels where as classically it is because of the acceleration. Now if a radiation is emitted due to jump from nth to n minus tau th level the frequency nu n, n minus tau is given as E n minus E n minus tau divided by h. This is given by finite difference, right where as we recall the previous lecture in classically, it is given by a differentiation with respect to j.

Now, as n tends to infinity, nu n t n minus tau go to tau nu n where nu n is the classical frequency. So, you see that classically since all these frequencies are observed, right. So, let us write this classically. Since all the frequencies tau nu n are observed right; that means, quantum mechanically a particle can jump to any n minus tau.

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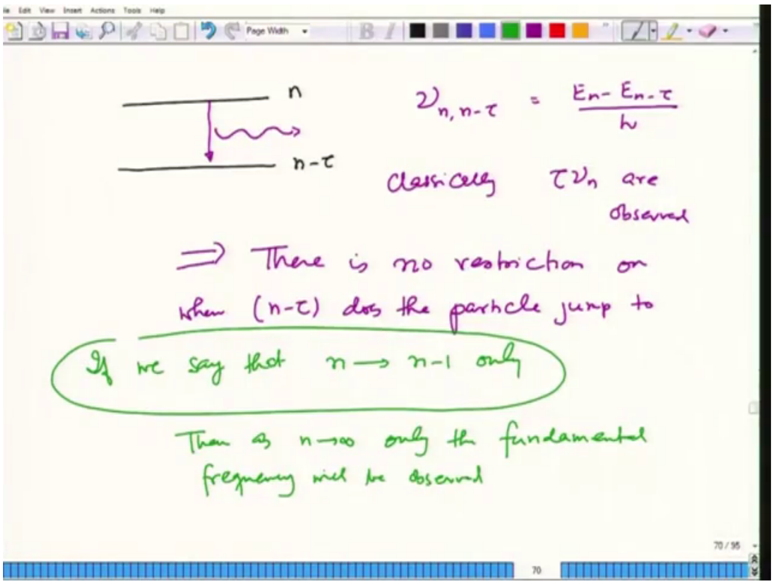

Let me write this again. So, this is nth level n minus tau level and here is this jump coming in and some radiation comes out. So, radiation frequency n, n minus tau is E n minus E n minus tau over h and you see that classically all tau nu n are observed because the particle makes a motion periodic motion of basic frequency nu n, but then also the all this harmonics exist. So, this means there is no restriction on where that is n minus tau does the particle jump to.

If let us see a count example, if we say that n can go to n minus one only and this is just a counter example if we say this, then as n goes to infinity only the fundamental frequency will be observed; that means, it is just not n minus one it can go to any frequency.

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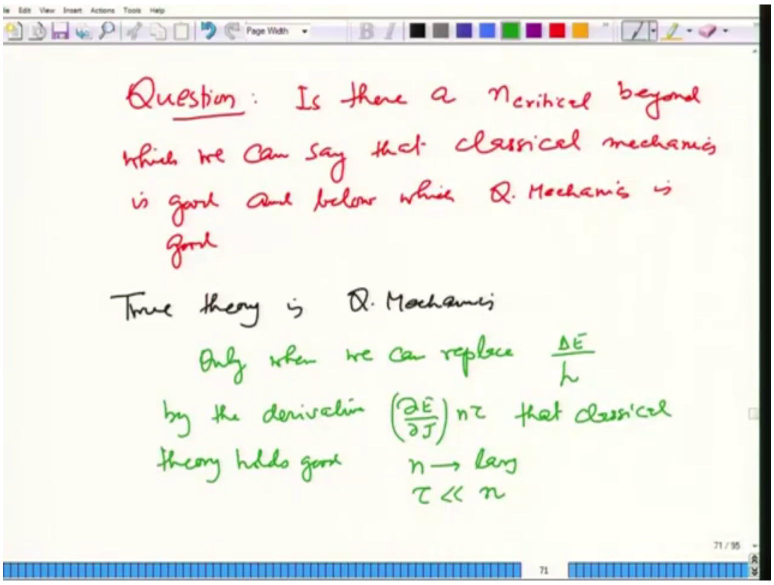

It likes is now the question arises question is there a n critical beyond which we can say that classical mechanics is good and below which quantum mechanics is good.

In other words is there a boundary between classical and quantum mechanics the answer is no actually the true theory is quantum mechanics it is only as we saw in the previous lecture only when we can replace delta E by h by the derivative d E by d j times n tau that classical theory holds good and; that means, basically that n has to be large enough and tau has to be much much smaller than n that is the only condition, but what this principle is telling you is that you should actually be approaching classical level when you can replace that the difference by derivative effectively and for that the condition is n is very large and tau is much much smaller than n.

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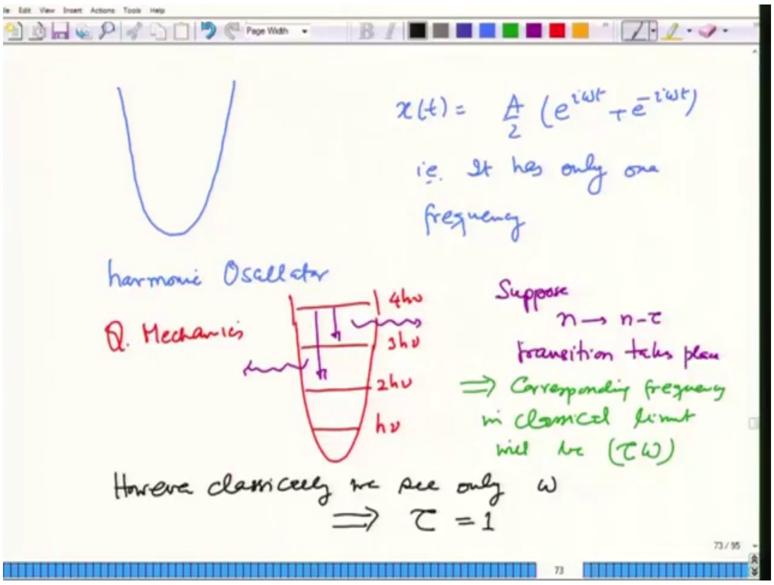

Now, let us take the other example the other example is this harmonic oscillator and classically its motion is given by x t equals a by 2 e raise to i omega t plus e raise to minus i omega t that is it has only one frequency. Let us look at it quantum mechanically, there are these levels h nu 2 h nu 3 h nu 4 h nu and. So, on and the radiation is given out when there is a jump. Now you see how we see by comparing with the classical results there is a selection rule suppose n to n minus tau transition takes place, then the corresponding frequency right the corresponding frequency in classical limit will be tau times omega.

However, classically we see only omega this implies tau at max can be one and this gives you a restriction that in a harmonic oscillator the only jumps that takes place are where n goes the particle makes the jump n goes to n minus 1 or it absorbs n plus 1.

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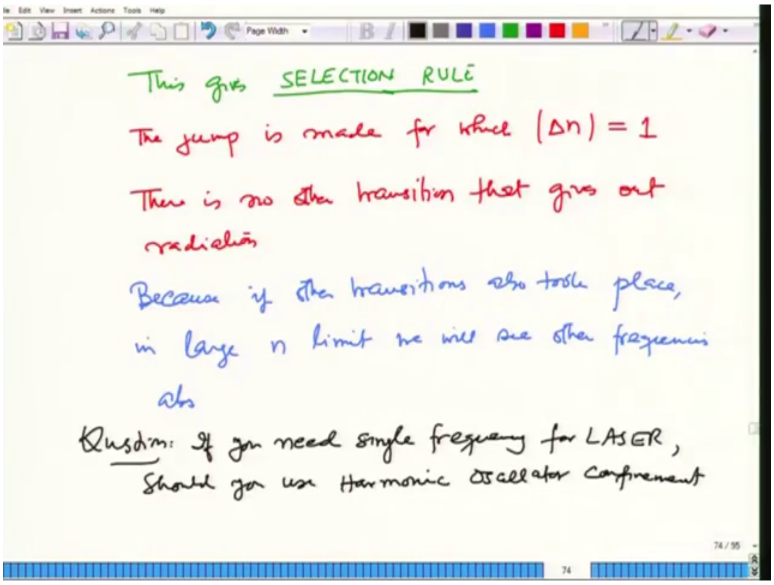

So, this gives what is called a selection rule and that says selection rule says that the jump is made or the transition takes place for which delta n magnitude is one; otherwise for all of the transition there will be no radiation.

So, there is no other transition that gives out radiation and why is that let me repeat because if other transitions also took place in large n limit, we will see other frequencies also and that does not happens. So, to be consistent with correspondence principle there is only one transition allowed in quantum level.

So, a question that has arisen just now is question; if you need single frequency for laser should you use harmonic oscillator confinement actually the 2 questions are not related because in a laser, you choose the frequency that causes the transition to be precisely that frequency. So, you can choose any system and the stimulated emission will take place only for that particular frequency with which you are doing this stimulated emission. So, this question does not really arise.