Introduction to Quantum Mechanics
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Lecture - 02
Black Body Radiation II- Intensity of radiation in terms of energy density

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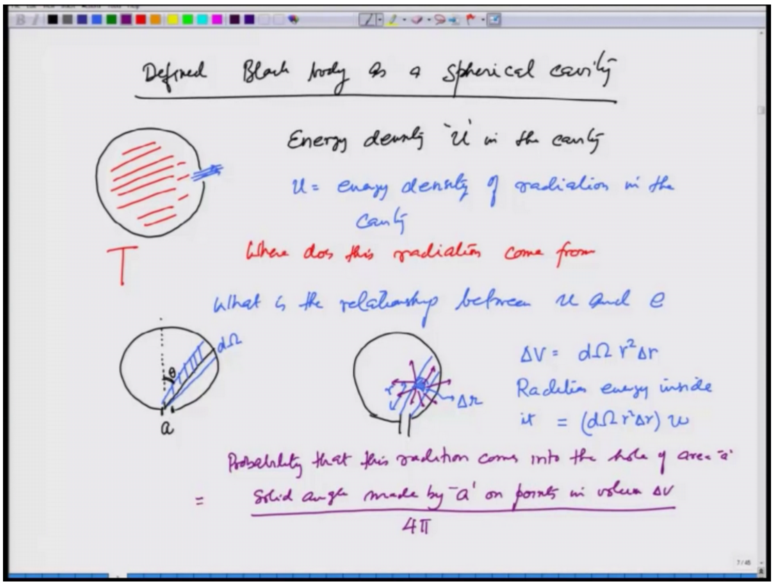

In the previous lecture we defined black body as a spherical cavity, and concluded that if I want to study black body radiation I can study the radiation coming out of a spherical cavity.

Now when I take a spherical cavity I am going to define something called the energy density u in the cavity. And when I say energy density I mean u is the energy density of radiation in the cavity. Where does this radiation come from? So, what is seen is that if the body is heated to a temperature T, it fills the cavity with radiation. As you saw the in example of coals in the previous our lecture when I burn these coals there is a cavity from which you can see red light coming out. So, with temperature there is this radiation is generated right and this then gets filled with radiation, all the radiation. And if I make a small hole in the cavity radiation starts coming out of here and it is this radiation that we study.

So next question I ask is; what is the relationship between u and the immersive power of the cavity? Why I am doing that is; slowly we will build up that the immersive power which will depend on u, and finally what we are going come is study u that is as good as studying the immersive power if you are going to understand the radiation coming out the cavity and u would be related to oscillators that are emitting energy inside the cavity. So, we want to relate all these quantities. So, let us find this relationship.

So, for this what I will do is I will take this cavity and this hole has an area a, and consider how much radiation comes out. So, let me look at an angle theta for perpendicular to the area, look at an angle theta into solid angle d omega and see how much radiation do I collect. So, let me build it up slowly. So, I will make this picture again. This is the cavity and I am looking into the solid angle. If I look at a distance r and consider a small volume of length delta r then this volume of the shaded region delta v is nothing but d omega r square delta r. The radiation energy inside it is equal d omega r square delta r times u; that is the radiation density inside it. And this radiation is going all around because isotropic.

So, this radiation is going all around from this area into the solid angle all around. So, the probability that this radiation comes into the hole of area a is going to be equal to the solid angle made by a on points in volume delta v divided by 4 pi. Because 4 pi is the solid angle into which radiation all this is going. So, if I calculate the solid angle made by a on points and volume v and divided by 4 pi that is going to be the probability. So, let us do that.

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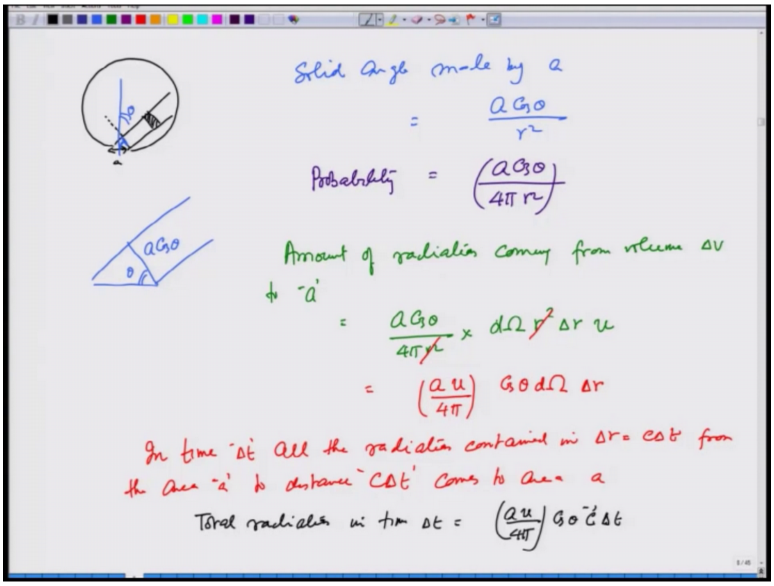

So here is this cavity, this was the shaded region and this is a. So, the perpendicular cross section of this a, because this angle is theta is this perpendicular cross section this is also theta out here, so if I make it bigger this is a and this is the perpendicular cross section this angle is theta. So, this is going to be a cosine of theta. So, the solid angle made by a is going to be a cosine of theta over r square. So, the probability that I was talking about that radiation comes into this area is going to a cosine theta divided by 4 pi r square, alright.

So, now we have the amount of radiation coming from volume delta v to area a is going to be probability a cosine theta divided by 4 pi r square times the radiation; that is contained there which is going out and amount of radiation contained there is nothing but d omega r square delta r u. I am going to cancel this r square. This comes out to be a over a u over 4 pi cosine theta d omega delta r, alright.

So now, in time delta T all the radiation contained in delta r equals c delta T from the area a to distance c delta T comes to area a. So, the total radiation coming out in time delta T is going to be a u over 4 pi cosine theta c delta T; where c is speed of light.

So, this is what I figured out.

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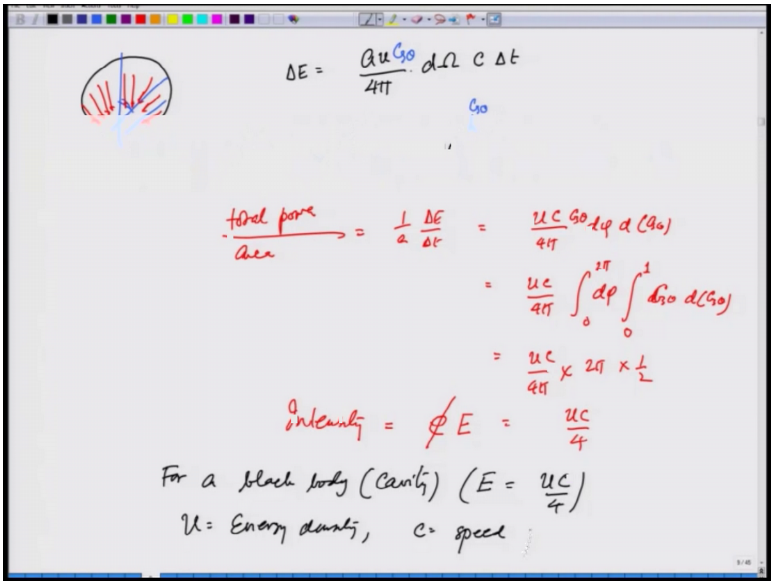

We figured out that if I have a cavity like this, this is area a then the radiation coming from angle theta from the omega is delta e equals a u over 4 pi times d omega c delta T power is going to be delta e over delta T by the unit area is going to be 1 over a is equal to u over 4 pi c d omega.

And if I want to calculate the power coming from all sides, so here, here, here, here, here, here, here, here, I got to integrate over d omega. So, total power per unit area is going to be 1 over a delta e delta T integrated over this whole thing u c over 4 pi; d omega is going to be d phi d cosine of theta; I had forgotten a cosine theta over here cosine theta here cosine theta here which is nothing but u c over 4 pi d phi integrates on 0 to 2 pi and cosine theta integrates from 0 to 1 cos theta d cos theta; which is nothing but u c over 4 pi times 2 pi times 1 by 2 which is u c by 4. Total power coming out per unit area is but the intensity, and this is also equal to the emissive power and I can write this as cavity e because this is a emissive power of a black body.

So, what we conclude is for a black body represented by a cavity emissive power over area per unit time is u c by 4; where u is the energy density and c is the speed of light.